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[PREPARED IN THE ORDNANCE COLLEGE.]

TEXT BOOK
OF
GUNNERY.

PART II.



LONDON:
PRINTED FOR HIS MAJESTY'S STATIONERY OFFICE.
BY HARRISON AND SONS, ST. MARTIN'S LANE,
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1911.

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Artillery, field and mountain

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ERRATA.

Page 15, line 4.— Δ^{2r-1} should read $\Delta^{2r-1}t_{s-(r-1)}t$.

Page 15, line 9 from bottom.—(3) should read (20).

Page 24, line 5.—0.7799 should read 0.07809.

Page 26, line 13 from bottom.— t_1 should read t_2 .

Page 28, line 4.—(14) should read (17).

Page 36, line 7. $\frac{180g}{\pi} \frac{y}{v} dt$ should read $-\frac{180}{\pi} \frac{y}{v} .dt$.

Page 47, line 9.—Dele “ $-B_a$.”

Page 48, line 16.—For $+B_a$ read $-B_a$.

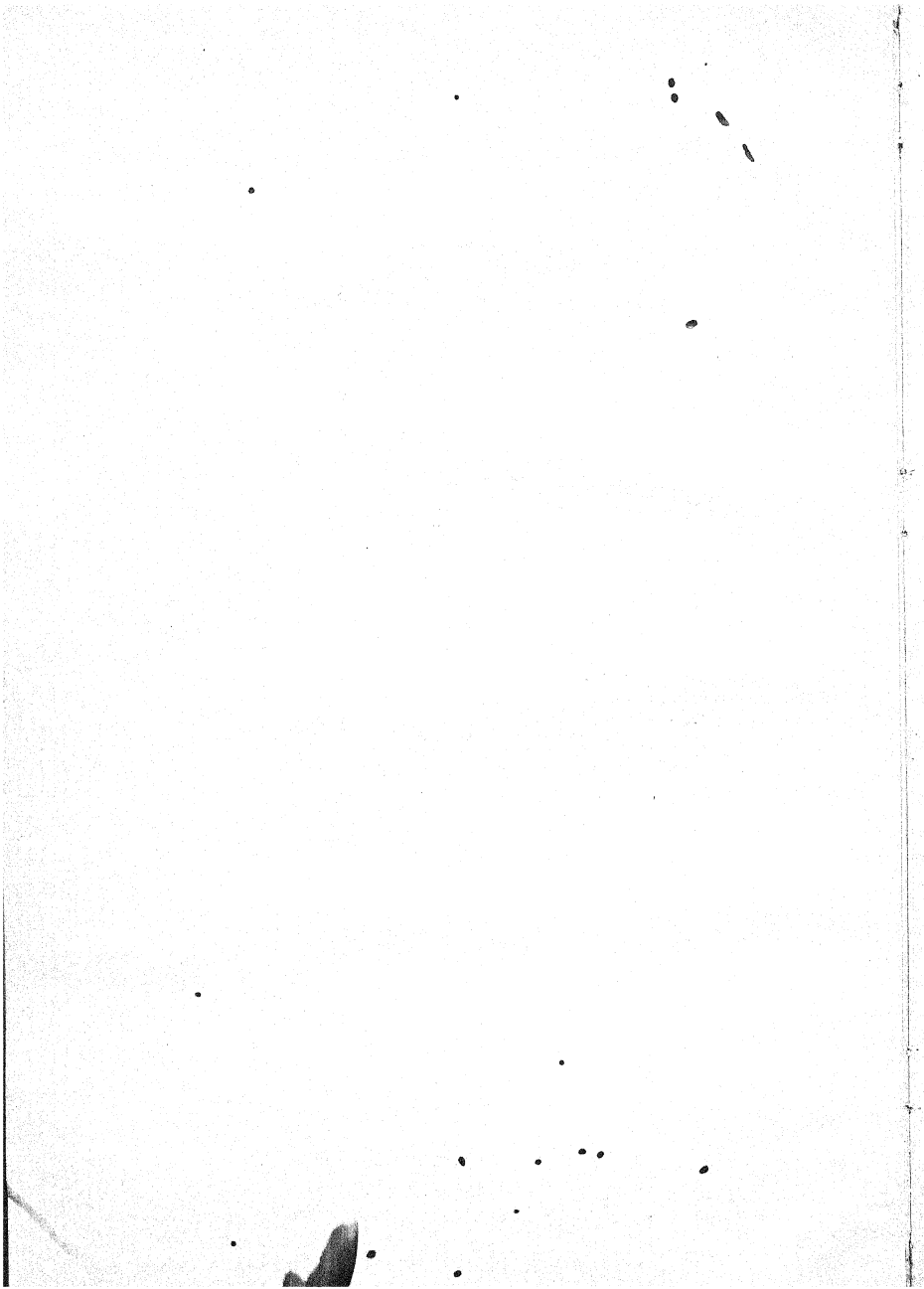
Page 66, in diagram.— y_1 is length BC and not BE.

Page 72, line 2 from bottom.—Read “U = ” instead of “V = .”

Page 76, line 9.—Dele “pseudo.”

Page 88, line 15 from bottom.—“Entering” should read “referring to.”

Page 94, line 21.—Read 0.7987 ϵ instead of 0.9024 ϵ .



TEXT BOOK OF GUNNERY.

CHAPTER I.

THE RESISTANCE OF THE AIR.

IN Chapter II., Part I., of the Text Book of Gunnery, the practical importance of a Ballistic Table has been illustrated by various examples, in which, by the use of the table, the solution was effected of a Ballistic Problem; it is necessary now to examine the theory upon which a Ballistic Table is based, and the experimental data upon which the calculation is founded.

The first requirement is the experimental determination of the *Resistance of the Air* to a projectile moving with a velocity within the limit of that found useful in artillery fire.

The most important series of experiments carried out in this country are those of the Rev. F. Bashforth, B.D., the first Professor of Mathematics to the Advanced Class of Artillery Officers.

These experiments were conducted in 1865-1870 and in 1878-1879, and the results are tabulated in the *Reports on the Experiments made with the Bashforth Chronograph, &c., 1865-1870*; and *Final Report on Experiments with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Elongated Projectiles, 1878-1880*.

The projectiles employed in these experiments were of various weights and sizes, and were fired from guns of 3, 5, 6, 7, and 9 inches calibre; the external shape was nearly uniform for all, consisting of a cylindrical body with a flat or slightly rounded base, and provided with an ogival-pointed head, struck with a radius of $1\frac{1}{2}$ diameters.

In 1902-1906 the Ordnance Committee (now the Ordnance Board), with Sir G. Greenhill as technical adviser, carried out experiments to determine the resistance of the air to the motion of elongated projectiles, struck with a radius of 2 diameters. The Ballistic Tables at the end of this book are founded upon the data obtained from these experiments, and these tables are used in the present text. Previous editions (containing the Bashforth ballistic tables) were written by Sir G. Greenhill, to whom the scientific development of gunnery owes so much.

As a first result of the experiments it was found that the resistance was proportional, at the same velocity, to the cross-sectional area or to the square of the diameter.

The resistance R can thus be split up into two factors, one of which is d^2 , where d denotes the diameter of the shot in inches; and the other is the resistance of the air at the same velocity to a similar 1-inch projectile; this is denoted by p , so that

$$R = d^2 p.$$

These values of p refer to a certain standard density of the air, of 534.22 grains per cubic foot, which is the density of dry air at sea-level, in the latitude of Greenwich, at a temperature of 62° F. and a barometric height of 30 inches.

Assuming that a projectile travels during its flight so that its axis coincides with the tangent of the trajectory, then the forces acting on the projectile are the force of gravity, vertically downwards, and considered as constant; and the variable resistance of the air which acts continually in a direction opposite to that of the motion of the projectile. Experiment

shows that this resistance is proportional to the cross-section area or square of the diameter of the projectile; hence, putting $p = f(v)$, where $f(v)$ is a function of the velocity of the shot,

$$R = d^2 p = d^2 f(v).$$

It is assumed further that the resistance is proportional to the density of the air; so that if the density changes to δ grains per cubic foot, we must put $R = \tau_0 d^2 p$, where

$$\tau_0 = \frac{\delta}{\Delta},$$

and

$$\Delta = 534 \cdot 22.$$

Therefore

$$\tau_0 = \frac{\delta}{534 \cdot 22},$$

and Table VIIA, calculated from the formula

$$\tau_0 = \frac{h}{30} \cdot \frac{460 + 62}{460 + F} = \frac{h}{30} \cdot \frac{1}{1 + \frac{F - 62}{522}} \dots \dots \dots (1),$$

derived from the laws of Boyle and Charles, gives the value of τ for different Fahrenheit temperatures F and barometric heights h inches; this applies to dry air, so that a further correction is required from the hygrometrical tables given by the readings of the wet- and dry-bulb thermometers, as damp air is perceptibly lighter than dry air at the same temperature and pressure; the air is supposed to be two-thirds saturated, so that a pressure two-thirds of the pressure in inches of mercury of the aqueous vapour at the temperature F is added. The effect of the vapour correction is to reduce the standard temperature from 62° to 60° F.

This factor τ_0 is called the *coefficient of tenacity at the ground level*, but if a projectile attain any considerable height above ground, then a correction in τ_0 must be made to allow for the decrease in density (and temperature) in the air above the ground; a good average, as shown in Chapter III, for the mean density of air of the whole trajectory is obtained by considering the average height h of the trajectory to be at two-thirds of the maximum height, hence the working formula for the coefficient of tenacity for a trajectory is

$$\tau_h = \frac{\tau_0}{f}, \text{ so that } f = \frac{\tau_0}{\tau_h},$$

where f is the *altitude factor*, which corrects for a mean height above ground of h feet, f is 1 at the ground level or near it, and is greater than 1 for higher levels. Hence

$$R = \frac{\tau_0}{f} d^2 p = \tau_h d^2 p \dots \dots \dots (2).$$

In all carefully conducted experiments the value of τ_0 should be calculated and allowed for from day to day.

For shooting under water, the coefficient of tenacity becomes 800, because the density of water is about 800 times that of ordinary air.

The extract from Glaisher's *Hygrometrical Tables*, Table VII at the end of the book, gives δ , the density in grains/ft.³, for the reading of the barometer in inches, and of the wet- and dry-bulb thermometer in degrees Fahrenheit of the meteorological record on the day of an experiment, and hence the value of

$$\tau_0 = \frac{\delta}{\Delta}.$$

In using the hygrometric table, look out the number corresponding to the readings of the thermometer, wet and dry bulb; this gives the grains per cubic foot at a barometric height of 29 inches; and then the table of proportional parts shows the addition to be made

for the extra height of the barometer above 29 inches. The standard barometric height in the table is taken very low, at 29 inches, so as to avoid negative proportional parts.

As examples, find the value of τ_0 when the meteorological record is

Barometer	29.95
Wet bulb	37° F.
Dry bulb	39° F.

$$\begin{cases} 538.9 \text{ for 29 inches,} \\ + 16.7 \text{ for 0.9 difference,} \\ + 0.9 \text{ for 0.05 difference,} \end{cases}$$

$$\text{Value of } \delta = 556.5.$$

Therefore

$$\tau_0 = \frac{\delta}{\Delta} = \frac{556.5}{534.22} = 1.042.$$

For

Barometer	30.25
Wet bulb	42° F.
Dry bulb	45° F.

$$\begin{cases} 532.3 \\ 18.3 \\ 3.7 \\ 0.9 \end{cases}$$

$$\delta = 555.2$$

$$\tau_0 = \frac{555.2}{534.22} = 1.039.$$

To find the value of the altitude factor f (see Note at end of Chapter).

1st Case.—If the decrease in density of the air at a height h feet is taken as due only to the decrease in the pressure of the air, the temperature being supposed uniform, then

$$f = e^{\frac{h}{k}} \text{ (see Note at end of Chapter) } \quad (3),$$

where k is the height of the homogeneous atmosphere, that is, the height of an atmosphere of uniform density which will give the barometric pressure, and it is equal to 27,800 feet on the average, but varying with the temperature from 26,000 to 30,000 feet.

For a moderate value of h (3) may be written

$$f = 1 + \frac{h}{k} \quad \text{and} \quad \frac{1}{f} = 1 - \frac{h}{k} \quad (4),$$

where $k = 27,800$.

2nd Case.—Experiments show that the temperature, as well as the pressure, decreases with the increase of altitude. Considering *dry air only*, suppose that the temperature at any height is that which the surface air would have if it expanded adiabatically (*i.e.*, without loss or gain of heat) from the surface, then the value of f is given by

$$f = \left(1 - \frac{\gamma - 1}{\gamma} \frac{h}{k}\right)^{\frac{\gamma}{\gamma - 1}} \quad (5),$$

where $\gamma = 1.4$ is the ratio of the specific heat of dry air at constant pressure to its specific heat at constant volume (see Note at end of Chapter), and with $\gamma = 1.4$,

$$f = \left(1 - \frac{h}{97,300}\right)^{-2.5} \quad (6).$$

Table VIIA gives the value of τ_0 , and the value of f found for values up to $h = 10,000$ feet, these are plotted in fig. 1, when the ground temperature is 60° F.

For moderate values of h (5) may be written

$$f = 1 + \frac{h}{\gamma k} = 1 + 0.000026h,$$

where h is in feet.

3rd Case.—In 1st case no account is taken of change of temperature when ascending in the atmosphere; in the 2nd case no account is taken of the humidity of the atmosphere and the correct value for γ is probably less than 1.4 ; a smaller value of γ in the 2nd case would bring closer together the values of f obtained by the 1st and 2nd cases.

A well-known and practical rule* is that there is:—

1 inch in fall of barometer per 1000 feet altitude,
 1° F. fall of temperature „ 300 „ „

Working upon this and employing Table VIIA, the value of $f = \frac{\tau_0}{\tau_h}$ is readily obtained and then plotted for various values of h feet.

On fig. 1 the curve of f for various values of h is seen as obtained from the three cases considered, that got from the 3rd case is seen to be a mean between the other two, it has therefore claims for being considered suitable, and, if employed, there is no need of a separate table for f , but it is got direct from Table VIIA as soon as the barometer and thermometer readings at the ground surface are known.

As an example, suppose a shot attain a maximum height of 900 feet and that the mean height of the trajectory is taken as 600 feet. To correct for this, when the readings at the earth's surface are

30''
 56° wet bulb }
 60° dry bulb }

neglecting the wet-bulb reading, $\tau_0 = 1$ from Table VIIA, and from the same table τ_h for

$$\left. \begin{array}{l} 30 - .6 = 29'' .4 \\ 60 - 2 = 58^\circ \end{array} \right\} \text{ is equal to } 0.9836.$$

So that

$$f = \frac{\tau_0}{\tau_h} = \frac{1}{0.9836} = 1.016.$$

The following tabulated values of f are now set out for each rise of 1000 feet.

* Captain R. K. Hezlet, R.A., finds that, from an examination of the British Association Report on the upper atmosphere (Winnipeg, 1909), a better value for the mean temperature gradient up to 5000 feet altitude would appear to be 2.5° F. per 1000 feet. Taking this rate of decrease of temperature and 1 inch fall of barometer per 1000 feet, it will be found that for barometer 30'', thermometer 60° F. on ground, up to 5000 feet,

$$\frac{1}{f} = \tau_h = 1 - 0.000027 h.$$

This gives slightly higher values than in the table below.

Altitude Correction f in Ballistic Coefficient.

$$C = f \frac{w}{K \sigma \tau_0 \rho^{1/2}}; \quad f = \frac{\tau_0}{\tau_h}$$

Value of f at 60° F. calculated from τ_0 and from Table VIIA, allowing:—

1 inch fall of pressure per 1000 feet of altitude,
1° F. fall of temperature per 300 feet of altitude.

h feet.	f .	Differences. Δ
0	1.000	
1,000	1.028	0.028
2,000	1.057	0.029
3,000	1.089	0.032
4,000	1.123	0.034
5,000	1.160	0.037
6,000	1.200	0.040
7,000	1.244	0.044
8,000	1.291	0.047
9,000	1.344	0.053
10,000	1.400	0.056

Working with Glaisher's table to find f for a height of 600 feet, and supposing the wet-bulb fall of temperature to be the same as that of the dry-bulb temperature, then for surface readings of

$$\left. \begin{array}{l} 30'' \\ 56'' \\ 60'' \end{array} \right\} \delta_0 = 515.9 + 17.8 = 533.7,$$

and these readings at 600 feet elevation become

$$\left. \begin{array}{l} 29''.4 \\ 54'' \\ 58'' \end{array} \right\} \text{for which } \delta = 518.1 + 7.1 = 525.2;$$

so that

$$f = \frac{533.7}{525.2} = 1.016.$$

The resistance of the air is reduced considerably in a modern projectile by giving it a greater length and a sharper point; and a factor κ , called the *coefficient of shape*, is brought in to allow for this change.

For an ogival head of 2 calibres $\kappa = 1$ in the present new ballistic tables (at end of book) and for other shaped heads

$$\kappa = \frac{2}{m} \sqrt{\frac{4m-1}{4}}, \text{ approximately,}$$

where m is the number of calibres in the radius of the ogival head.

For a 3-calibre ogive

$$\kappa = \frac{2}{3} \sqrt{\frac{11}{4}} = 0.835.$$

Bashforth's tables were constructed for a $1\frac{1}{2}$ -calibre ogive, whereas the present tables are for a 2-calibre ogive.

For a so-called flat-headed *proof* projectile $\kappa = 2$ on the average; this has rounded edges, and were it really flat, κ would be greater than 2.

For spherical shot, such as a shrapnel bullet, κ is not constant, and a separate ballistic table (Table IX) is constructed; but $\kappa = 1.7$, on the average.

Lastly, to allow for the superior centring of the projectile obtained with breech-loading guns, Bashforth introduced a factor σ , called the *coefficient of steadiness*.

This steadiness may vary during the flight of the projectile, as the shot is often unsteady for some distance after leaving the muzzle, and finally steadies down afterwards, sometimes becoming unsteady again in high-angle howitzer fire.

With the present new tables and with modern breech-loading guns σ may be taken as unity ($\sigma = 1$).

Collecting all the coefficients, κ , σ , τ , we now put the resistance of the air in pounds,

$$R = m^2 p. \quad (7),$$

where

$$n = \kappa \sigma \tau$$

is called the *coefficient of reduction*.

Denoting the weight of the shot by w lbs., and the retardation of the shot by r f/s per second,

$$\frac{R}{w} = \frac{r}{g},$$

$$r = R \frac{g}{w} = m^2 p \frac{g}{w} = \frac{m^2}{w} p g \quad (8);$$

put

$$\frac{w}{m^2} = C,$$

then C is called the *ballistic coefficient* of the shot, and from (8)

$$r = \frac{p g}{C},$$

that is

$$C r = p g \quad (9)$$

Until the time of Benjamin Robins, and of his invention of the Ballistic Pendulum (1740), the vaguest ideas prevailed as to the velocity of shot and the resistance of the air.

It was never realised that such an attenuated elastic medium could offer so enormous a resistance, in spite of Newton's caution (Ex Medii subtilitate resistentia projectilium celerime motorum non multum diminuitur. *Philosophiæ Naturalis Principia Mathematica*, lib. ii, prop. xxxiii, cor. 5), so that artillerists were in the habit of neglecting this resistance, and of employing Galileo's parabolic theory for unresisted motion; and thereby the velocity of the shot was considerably under-estimated.

Thus, for instance, the velocity V required with an elevation of 9° to attain a range of 3500 yards is, according to this parabolic theory (Chapter II, § 4, Part I).

$$V = \sqrt{(gX \operatorname{cosec} 2z)},$$

where $X = 10,500$, the range in feet, and $2z = 18^\circ$; hence

$$V = 1047 \text{ f/s.}$$

But it is found that the modern magazine rifle, with an initial velocity of 2000 f/s, can hardly attain a range of 3500 yards, whatever elevation is given; and the resistance of the air to the bullet at the outset is now estimated at about $1\frac{1}{2}$ lbs., or 40 times the weight of the bullet.

So also Robins found, in an experiment (*New Principles of Gunnery*, 1742, Chap. II, Prop. II) by firing at his ballistic pendulum at ranges of 25, 75, and 125 feet, that the mean velocities of impact were 1670, 1550, and 1425 f/s.

The musket employed was a 12-bore, so that the bullets weighed 12 to the pound; and the charge of powder was half the weight of the bullet.

Denoting by R the average resistance in pounds over the range of 100 feet during which the velocity fell from $V = 1670$ to $v = 1425$,

$$R = \frac{w(V^2 - v^2)}{2g \times 100} = 10 \text{ lbs., about,}$$

or 120 times the weight of the bullet; this may be taken as the resistance of the air to a spherical bullet of this description, $\frac{3}{4}$ of an inch in diameter, moving with the velocity of 1550 f/s, at the mean range of 75 feet.

Mr. A. Mallock, F.R.S., has repeated the Robins experiment with a ballistic pendulum similar to that described in Part I, using a 0.303-inch calibre, and attaining a maximum velocity of 4500 f/s with a light aluminium bullet weighing 23.3 grains or 300 to the lb. (*Proceedings, Royal Society*, Nov. 1904).

The loss of velocity Δv at velocity v in a range of 5 yards, or $\Delta s = 15$ feet, was about 700 f/s.

So that with $v = 4500$, $\Delta v = 700$, the retardation

$$r = \frac{\Delta v}{\Delta t} = v \frac{\Delta v}{\Delta s} = 4500 \frac{700}{15} = 210,000.$$

Also

$$\frac{r}{g} = \frac{R}{w} = 6500,$$

hence, at the velocity of 4500 f/s the retardation is found to be 210,000 f/s², and the resistance about 6500 times the weight of the bullet.

The conclusions of Robins naturally met with great opposition from the teachers of the ancient theory; thus, for instance, Professor Müller, in his *Treatise of Artillery Supplement*, 1768, p. 110, proves that "the velocity from a 42-pr. can never amount to 914.7 f/s, and consequently much less in a smaller calibre."

But the experimental results, obtained by the modern method of shooting through electric screens, confirm Robins' results; and, according to Bashforth, these results of Robins, obtained from experiments with musket balls, are more accurate than those obtained 50 years later in Hutton's experiments with cannon balls and a larger ballistic pendulum.

The practical details of the construction and use of modern electro-ballistic apparatus are given in Chapter IV, § 2, Part I.

An electro-ballistic experiment consists essentially in recording the instant \hat{t} of time,

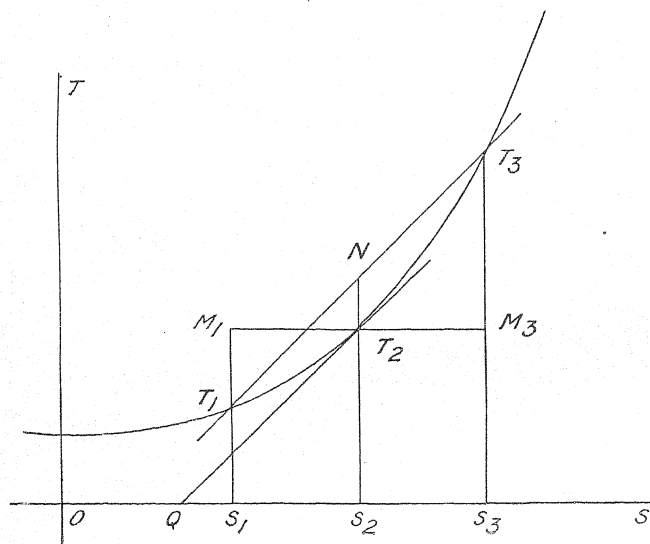
$$t_1, t_2, t_3 \dots \text{seconds,}$$

at which an electric screen at distance

$$s_1, s_2, s_3 \dots \text{feet,}$$

measured from a fixed point, is cut by the passage of a shot flying nearly horizontally.

Fig. 2.



Taking s and t as co-ordinates, a fair curve is drawn through the points

$$(s_1, t_1), (s_2, t_2), (s_3, t_3) \dots \quad (\text{fig. 2}).$$

And now the problem is to determine the most appropriate analytical expression for this curve, in the form

$$t = f(s);$$

and thence to derive

$$\frac{dt}{ds} \quad \text{and} \quad \frac{d^2t}{ds^2} \quad \dots \quad \dots$$

The distance s must be taken as the independent variable in screen records, and not the time t , so that v denoting the velocity, and r the retardation,

$$\frac{dt}{ds} = \frac{1}{v},$$

$$\frac{d^2t}{ds^2} = \frac{d}{ds} \left(\frac{dt}{ds} \right) = -\frac{1}{v^2} \frac{dv}{ds} = -\frac{1}{v^3} \frac{dv}{dt} = -\frac{1}{v^3} \frac{dv}{dt} = \frac{r}{v^3},$$

so that

$$r = -\frac{dv}{dt} = -v \frac{dv}{ds} = \frac{d^2t}{ds^2} v^3 \quad (10),$$

and if w lbs. be the weight of the shot, and R the resistance of the air in pounds,

$$wt^2 p = R = w \frac{r}{g} = w \frac{d^2t}{ds^2} \cdot \frac{v^3}{g} \quad (11).$$

Writing $\frac{v}{wt^2} = C$, the ballistic coefficient,

$$pg = Cr = C \frac{d^2t}{ds^2} v^3 = C \frac{d^2t}{ds^2} 10^9 \left(\frac{v}{1000} \right)^3 = K \left(\frac{v}{1000} \right)^3 \quad (12),$$

where

$$K = C \frac{d^2t}{ds^2} \times 10^9 \quad (13).$$

This value of K is as defined by Bashforth, and the definition still holds good. The advantage of the notation is that in the reduction of screen records the retardation r and the resistance R is divided into two factors, one of which is the cube of the velocity, the other factor is

$$\frac{d^2t}{ds^2}, \quad \text{and} \quad w \frac{d^2t}{ds^2} \cdot \frac{1}{g} \quad (14),$$

which are given immediately by the difference of the screen records, but as they are very small decimals, beginning with seven or eight zeroes, Bashforth found it more convenient to reckon velocity in thousands of f/s, and to multiply by 10^9 ; the formulas for r , R , are thus

$$pg = Cr = K \left(\frac{v}{1000} \right)^3 \quad (15),$$

$$R = w \frac{r}{g} = w \frac{K}{Cg} \left(\frac{v}{1000} \right)^3 \quad (16),$$

and

$$K = C \frac{d^2t}{ds^2} \times 10^9 \quad (17),$$

where v is in f/s, r in f/s².

The value of $\frac{d^2t}{ds^2}$ and hence that of K must be determined from the reduction of the screen records. The method employed by Bashforth is that of *Finite Differences*, and his method is the one now in use for the determination of v and K from the screen records.

METHOD OF FINITE DIFFERENCES.

In the notation of this subject, t_s or $f(s)$ denotes the value of t from a fixed point, say one of the screens, to any distance s , to a given screen, for instance; and then t_{s+l} or $f(s+l)$ will denote the value of t to any extra distance $s+l$, say to the next screen, l feet beyond; and generally, as required for the problem in hand, t_{s+nl} or $f(s+nl)$ will denote the time to the n th screen beyond the given screen, and t_{s-nl} or $f(s-nl)$ will denote the time to the n th screen in front of the given screen, the screens being spaced equally l feet apart.

Again, in the subject of Finite Differences, the symbol Δ is employed as a prefix (not as a factor) to denote the operation of differencing; and thus

$$t_{s+l} - t_s \text{ is denoted by } \Delta t_s;$$

or

$$f(s+l) - f(s) \text{ is denoted by } \Delta f(s);$$

while

$$\Delta t_{s+l} - \Delta t_s \text{ is denoted by } \Delta^2 t_s;$$

or

$$\Delta f(s+l) - \Delta f(s) \text{ is denoted by } \Delta^2 f(s);$$

and so on.

Then, since

$$\Delta t_s = t_{s+l} - t_s,$$

therefore

$$\begin{aligned} \Delta^2 t_s &= \Delta t_{s+l} - \Delta t_s \\ &= t_{s+2l} - t_{s+l} - t_{s+l} + t_s \\ &= t_{s+2l} - 2t_{s+l} + t_s, \end{aligned}$$

and similarly,

$$\begin{aligned} \Delta^3 t_s &= \Delta t_{s+2l} - 2\Delta t_{s+l} + \Delta t_s \\ &= t_{s+3l} - 3t_{s+2l} + 3t_{s+l} - t_s; \end{aligned}$$

and generally, by induction,

$$\Delta^n t_s = t_{s+nl} - nt_{s+(n-1)l} + \frac{n(n-1)}{2} t_{s+(n-2)l} - \dots \dots \dots (18),$$

analogous to the Binomial Theorem.

Again—

$$\begin{aligned} t_{s+l} &= t_s + \Delta t_s, \\ t_{s+2l} &= t_{s+l} + \Delta t_{s+l} \\ &= t_s + 2\Delta t_s + \Delta^2 t_s, \\ t_{s+3l} &= t_s + 3\Delta t_s + 3\Delta^2 t_s + \Delta^3 t_s, \end{aligned}$$

and generally, by induction,

$$t_{s+nl} = t_s + n\Delta t_s + \frac{n(n-1)}{2!} \Delta^2 t_s + \frac{n(n-1)(n-2)}{3!} \Delta^3 t_s + \dots \dots \dots (19),$$

again analogous to the Binomial Theorem.

But if t_{s+nl} or $f(s+nl)$ is expanded by Taylor's Theorem in ascending powers of nl , then

$$t_{s+nl} = f(s) + nl \frac{df(s)}{ds} + \frac{n^2 l^2}{2!} \frac{d^2 f(s)}{ds^2} + \frac{n^3 l^3}{3!} \frac{d^3 f(s)}{ds^3} + \frac{n^4 l^4}{4!} \frac{d^4 f(s)}{ds^4} + \dots \quad (20).$$

The general, $(r+1)$ th, term in the series (19) can be written

$$\begin{aligned} & \frac{n(n-1)\dots(n-r+1)}{r!} \Delta^r t_s \\ &= -(-1)^r \frac{n(n-1)\dots(n-r+1)}{r!} \left(1 - \frac{n}{2}\right) \dots \left(1 - \frac{n}{r-1}\right) \frac{\Delta^r t_s}{r} \\ &= -(-1)^r \left\{ n - n^2 \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r-1}\right) + \dots \right\} \frac{\Delta^r t_s}{r}. \end{aligned}$$

Collecting the coefficients of n , n^2 and n^3 in (19),

$$\begin{aligned} t_{s+nl} &= t_s + n \left\{ \Delta t_s - \frac{1}{2} \Delta^2 t_s + \frac{1}{3!} \Delta^3 t_s - \frac{1}{4!} \Delta^4 t_s + \dots + (-1)^r \frac{\Delta^r t_s}{r} + \dots \right\} \\ &+ n^2 \left\{ -\frac{\Delta^2 t_s}{2} - \frac{\Delta^3 t_s}{3} \left(1 + \frac{1}{2}\right) + \frac{\Delta^4 t_s}{4} \left(1 + \frac{1}{2} + \frac{1}{3}\right) - \frac{\Delta^5 t_s}{5} \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) + \dots \right\} \\ &+ (-1)^{r-1} \frac{\Delta^r t_s}{r} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r-1}\right) + \dots \left\{ \right. \\ &+ n^3 \left\{ -\frac{\Delta^3 t_s}{6} - \frac{\Delta^4 t_s}{4} + \dots \right\} + n^4 \left\{ -\frac{\Delta^4 t_s}{24} + \dots \right\} + \dots \quad (21); \end{aligned}$$

so that, equating the coefficients of n and n^2 in these two different expressions for t_{s+nl} given in (20) and (21),

$$l \frac{dt_s}{ds} = \Delta t_s - \frac{1}{2} \Delta^2 t_s + \frac{1}{3!} \Delta^3 t_s - \dots - (-1)^r \frac{\Delta^r t_s}{r} + \dots \quad (22),$$

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_s - \Delta^3 t_s + \frac{11}{2} \Delta^4 t_s - \dots + 2(-1)^r \frac{\Delta^r t_s}{r} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{r-1}\right) + \dots \quad (23).$$

With an unlimited number of screens, l feet apart, and their time records, the successive differences of the records can be found according to the following scheme.

It will be noticed that the series of numbers

$$t_s, \Delta t_s, \Delta^2 t_s, \Delta^3 t_s, \Delta^4 t_s, \dots,$$

run in a diagonal line slanting downwards, so that the preceding formulas (22) and (23) are suitable for employment at the initial screens of a series.

t_s	Δt_s	$\Delta^2 t_s$	$\Delta^3 t_s$	$\Delta^4 t_s$	$\Delta^5 t_s$	$\Delta^6 t_s$	$\Delta^7 t_s$	$\Delta^8 t_s$
$t_s - 7l$								
$t_s - 6l$	$\Delta t_s - 7l$	$\Delta^2 t_s - 7l$						
$t_s - 5l$	$\Delta t_s - 6l$		$\Delta^2 t_s - 7l$					
$t_s - 4l$	$\Delta t_s - 5l$	$\Delta^2 t_s - 6l$	$\Delta^3 t_s - 6l$	$\Delta^4 t_s - 7l$				
$t_s - 3l$	$\Delta t_s - 4l$	$\Delta^2 t_s - 5l$	$\Delta^3 t_s - 5l$	$\Delta^4 t_s - 6l$	$\Delta^5 t_s - 7l$			
$t_s - 2l$	$\Delta t_s - 3l$	$\Delta^2 t_s - 4l$	$\Delta^3 t_s - 4l$	$\Delta^4 t_s - 5l$	$\Delta^5 t_s - 6l$	$\Delta^6 t_s - 7l$	$\Delta^7 t_s - 7l$	
$t_s - l$	$\Delta t_s - 2l$	$\Delta^2 t_s - 3l$	$\Delta^3 t_s - 3l$	$\Delta^4 t_s - 4l$	$\Delta^5 t_s - 5l$	$\Delta^6 t_s - 6l$	$\Delta^7 t_s - 6l$	$\Delta^8 t_s - 7l$
t_s	$\Delta t_s - l$	$\Delta^2 t_s - 2l$	$\Delta^3 t_s - 2l$	$\Delta^4 t_s - 3l$	$\Delta^5 t_s - 4l$	$\Delta^6 t_s - 5l$	$\Delta^7 t_s - 5l$	$\Delta^8 t_s - 6l$
$t_s + l$	Δt_s	$\Delta^2 t_s - l$	$\Delta^3 t_s - l$	$\Delta^4 t_s - 2l$	$\Delta^5 t_s - 3l$	$\Delta^6 t_s - 4l$	$\Delta^7 t_s - 4l$	$\Delta^8 t_s - 5l$
$t_s + 2l$	$\Delta t_s + l$	$\Delta^2 t_s$	$\Delta^3 t_s$	$\Delta^4 t_s - l$	$\Delta^5 t_s - 2l$	$\Delta^6 t_s - 3l$	$\Delta^7 t_s - 3l$	$\Delta^8 t_s - 4l$
$t_s + 3l$	$\Delta t_s + 2l$	$\Delta^2 t_s + l$	$\Delta^3 t_s + l$	$\Delta^4 t_s$	$\Delta^5 t_s - l$	$\Delta^6 t_s - 2l$	$\Delta^7 t_s - 2l$	$\Delta^8 t_s - 3l$
$t_s + 4l$	$\Delta t_s + 3l$	$\Delta^2 t_s + 2l$	$\Delta^3 t_s + 2l$	$\Delta^4 t_s + l$	$\Delta^5 t_s$	$\Delta^6 t_s - l$	$\Delta^7 t_s - l$	$\Delta^8 t_s - 2l$
$t_s + 5l$	$\Delta t_s + 4l$	$\Delta^2 t_s + 3l$	$\Delta^3 t_s + 3l$	$\Delta^4 t_s + 2l$	$\Delta^5 t_s + l$	$\Delta^6 t_s$	$\Delta^7 t_s$	$\Delta^8 t_s - l$
$t_s + 6l$	$\Delta t_s + 5l$	$\Delta^2 t_s + 4l$	$\Delta^3 t_s + 4l$	$\Delta^4 t_s + 3l$				
$t_s + 7l$	$\Delta t_s + 6l$	$\Delta^2 t_s + 5l$						

At the final screens the numbers end off in a diagonal line sloping upwards, containing the typical terms

$$t_s, \Delta t_{s-l}, \Delta^2 t_{s-2l}, \Delta^3 t_{s-3l}, \Delta^4 t_{s-4l}, \dots$$

But

$$t_{s-l} = t_s - \Delta t_{s-l},$$

$$t_{s-2l} = t_{s-l} - \Delta t_{s-2l}$$

$$= t_s - \Delta t_{s-l} - \Delta(t_{s-l} - \Delta t_{s-2l})$$

$$= t_s - 2\Delta t_{s-l} + \Delta^2 t_{s-2l},$$

and so on; so that generally

$$t_{s-nl} = t_s - n\Delta t_{s-l} + \frac{n(n-1)}{2!} \Delta^2 t_{s-2l} - \dots$$

$$= t_s - n(\Delta t_{s-l} + \frac{1}{2}\Delta^2 t_{s-2l} + \frac{1}{3}\Delta^3 t_{s-3l} + \dots)$$

$$+ n^2(\frac{1}{2}\Delta^2 t_{s-2l} + \frac{1}{2}\Delta^3 t_{s-3l} + \frac{1}{2}\Delta^4 t_{s-4l} + \dots) \quad (24)$$

and therefore, as before,

$$l \frac{dt_s}{ds} = \Delta t_{s-l} + \frac{1}{2}\Delta^2 t_{s-2l} + \frac{1}{3}\Delta^3 t_{s-3l} + \dots \quad (25)$$

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-2l} + \Delta^3 t_{s-3l} + \frac{1}{1\frac{1}{2}} \Delta^4 t_{s-4l} + \dots \quad (26)$$

the formulas appropriate at the final screens of a series.

But at the middle screens the numbers which run horizontally are typified by

$$t_s, \frac{\Delta t_{s-l}}{\Delta t_s}, \Delta^2 t_{s-l}, \frac{\Delta^2 t_{s-2l}}{\Delta^2 t_{s-l}}, \Delta^4 t_{s-2l}, \dots$$

The formulas required are now

$$\begin{aligned}
 l \frac{dl_s}{ds} &= \frac{1}{2} (\Delta t_{s-l} + \Delta t_s) - \frac{1}{3!} \frac{1}{2} (\Delta^3 t_{s-2l} + \Delta^3 t_{s-l}) \\
 &+ \frac{1^2 \cdot 2^2}{5!} \frac{1}{2} (\Delta^5 t_{s-3l} + \Delta^5 t_{s-2l}) \dots \\
 &- (-1)^r \frac{1^2 \cdot 2^2 \cdot 3^2 \dots (r-1)^2}{(2r-1)!} \frac{1}{2} (\Delta^{2r-1} t_{s-rl} + \Delta^{2r-1}) \dots \dots \dots (27) \\
 &+ \dots \\
 l^2 \frac{d^2 l_s}{ds^2} &= \Delta^2 t_{s-l} - \frac{1}{3!} \frac{\Delta^4 t_{s-2l}}{2} + \frac{1^2 \cdot 2^2}{5!} \frac{\Delta^6 t_{s-3l}}{3} - \dots \\
 &- (-1)^r \frac{1^2 \cdot 2^2 \cdot 3^2 \dots (r-1)^2}{(2r-1)!} \frac{\Delta^{2r} t_{s-rl}}{r} + \dots \dots \dots (28),
 \end{aligned}$$

the first (27) involving odd differences, and the second (28) even differences only (De Morgan, *Differential and Integral Calculus*, p. 544).

This is proved, if equation (19) is replaced by an equivalent formula,

$$\begin{aligned}
 t_{s+nl} &= t_s + n \Delta t_s + \frac{n(n-1)}{2!} \Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3!} \Delta^3 t_{s-l} + \dots \\
 &+ \frac{(n+r-1) \dots (n-r)}{(2r)!} \Delta^{2r} t_{s-rl} + \frac{(n+r) \dots (n-r)}{(2r+1)!} \Delta^{2r+1} t_{s-(r+1)l} + \dots \dots \dots (29).
 \end{aligned}$$

Putting

$$\Delta t_s = \Delta t_{s-l} + \Delta^2 t_{s-l},$$

and, generally,

$$\Delta^{2n+1} t_{s-nl} = \Delta^{2n+1} t_{s-(n+1)l} + \Delta^{2n+2} t_{s-(n+1)l}$$

formula (29) is equivalent to

$$\begin{aligned}
 t_{s+nl} &= t_s + n \Delta t_{s-l} + \frac{n(n-1)}{2!} \Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3!} \Delta^3 t_{s-2l} + \dots \\
 &+ \frac{(n-r+1) \dots (n-r)}{(2r)!} \Delta^{2r} t_{s-rl} + \frac{(n-r) \dots (n-r)}{(2r+1)!} \Delta^{2r+1} t_{s-(r+1)l} + \dots \dots \dots (30).
 \end{aligned}$$

Taking the half sum of (29) and (30),

$$\begin{aligned}
 t_{s+nl} &= t_s + n \frac{1}{2} (\Delta t_{s-l} + \Delta t_s) + \frac{n^2}{2!} \Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3!} \frac{1}{2} (\Delta^3 t_{s-2l} + \Delta^3 t_{s-l}) + \dots \\
 &+ \frac{n(n-r+1)(n-r+2) \dots (n-r-1)}{(2r)!} \Delta^{2r} t_{s-rl} \\
 &+ \frac{(n-r) \dots (n-r)}{(2r+1)!} \frac{1}{2} (\Delta^{2r+1} t_{s-(r+1)l} + \Delta^{2r+1} t_{s-rl}) + \dots \dots \dots (31),
 \end{aligned}$$

and equating the coefficients of n and n^2 in this equation and in (3) will lead to the two required formulas (27) and (28), already stated.

Having thus determined

$$l \frac{dl_s}{ds} \quad \text{and} \quad l^2 \frac{d^2 l_s}{ds^2}$$

by the successive differences of the screen records, the velocity v is the reciprocal of $\frac{dl}{ds}$ while the retardation r is given by

$$r = -\frac{dv}{dt} = -v \frac{dv}{ds} = \frac{d^2 l}{ds^2} v^3, \text{ as on page 13.}$$

Numerical illustrations taken from the *Reports on Experiments made with the Bashforth Chronograph, to determine the Resistance of the Air to the Motion of Projectiles, 1865-1870* and

1878-1880, or from *A Revised Account of the Experiments made with the Bashforth Chronograph* (Cambridge, 1890), will make the preceding theory more clear.

The following examples are given by Sir G. Greenhill, F.R.S., to whom is due the explanation of the method of Finite Differences given in this chapter and in previous editions of the present work, also a great many examples in the 1902 edition of the *Text Book of Gunnery*.

Round 463, Report VIII, Table I, has been selected as a specimen for showing the nature of reduction employed, arranged in the scheme annexed, the chronograph records being taken to five decimals from the *Bashforth Chronograph*, p. 41, 1890; the date must be added for the meteorological record in the calculation of the tenuity factor T , by means of the extract from the Hygrometrical Table.

To obtain velocity to one decimal, five-figure logarithms are required; but the Slide Rule will give the accuracy permissible in the calculation of K , r , R , . . . which depend on the two-figure observed values of $\Delta^2 t$.

Round 1 is taken also for its historical interest, October 7th, 1867, in which a solid shot, weighing 12 lbs., was fired from a 3-inch gun, with a charge of 2 lb. of powder; the instants of time at which the 10 screens, 150 feet apart, were cut by the shot, are recorded to 4 decimals in the following table, where the time differences are also given.

Round 1, Report III, Table I, p. 27, Table X, p. 54.

October 7th, 1867.	Barometer.	Thermometer.	
		Dry.	Wet.
10 a.m.	29.66	53	53
3 p.m.	29.62	52	48

and Bashforth takes $\tau = 1.002$, $w = 12$, $d = 2.92$,

$$C = \frac{w}{\tau d^2}, \quad \log C = 0.1476, \quad C = 1.405.$$

Screen	t	Δt	$\Delta^2 t$
1	0.0000		
2	1247	0.1247	
3	2513	1266	0.0019
4	3800	1287	21
5	5109	1309	22
6	6439	1330	21
7	7789	1350	20
8	0.9162	1373	23
9	1.0559	1397	24
10	1.1979	0.1420	0.0023

It will be noticed that the second difference is very nearly constant, and on the average equal to 0.0021, or 0.0022, and that the higher differences are illusory; this is because the chronograph does not record smaller intervals of time than the ten-thousandth of a second, recorded in the fourth place of decimals; and this figure is therefore subject to a correction, which may reach to nearly ± 5 in the fifth place.

The calculation of v , depending chiefly on Δt , will be given within the unit place by the four decimals; but in calculating K the irregularity in Δ^2 must be smoothed down by a smoothing figure in the fifth decimal place, to obtain a result in accordance with Bashforth.

No fixed method can be laid down for this smoothing operation, but a convenient way is to plot $\Delta^2 t$ to a large scale, say one inch to one ten-thousandth of a second, join up the points by a broken line, and then draw a straight line by eye to run as evenly as possible between the points; the corrected values of Δ^2 being measured to this line, then Δ^2 will be constant; and Δt is derived by the successive additions of Δ^2 and Δ^2 .

This operation may show an over-correction of t at some screen; but this can be cancelled by sharing it between the screen intervals by a suitable correction in Δt , left untouched before, thus making a constant correction throughout of Δt .

On this method, with Round 1, we shall adopt the corrections of Δ^2 as follows:—

Screen.	t .	Δt .	Δ^2 .	Correction adopted in Δ^2 .	Δ^2 .
1	0·00000				
2	12470	0·12470			
3	25151	12681	0·00211	+ 0·00021	
4	38044	12893	212	+ 2	
5	51150	13106	213	— 7	
6	64470	13320	214	+ 4	
7	78005	13535	215	+ 15	
8	0·91756	13751	216	— 14	
9	1·05724	13968	217	— 23	
10	1·19910	0·14186	0·00218	— 0·00012	0·00001

This makes $t_{10} = 1·19910$, or 0·00120 above the measured value, and this difference can be shared between 9 intervals by a correction — 0·00013 in Δt ; and now we find the corrected values of t for Round 1 given in the *Chronograph*, p. 33, as shown here.

ROUND 1.

Number of screen.	t .	Δt .	$\Delta^2 t$.	$\Delta^2 t$.
1	0·00000			
2	12457	0·12457		
3	25125	12668	0·00211	
4	38005	12880	00212	
5	51098	13093	00213	
6	64405	13307	00214	
7	77927	13522	00215	
8	0·91665	13738	00216	
9	1·05620	13955	00217	
10	1·19793	0·14173	0·00218	0·00001

The calculation of v and K can proceed now as in Round 463.

Round 463 is now set out completely; \bar{K} and \bar{p} represent the mean value of K and of p obtained from the results of a large number of rounds.

Since

$$pg = Cr = K \left(\frac{v^6}{1000} \right)^3,$$

$$\frac{p}{\bar{p}} = \frac{r}{\bar{r}} = \frac{K}{\bar{K}}.$$

for a definite velocity $= v$, and

$$\frac{p}{r} = \frac{C}{g};$$

therefore

$$\frac{p}{\bar{p}} = \frac{r}{\bar{r}} = \frac{K}{\bar{K}} = \frac{\bar{C}}{C} = \frac{\sigma}{\bar{\sigma}},$$

since

$$C = \frac{w}{\kappa \sigma \tau d^2},$$

and here σ is the uncertain factor, hence since $\sigma > \bar{\sigma}$ in Round 463, it shows that the coefficient of steadiness σ is above the normal.

ROUND 463, MARCH 7TH, 1879. REPORT VIII, TABLE I.

Thermometer { Wet, 37° F. } Barometer, 29.95 { 538.9 }
 { Dry, 39° F. } { 16.7 }
 { 0.9 }
 $\delta = 556.5$.

Barom.	z	Δz	$\Delta^2 z$	$\Delta^3 z$	$z \frac{dz}{ds}$	v	$p \frac{d^2 z}{ds^2}$	$K = C \frac{d^2 z}{ds^2} \cdot 10^9$	$r = \frac{d^2 z}{ds^2} \times e^3$	$\frac{R}{w} = \frac{r}{g}$	$R = w \frac{r}{g}$	$p = \frac{R}{\tau d^2}$	\bar{p}	$\sigma = \frac{p}{\bar{p}} = \frac{K}{\bar{K}}$
1	0.00000	0.07724	0.00091		0.07679	1953.4	0.00060	74.6	298.2	9.36	648.5	17.28	16.19	1.068
2	.07724	7815		0.00001	7770	1930.6	90	74.6	287.8	8.94	625.5	16.06	15.81	1.064
3	.15539	7907	92	2	7860	1908.4	94	77.9	280.4	9.03	632.3	16.85	15.47	1.068
4	.23446	8001	94	0	7954	1885.8	94	77.9	280.3	8.71	610.0	16.24	15.17	1.070
5	.31447	8095	94	0	8048	1863.8	94	77.9	270.5	8.40	588.0	15.67	14.88	1.053
6	.39542	8189	94	0	8142	1842.3	54	77.9	261.3	8.11	568.0	15.13	14.59	1.037
7	.47731	8283	94	1	8236	1821.3	94	77.9	252.4	7.85	549.5	14.63	14.29	1.024
8	.56014	8378	95	1	8330	1800.6	95	78.8	246.4	7.66	536.0	14.27	14.01	1.018
9	.64392	8474	96	1	8426	1780.2	96	79.7	240.7	7.47	522.9	13.94	13.74	1.014
10	.72866	8571	97	0.00001	8522	1760.1	97	80.4	235.8	7.44	520.5	13.86	13.47	1.029
11	.81437	8668	0.00098		8620	1740.2	98	81.3	229.4	7.14	499.5	13.60	13.19	1.029
12	0.90106				0.08718	1720.5	0.00089	82.1	224.0	6.96	487.3	12.96	12.91	1.002

In the Bashforth system of measurement two parallel lines are traced on a cylindrical drum, which is spun round a vertical axis with a peripheral speed of about 10 to 12 inches a second. On one line an astronomical clock marks the seconds, and on the other line the shot registers its passage through each screen as it is broken, each record being made by an electric signal actuating a stylus.

The clock mark gives the time scale of the movement of the cylinder, and the problem is to determine the intermediate instant of time corresponding to the mark caused by the fracture of a screen by the passage of the shot.

For Round 479 the distance measured on the paper was as follows, on a linear scale of about 3 units to 1 inch, the record being measured carefully under a microscope :—

ROUND 479.

Seconds.	Seconds— linear measurement.	Screens.	Screens— linear measurement.
1	16·576		
2	49·130	1	73·856
		2	76·000
		3	78·180
		4	80·390
3	81·410	5	82·640
		6	84·910
		7	not observed
		8	89·210
		9	91·994
		10	94·376
		11	96·814
		12	99·314
4	113·454		
5	145·350		
6	176·994		

The screen records extend from 73 to 100, lying between the 2-second and 4-second record, and it is sufficient to form a complete record in this interval of time to every tenth of a second.

First, the half-second record is found by interpolation, just as well as if the clock had given a record every half-second.

The record gives—

t	s	Δs	$\Delta^2 s$	$\Delta^3 s$
1	16·576			
2	49·130	32·554		
3	81·410	32·280	-0·274	
4	113·454	32·044	-0·236	0·038
5	145·350	31·896	-0·148	0·088
6	176·994	31·644	-0·252	

in which the negative value of Δ^2 indicates the retardation of the drum.

Putting $n = \frac{1}{2}$ in formula (19)

$$s_{t+n} = s_t + n\Delta s + \frac{n(n-1)}{2!}\Delta^2 + \frac{n(n-1)(n-2)}{3!}\Delta^3 + \dots,$$

$$s_{2+\frac{1}{2}} = s_2 + \frac{1}{2}\Delta s - \frac{1}{8}\Delta^2 + \frac{1}{16}\Delta^3,$$

$$s_{3+\frac{1}{2}} = s_3 + \frac{1}{2}\Delta s - \frac{1}{8}\Delta^2 + \frac{1}{16}\Delta^3.$$

	$s_{2.5}$	$s_{3.5}$
s	49.1300	81.4100
$\frac{1}{2}\Delta$	16.1400	16.0220
$-\frac{1}{8}\Delta^2$	0.0295	0.0185
$\frac{1}{16}\Delta^3$	0.0055	0.0055
	65.3050	97.4560

The time-table now reads :-

t	s	Δs	$\Delta^2 s$	$\Delta^3 s$
2.0	49.130			
2.5	65.305	+16.175		
3.0	81.410	+16.105	-0.070	
3.5	97.456	+16.046	-0.059	0.011
4.0	113.454	+15.998	-0.048	0.011

A further application of the interpolation formula (19), with $n = \pm \frac{1}{5}, \pm \frac{2}{5}, \dots$, will lead to the clock record for every tenth of a second, between 2.5 and 3.6, between which the screen records are found.

Thus for $s_{2.6}$ the expression is

$$s_{2.5+\frac{1}{5}} = s_{2.5} + \frac{1}{5}\Delta + \frac{\frac{1}{5}(\frac{1}{5}-1)}{2!}\Delta^2 + \frac{\frac{1}{5}(\frac{1}{5}-1)(\frac{1}{5}-2)}{3!}\Delta^3,$$

$$s_{2.6} = s_{2.5} + 0.2\Delta - 0.08\Delta^2 + 0.0048\Delta^3;$$

whilst

$$s_{2.7} = s_{2.5} + \frac{2}{5}\Delta + \frac{\frac{2}{5}(\frac{2}{5}-1)}{2!}\Delta^2 + \frac{\frac{2}{5}(\frac{2}{5}-1)(\frac{2}{5}-2)}{3!}\Delta^3$$

$$= s_{2.5} + 0.4\Delta - 0.12\Delta^2 + 0.0064\Delta^3,$$

and so on, so that

	$s_{2.6}$	$s_{2.7}$	$s_{2.8}$	s
$s_{2.5}$	65.30500	65.30500		
$n\Delta$	3.22100	6.44200		
$\frac{n(n-1)}{2!}\Delta^2$	+0.00472	0.00708		
$\frac{n(n-1)(n-2)}{3!}\Delta^3$	+0.00058	0.00070		

M-3071

To find the corresponding record for $t = 3.5, 3.6, 3.7 \dots 3.9$, make use of formula (24). For example, to find $s_{3.6}$:—

$$3.6 = 4.0 - 0.5 \left(\frac{1}{5}\right), \text{ where } 0.5 = l, \frac{1}{5} = n,$$

then

$$s_{4-nl} = s_{3.6} = s_4 - n\Delta s_3 + \frac{n(n-1)}{2!} \Delta^2 s_2 - \frac{n(n-1)(n-2)}{3!} \Delta^3 s_1,$$

$$s_{3.6} = 113.454 - \frac{1}{5} \text{ of } 15.998 + \frac{1}{5 \cdot 0} \text{ of } 0.048 - \frac{1}{1 \cdot 2 \cdot 5} \text{ of } 0.011$$

$$= 113.454 - 12.7984 + 0.00384 - 0.0004$$

$$= 100.659,$$

and then the table reads :—

Time.	Record.	Δ^1	Δ^2	Time.	Record.	Δ^1	Δ^2
2.5	65.305			3.1	84.624		
		+ 3.226				+ 3.211	- 0.003
2.6	68.531		- 0.002	3.2	87.835		- 0.002
		+ 3.224				+ 3.209	- 0.002
2.7	71.755		- 0.003	3.3	91.044		- 0.002
		+ 3.221				+ 3.207	- 0.002
2.8	74.976		- 0.003	3.4	94.251		- 0.002
		+ 3.218				+ 3.205	- 0.002
2.9	78.194		- 0.002	3.5	97.456		- 0.002
		+ 3.216				+ 3.203	
3.0	81.410		- 0.002	3.6	100.659		
		+ 3.214					

Here Δ^2 is so small that its influence becomes insensible, and the interpolation formula reduces to the rule of proportional parts, equivalent to neglecting the retardation of the drum in so short an interval of time as one-tenth of a second.

The smoothed record of Round 479 is—

Screen.	Smoothed Record.	Δ^1	Δ^2
1	73.856		
2	76.000	2.144	
3	78.178	2.178	
4	80.390	2.212	
5	82.636	2.246	
6	84.916	2.280	
7	87.230	2.314	
8	89.578	2.348	
9	91.960	2.382	
10	94.376	2.416	
11	96.826	2.450	
12	99.310	2.484	

The screen record in length of travel of the drum must now be converted into decimals of a second, using proportional parts for the interval of one-tenth of a second.

Thus the record 73·856 of the first screen lies in the interval 2·7 to 2·8 seconds, and closer to 74·976, the 2·8-second record, than to 71·755, the 2·7-second record; using proportional parts,

$$\begin{array}{r} 73\cdot856 \\ 74\cdot976 \\ \hline - 1\cdot120 \\ 3\cdot221 \end{array} = 0\cdot3477$$

$$\begin{array}{r} 2\cdot800 \\ - 0\cdot03477 \\ \hline t_1 = 2\cdot76523 \end{array}$$

and so for t_2, t_3, \dots

The resulting table of time is given by Captain J. H. Hardcastle, late R.A. :—

Screen.	t'' Smoothed records trans- formed to seconds by scale.	These are the figures used in 1879 reduction $t'' - t_1$	Figures published in 1879. t''	Figures published in 1890.	Δ^1	Δ^2
1	2·76523	0·00000	0·0000	0·00000	0·06659	
2	83182	0·06659	0·0666	0·06659	0·06768	0·00109
3	89950	0·13427	0·1343	0·13427	0·06878	0·00110
4	96828	0·20305	0·2031	0·20305	0·06987	0·00109
5	3·03815	0·27292	0·2729	0·27292	0·07096	0·00109
6	10909	0·34386	0·3439	0·34388	0·07205	0·00109
7	18116	0·41593	0·4159	0·41593	0·07314	0·00109
8	25432	0·48909	0·4891	0·48907	0·07424	0·00110
9	32856	0·56333	0·5633	0·56331	0·07534	0·00110
10	40390	0·63867	0·6387	0·63865	0·07644	0·00110
11	48084	0·71511	0·7151	0·71509	0·07754	0·00110
12	55788	0·79265	0·7927	0·79263		

Second differences slightly
irregular = 0·00107 or 113, 10
or 12. This is due to the trans-
formation of space into time.

These second differences
of the five-figure records
being just smoothed as
shown account for the
difference between the
1879 and the 1890 reports.

The figures published in 1890 and 1879 are slightly discrepant, depending upon different estimates of the fifth decimal in the screen times or on one hundred-thousandth of a second.

See a paper published by Captain J. H. Hardcastle, late R.A., concerning Round 479 in the *Proceedings of the R.A. Institution*, vol. XXX, 1903.

Taking the figures published in 1890 for Round 479, with $d = 6$, $w = 50$, $\tau = 1\cdot014$, $kr = 1$, and screen 150 feet apart,

$$C = \frac{w}{\kappa \tau d^2} = 1\cdot37,$$

$$l = 150', \quad \Delta t_s = 0.$$

For a middle screen, say the 6th, from (27) and (28)

$$t \frac{dt}{ds} = \frac{1}{2} (\Delta t_{s-1} + \Delta t_s) = \frac{1}{2} (\Delta t_5 + \Delta t_6),$$

i.e.,

$$\frac{l}{v} = \frac{1}{2} (0.07096 + 0.07205) = 0.0715$$

and

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-l} = \Delta^2 t_s = 0.00109.$$

For a final screen, say the 12th, from (25)

$$l \frac{dt_s}{ds} = \Delta t_{s-l} + \frac{1}{2} \Delta^2 t_{s-2l}$$

i.e.,

$$\frac{l}{v} = 0.07754 + \frac{0.00110}{2} = 0.7799;$$

from (26)

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-2l} = 0.00110,$$

and for an initial screen, say the 1st, from (22)

$$l \frac{dt_s}{ds} = \Delta t_s - \frac{1}{2} \Delta^2 t_s + \frac{1}{6} \Delta^3 t_s - \dots,$$

$$\frac{l}{v} = 0.06659 - \frac{0.00109}{2} = 0.06605;$$

from (23),

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_s - \Delta^3 t_s + \dots = 0.00109;$$

the results from Round 479 for each screen are tabulated thus:—

Number of screen . . .	1	2-5	6	7-11	12
$\frac{l}{v}$	0.06605		0.0715		0.7799
$\log \frac{l}{v}$	2.8199				
$\log l (= \log 150)$	2.1761				
$\log v$	3.3562				
v f/s	2271		2098		1923
$l^2 \frac{d^2 t}{ds^2}$	0.00109		0.00109		0.00110
$\log \frac{d^2 t}{ds^2} 10^9$	1.6852				
$\log C$	0.1367				
$\log K = \log \left(C \frac{d^2 t}{ds^2} 10^9 \right)$	1.8219				
K	66.36		66.36		67
$p = \frac{K}{q} \left(\frac{v}{1000} \right)^3$	24.14		19.02		14.65
$pg = Cr = K \left(\frac{v}{1000} \right)^3$	776.9		612.7		471.7
$R = n d^2 p$	881.2		695		535

The method of finite differences is very useful for detecting any irregularity in the records or any error in transcribing them; also it often enables a general formula to be deduced for connecting variables in an experiment.

Thus, to find the formula connecting the figures 4, 11, 22, 37, 56, 79, 106 :—

n	t	Δt	$\Delta^2 t$	$\Delta^3 t$
0	4 ($=t_0$)	7 ($=\Delta t_0$)		
1	11 ($=t_1$)	11 ($=\Delta t_1$)	4 ($=\Delta^2 t_0$)	
2	22 ($=t_2$)	15	4 ($=\Delta^2 t_1$)	
3	37	19	4	0
4	56	23	4	
5	79	27	4	
6	106			

From (19),

$$t_{s+nl} = t_s + n \Delta t_s + \frac{n(n-1)}{2!} \Delta^2 t_s + \dots;$$

here $s = 0$, and

$$t_{nl} = t_n = t = t_0 + n \Delta t_0 + \frac{n(n-1)}{2!} \Delta^2 t_0 + \dots$$

$$= 4 + n \cdot 7 + \frac{n(n-1)}{2!} 4 + 0,$$

giving

$$t = 4 + 5n + 2n^2.$$

This result can be obtained direct from (21), thus

$$t_n = t_0 + n \left(\Delta t_0 - \frac{1}{2} \Delta^2 t_0 + \frac{1}{3} \Delta^3 t_0 \right) + n^2 \left(\frac{\Delta^2}{2} - \frac{\Delta^3}{2} + \dots \right) + n^3 \left(\frac{\Delta^3}{6} - \frac{\Delta^4}{4} \right)$$

$$= 4 + n \left(7 - \frac{1}{2} 4 + 0 \right) + n^2 \left(\frac{4}{2} - 0 \right),$$

i.e.,

$$t = 4 + 5n + 2n^2.$$

The second differences were constant, and the equation connecting n and t is a parabola.

To find a formula giving the pressure p in tons/in.² in a closed vessel for any density of loading δ from the results of the following experiment :—

δ	p
0.05	3.40
0.10	6.80
0.15	10.47
0.20	14.68
0.25	19.70

Writing n for 20δ and employing formula (21), replacing t by p ,

$$p_n = p_0 + n \left(\Delta t_0 - \frac{1}{2} \Delta^2 t_0 + \frac{1}{3} \Delta^3 t_0 + \dots \right) + n^2 \left(\frac{\Delta^2 t_0}{2} - \frac{\Delta^3 t_0}{2} + \Delta^4 t_0 \frac{11}{24} \right)$$

$$+ n^3 \left(\frac{\Delta^3 t_0}{6} - \frac{\Delta^4 t_0}{4} + \dots \right) + n^4 \left(-\frac{\Delta^4 t_0}{24} + \dots \right).$$

n	δ	p	Δp	$\Delta^2 p$	$\Delta^3 p$	$\Delta^4 p$
0	0	0 ($=p_0$)				
1	0.05	3.40 ($=p_1$)	3.40 ($=\Delta p_0$)	0 ($=\Delta^2 p_0$)		
2	0.10	6.80	3.40 ($=\Delta p_1$)	0.27 ($=\Delta^2 p_1$)	0.27 ($=\Delta^3 p_0$)	
3	0.15	10.47	3.67	0.54	0.27	0
4	0.20	14.68	4.21	0.81	0.27	
5	0.25	19.70	5.02			

$$p_n \text{ or } p = 0 + n(3.40 + 0.09)n + n^2(-0.135) + n^3\left(\frac{0.27}{6}\right) + 0$$

$$= 0.045n^3 - 0.135n^2 + 3.49n.$$

Replacing 208 for n gives

$$p = 360\delta^3 - 54\delta^2 + 69.8\delta,$$

which gives the formula for finding the pressure in a closed vessel for any density of loading δ .

If the screens, instead of being equidistant, were placed at distances

$$s_1, s_2, s_3, \dots, s_n,$$

from a fixed origin, and if

$$t_1, t_2, t_3, \dots, t_n$$

denoted the corresponding time records, then, according to Lagrange's Interpolation Formula, the simplest algebraical expression for t may be written

$$\begin{aligned}
 t = & \frac{(s-s_2)(s-s_3)\dots(s-s_n)}{(s_1-s_2)(s_1-s_3)\dots(s_1-s_n)} t_1 \\
 & + \frac{(s-s_1)(s-s_3)\dots(s-s_n)}{(s_2-s_1)(s_2-s_3)\dots(s_2-s_n)} t_2 \\
 & + \dots \\
 & + \frac{(s-s_1)(s-s_2)\dots(s-s_n)}{(s_r-s_1)(s_r-s_2)\dots(s_r-s_n)} t_r \\
 & + \dots \\
 & + \frac{(s-s_1)(s-s_2)\dots(s-s_{n-1})}{(s_n-s_1)(s_n-s_2)\dots(s_n-s_{n-1})} t_n
 \end{aligned}$$

a formula which agrees in giving

$$t = t_1, \text{ when } s = s_1;$$

$$t = t_2, \text{ when } s = s_2;$$

$$\dots\dots\dots$$

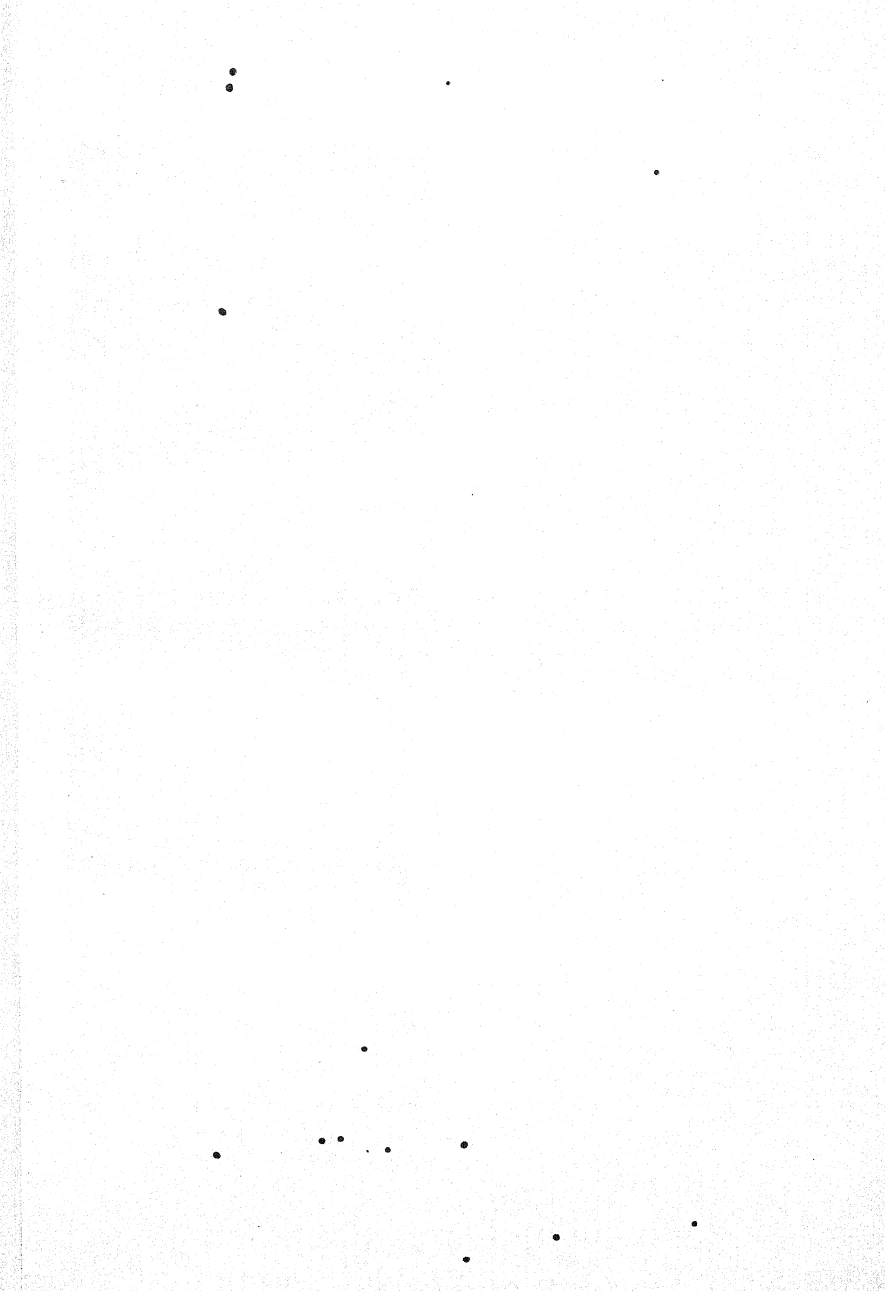
$$t = t_r, \text{ when } s = s_r;$$

$$\dots\dots\dots$$

$$t = t_n, \text{ when } s = s_n;$$

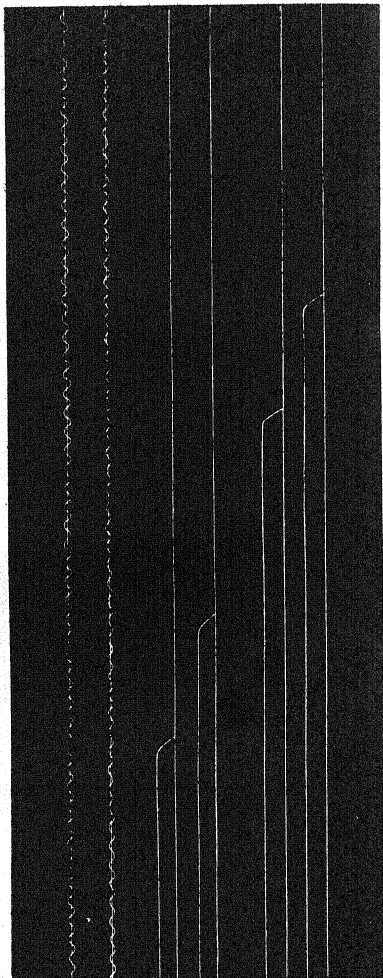
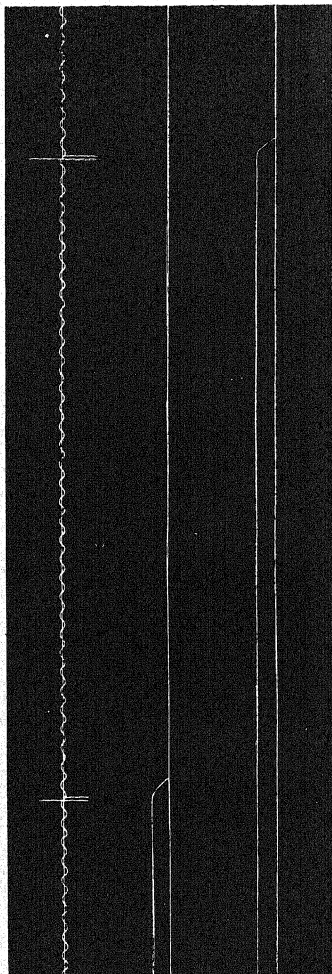
the asterisk * showing the position of the omitted vanishing factors.

Now to find $\frac{dt}{ds}$ and $\frac{d^2t}{ds^2}$ at the r th screen, and thence v and K , we make $s_n = 0$ by replacing any s_n by $s_m - s_m$, and then pick out the coefficient of s and $\frac{1}{2}s^2$ in t .



JERVIS SMITH CHRONOGRAPH TRACE.

Plate I.



130.7 V

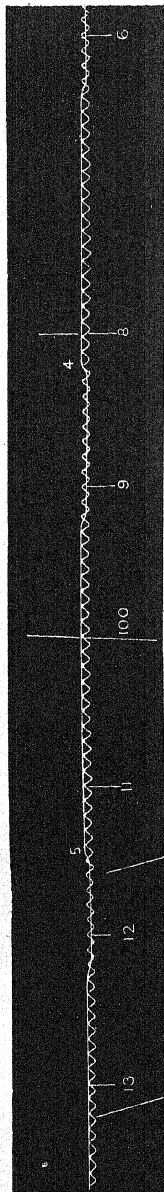
Δ^1
32.7

98.0 IV

Δ^1
31.2

66.8 III

MISSED 6TH SCREEN



Stylus to fork

Δ^2
1.5

Δ^2
1.7

37.3 II

Δ^1
29.5

Δ^1
28.7

8.6 NO.1 SCREEN RECORD

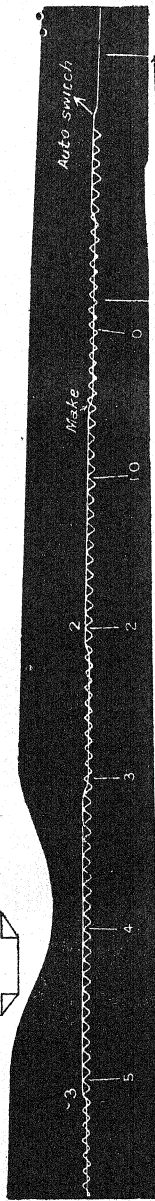
6 LBS POINTED
300S1003 CORDITE
MD SIZE II.

12 P8 18 CWT NO1957

NOON 6/4/05.



ROUND



BATCH N.Y.

Barometer
Dry bulb 30.12
Wet bulb 44.0 F
Wind 40.0 F
30 F's at VII

Δ^2
0.8

The heavy work formerly required in the conversion of the time scale is obviated now by the use of a timing-fork trace, shown in the diagrams of a record with the chronograph, which gives a continuous time record (see Plates I and II); a chronograph of this kind was employed for the 1902-1906 experiments for finding K and the resistance of the air, the time of a complete vibration of the tuning fork being $\frac{1}{348.5}$ second* (at 55° F.).

The upper figure gives the chronograph record of two screens, made each by its own stylus, and the measurement on the sinuous time record has been transferred from the first indication of the movement of a stylus. If more screens are employed than two, an additional stylus is required for each screen.

The lower figure shows the record of two rounds on the same plate of smoked glass; and so a number may be taken on the same plate, and measured up at the end of the day.

The new Ballistic Tables, founded upon the experiments carried out at Shoeburyness, are based upon a standard density of air of 534.2 grains per cubic foot, a standard ogival head of two calibres, so that κ , the coefficient of shape, is equal to unity for such ogival head.

The new Ballistic Table is calculated by integration, to be explained in the next chapter, by assuming a monomial law for K ($= C \frac{d^{3/2}}{ds^2} 10^9$) of the form

$$K = \lambda v^m,$$

in which the index m can be determined by plotting the experimental value of K on a logarithmic chart; and the value of m and K adopted in the Ballistic Table re-calculated is given in the following table, for each of eight regions of velocity.

A calculation can be made in one step from one end to the other of a region.

Region.	v	m	K	K
I	5000	-1.55	17.6044	[2.3290234] $\left(\frac{v}{1000}\right)^{-1.55}$
	4000		24.8789	
II	2600	-1.33	44.1221	[2.1965702] $\left(\frac{v}{1000}\right)^{-1.33}$
	2000		65.3989	
III	1460	-1.2	95.4076	[2.1788067] $\left(\frac{v}{1000}\right)^{-1.2}$
	1190		95.4076	
V	1040	0	59.9392	[1.7777107] $\left(\frac{v}{1000}\right)^0$
	840		59.9392	
VIII	400	-1.4	169.3627	[1.6717017] $\left(\frac{v}{1000}\right)^{-1.4}$

In the above the bracket [] signifies the antilogarithm of the number in the bracket (9263)

thus in Region VIII [$1.6717017 \equiv 46.957$, this being the antilogarithm of $\log 6717017$, and so for the other regions of velocity.

In Regions VII and V of the velocity, where $840 < v < 1040$, and $1190 < v < 1460$, a constant value of K exists, so that from (14)

$$K = C \frac{d^2 t}{ds^2} 10^9 = \text{constant},$$

or

$$\frac{d^2 t}{ds^2} = \text{constant} = 2b, \text{ say.}$$

Integrate with respect to s ,

$$\frac{dt}{ds} = \frac{1}{v} = 2bs + \text{const.},$$

when $s = 0$, $\frac{dt}{ds}$ is equal to $\frac{1}{V}$ if V denote the initial muzzle velocity, and v the remaining velocity at the end of a range s feet.

Therefore

$$\frac{dt}{ds} = 2bs + \frac{1}{V},$$

from which

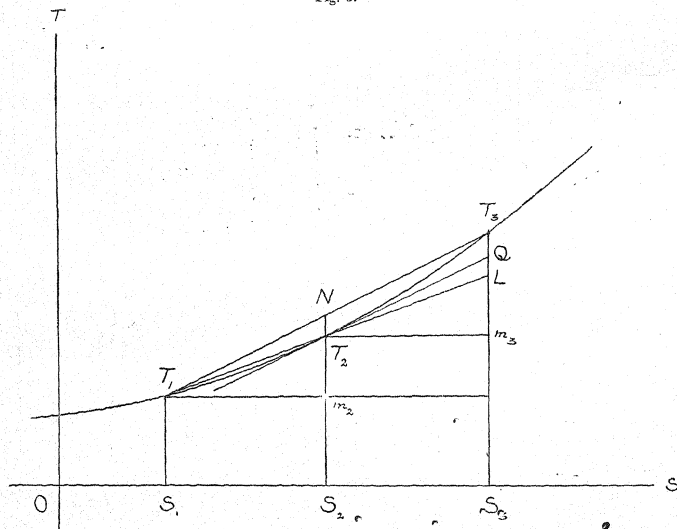
$$t = bs^2 + \frac{s}{V} + t_0,$$

where t_0 is a constant.

The curve $T_1 T_2 T_3$ on the figure (3), or the curve $f(s, t) = 0$, is a parabola.

Let $OS_1 = s$ feet and $S_1 S_2 = S_2 S_3 = l$ feet, then S_2 being the mid point between S_1 and S_3

Fig. 3.



and $T_1T_2T_3$ a parabola, the tangent T_2Q at T_2 is parallel to the chord T_1T_3 , so that with screen distance $S_1S_3 = 2l$,

$$\left. \begin{aligned} S_1T_1 &= t_s \\ S_2T_2 &= t_{s+l} \\ S_3T_3 &= t_{s+2l} \end{aligned} \right\} \text{also} \quad \begin{aligned} T_2m_2 &= t_{s+l} - t_s = \Delta t_s \\ T_3m_3 &= t_{s+2l} - t_{s+l} = \Delta t_{s+l} \end{aligned}$$

$$\Delta^2 t_s = \Delta t_{s+l} - \Delta t_s = T_3m_3 - T_2m_2.$$

Join T_1T_2 and produce it to L , then from similar triangles $T_1T_2m_2$, T_2Lm_3 , $Lm_3 = T_2m_2$.

Also $T_3L = 2NT_2$ and $T_3Q = NT_2$.

Therefore

$$\Delta^2 t_s = (T_3L + Lm_3) - T_2m_2 = T_3L = 2NT_2 = 2T_3Q,$$

or

$$\frac{\Delta^2 t_s}{2} = NT_2.$$

And therefore

$$T_3Q = QL = NT_2 = \frac{\Delta^2 t_s}{2}.$$

The initial velocity when $s = 0$ is V , and denoting by (v_1, t_1) , (v_2, t_2) , (v_3, t_3) , the velocity and time at s_1, s_2, s_3 respectively,

$$\frac{1}{v_1} = 2bs + \frac{1}{V}, \quad t_1 = bs^2 + \frac{s}{V} + t_0,$$

$$\frac{1}{v_2} = 2b(s+l) + \frac{1}{V}, \quad t_2 = b(s+l)^2 + \frac{s+l}{V} + t_0,$$

$$\frac{1}{v_3} = 2b(s+2l) + \frac{1}{V}, \quad t_3 = b(s+2l)^2 + \frac{s+2l}{V} + t_0.$$

The average velocity U over the distance $s_1s_3 (= 2l)$ is given by

$$\begin{aligned} \frac{1}{U} &= \frac{t_3 - t_1}{2l} = \frac{4bl(l+s) + \frac{2l}{V}}{2l} = 2b(s+l) + \frac{1}{V} \\ &= \frac{1}{2} \left(\frac{1}{v_1} + \frac{1}{v_3} \right) = \frac{1}{v_2} \text{ from above,} \end{aligned}$$

hence

$$U = v_2,$$

so that the average velocity U is the harmonic mean of the initial velocity and final velocity over any distance $2l$, and U is the actual velocity at the half distance.

This is the rule employed in determining velocity with a Boulangé chronograph at proof, where $2l$ is the distance in feet between the screens, and t_0, t_1 are the initial and final chronograph record of time, so that

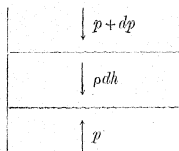
$$\frac{2l}{t_3 - t_1} = U$$

gives the average velocity of the shot between the screens, and this is taken to be the actual velocity at the point midway between the screens, but it depends upon the assumption that the resistance of the air varies as the cube of the velocity.

PROOF OF MATHEMATICAL FORMULAS ON PAGE 5 FOR FINDING f .

To find the tenuity factor $f = \frac{\tau_0}{\tau_h}$ for an altitude h feet above sea level.

Take the foot as the unit of length, the pound avoirdupois as the unit of weight; the density of a substance is the weight of the unit of volume, in this case unit volume is 1 cubic foot.



Let p_0 , ρ_0 denote the pressure and density in lbs/ft² and lbs/ft³ of the air at sea level; p , ρ the pressure and density at a height h feet.

Then, for a small increase of height dh feet, let the pressure at $h + dh$ feet be $p + dp$.

The pressure at height h is greater than at height $h + dh$ by the weight of the column of air, dh feet high (and a square foot in section), therefore

$$dp = -\rho dh \quad \text{I.}$$

Also, by definition,

$$f = \frac{\tau_0}{\tau_h} = \frac{\rho_0}{\rho} \quad \text{II.}$$

1st Case: If the temperature is constant.

$$pv = \text{const.}, \quad \text{or} \quad p = \lambda \rho$$

and

$$p_0 = \lambda \rho_0$$

therefore

$$p = \frac{p_0}{\rho_0} \rho.$$

Suppose the density of the atmosphere uniform throughout and equal to ρ_0 , then $p_0 = \rho_0 k$, where k is the height of the homogeneous atmosphere, that is, the height of an atmosphere of uniform density which will give the barometric pressure, p_0 .

Therefore

$$\frac{p_0}{\rho_0} = k = 27,800 \text{ about,}$$

and

$$p = k\rho \quad \text{III.}$$

from III.,

$$dp = k d\rho \quad \text{IV.}$$

from I. and IV.,

$$\int_{p_0}^p \frac{dp}{\rho} = - \int_0^h \frac{dh}{k};$$

hence

$$\log \frac{\rho}{\rho_0} = - \frac{h}{k},$$

therefore

$$\frac{\rho}{\rho_0} = e^{-\frac{h}{k}};$$

from II.,

$$f = \frac{\tau_0}{\tau_h} = \frac{\rho_0}{\rho} = e^{\frac{h}{k}} \quad \text{and} \quad \tau_h = \frac{\tau_0}{f} = \tau_0 \cdot e^{-\frac{h}{k}} \quad \text{V.}$$

2nd Case: Suppose the rate of diminution of temperature constant per foot upwards of the atmosphere.

In this case, assuming that the adiabatic law follows, namely:—

$$p\rho^\gamma = \text{const.} = \lambda,$$

or

$$p = \lambda\rho,$$

and

$$p_0 = \lambda\rho_0^\gamma.$$

Therefore

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma \quad \text{. VI.}$$

Differentiate VI.,

$$dp = \frac{p_0}{\rho_0^\gamma} \cdot \gamma \rho^{\gamma-1} d\rho \quad \text{. VII.}$$

From I. and VII.,

$$\gamma \cdot \frac{p_0}{\rho_0^\gamma} \rho^{\gamma-1} d\rho = -\rho dh,$$

or

$$\int_{\rho_0}^{\rho} \rho^{\gamma-2} d\rho = - \int_0^h \frac{\rho_0^\gamma}{\gamma p_0} dh,$$

$$\frac{\rho^{\gamma-1}}{\gamma-1} - \frac{\rho_0^{\gamma-1}}{\gamma-1} = - \frac{\rho_0^{\gamma-1}}{\gamma} \cdot \frac{h}{k} \quad \text{since} \quad \frac{p_0}{\rho_0} = k,$$

$$\left(\frac{\rho}{\rho_0}\right)^{\gamma-1} = 1 - \frac{\gamma-1}{\gamma} \cdot \frac{h}{k},$$

$$f = \frac{\rho_0}{\rho} = \left\{ 1 - \frac{\gamma-1}{\gamma} \cdot \frac{h}{k} \right\}^{-\frac{1}{\gamma-1}} \quad \text{. VIII.}$$

CHAPTER II.

CONSTRUCTION OF A BALLISTIC TABLE.

THE experimental determination of the *resistance of the air* having been obtained by the experiments carried out in 1902-1906, the calculation of a Ballistic Table can be carried out.

In the previous chapter the value of K is given for each region of velocity, and in the same chapter it is shown that

$$p^m = Cr = K \left(\frac{v}{1000} \right)^3$$

(see (15), (17), of Chapter I), where

$$K = C \frac{d^2 t}{ds^2} 10^n$$

and K is of the form (see p. 27)

$$[A_0] \left(\frac{v}{1000} \right)^m,$$

where $[A_0]$ denotes the antilogarithm of A_0 , hence Cr is of the form

$$[A_0] \left(\frac{v}{1000} \right)^{m+3} = \frac{[A_0]}{(1000)^n} v^n, \text{ where } n = m + 3$$

$$= A] v^n. \dots \dots \dots (1)$$

Thus, in Region VIII of velocity, for $v < 840$

$$K = [1.6717017] \left(\frac{v}{1000} \right)^{-14};$$

hence for this region

$$Cr = \frac{[1.6717017]}{(1000)^{16}} v^{-14+3} = [4.8717017] v^{11}.$$

In the Region III of velocity $2000 < v < 2600$,

$$K = [2.2671157] \left(\frac{v}{1000} \right)^{-15};$$

Therefore for this region

$$Cr = \frac{[2.2671157]}{(1000)^{15}} v^{-15+3}$$

$$= [3.7671157] v^{13}.$$

The following table gives the value of Cr for each region of velocity:—

Region.	v	$pg = Cr = K \left(\frac{v}{1600} \right)^3$
I	5000–4000	$[3.9790234] v^{1.45} = [A_1] v^{u_1}$
II	4000–2600	$[3.1865702] v^{1.07} = [A_2] v^{u_2}$
III	2600–2000	$[3.7671157] v^{1.5} = [A_3] v^{u_3}$
IV	2000–1460	$[4.7768067] v^{1.8} = [A_4] v^{u_4}$
V	1460–1190	$[8.9795830] v^3 = [A_5] v^{u_5}$
VI	1190–1040	$[18.3689459] v^{6.45} = [A_6] v^{u_6}$
VII	1040–840	$[3.7777107] v^3 = [A_7] v^{u_7}$
VIII	840–0	$[4.8717017] v^{1.6} = [A_8] v^{u_8}$

The above values of Cr are for a standard projectile of 2 calibres ogive, moving under standard conditions in air of standard density, and these values of Cr are required for computing Ballistic Tables given at the end of the book.

To find t , the time it takes in seconds for the velocity of a projectile, d inches in diameter and weighing w pounds, to fall from any initial velocity, V f/s, to any final velocity, v f/s.

If dt seconds is the time during which the resistance R of the air causes the velocity to fall dv f/s, so that the velocity drops from $v + \frac{1}{2} dv$ to $v - \frac{1}{2} dv$ in passing through the mean velocity v , then, in accordance with the laws of motion,

$R dt$ = loss of momentum of the shot, in second-pounds

$$= w \frac{v + \frac{1}{2} dv}{g} - w \frac{v - \frac{1}{2} dv}{g} = w \frac{dv}{g},$$

and from (7), Chapter I,

$$R = n d^2 p \quad \dots \dots \dots (2);$$

hence

$$dt = \frac{w}{n d^2} \frac{dv}{pg} = C \frac{dv}{pg},$$

$$\frac{dt}{C} = \frac{dv}{pg} = \frac{dv}{Cr} \quad \dots \dots \dots (3),$$

where C is the ballistic coefficient of the shot.

Since r is a retardation and v decreases as t increases, then with dt , dv very small,

$$\int_0^t \frac{dt}{C} = \int_V^v - \frac{dv}{Cr} = + \int_v^V \frac{dv}{[A] v^u} = \sum_v \frac{dv}{Cr},$$

giving

$$\frac{t}{C} = T(V) - T(v) = \sum_v \left(\frac{dv}{Cr} \right) \quad \dots \dots \dots (4),$$

which is employed for calculating Table II, and if V and v are both within the same region of velocity, the integration can be carried out at one step; if V and v are in different regions of velocity, two (or more) integrations will be required (see p. 37).

Next, if the shot advances a distance ds feet in the time dt seconds, during which the velocity drops from $v + \frac{1}{2} dv$ to $v - \frac{1}{2} dv$,

$R \cdot ds$ = loss of kinetic energy in foot-pounds

$$= w \frac{(v + \frac{1}{2} dv)^2}{2g} - w \frac{(v - \frac{1}{2} dv)^2}{2g} = w \frac{v dv}{g};$$

so that

$$ds = \frac{w}{u^2} \frac{v dv}{pg} = C \frac{v dv}{pg},$$

$$\frac{ds}{C} = \frac{v dv}{pg} = v \frac{dv}{Cr} \quad \dots \dots \dots (5),$$

and the distance s feet through which the shot advances whilst its velocity drops from the initial V f/s to the final v f/s is given by

$$\int_0^s \frac{ds}{C} = \int_V^v -v \frac{dv}{Cr} = \int_V^v + \frac{v dv}{[A]^{p+1}} = \int_V^v \frac{dv}{[A]^{p+1}},$$

giving

$$\frac{s}{C} = S(V) - S(v) = \sum_v \left(\frac{v dv}{Cr} \right) \quad \dots \dots \dots (6),$$

which is employed for calculating Table III; as before, if V and v are in the same region of velocity, one integration only is necessary, but more than one if these are not in the same region (see p. 41).

A third table (Table IV), is useful for determining the change in direction of motion of a projectile while the velocity drops from any initial value V to any final value v .

To explain the theory of this table, let the tangent at the point of the trajectory, where the velocity is v , make an angle i radians with the horizon.

Then, if di denotes the infinitesimal decrement of i in the infinitesimal increment of time dt , resolving normally in the trajectory (see *Notes on Dynamics*, 2nd edition, 1908, Sir G. Greenhill, published by Wyman and Sons),

$$v \frac{di}{dt} = g \cos i \quad \dots \dots \dots (7).$$

This may be proved in the following manner: Suppose that in passing through the point P on the trajectory, where the inclination is i radians, the velocity drops from

$$v + \frac{1}{2} \Delta v \text{ to } v - \frac{1}{2} \Delta v \text{ f/s,}$$

as the shot passes from Q to R , where the inclinations are

$$i + \frac{1}{2} \Delta i \text{ and } i - \frac{1}{2} \Delta i \text{ radians.}$$

Measure off the length TU and TV from T , the point of intersection of the tangents at Q and R , to represent to scale the velocity at Q and R ; then UV represents to the same scale the change in velocity in passing from Q to R .

Draw UW vertical, and VW parallel to the tangent at P , so as to form the triangle UVW ; then, on the assumption that the average resistance of the air acts in the direction of the tangent at P , the triangle of velocity UVW shows that UW represents the change in velocity due to gravity, and WV the change due to the resistance of the air; so that if the shot takes Δt seconds to pass from Q to R , we may put

$$UW = g\Delta t,$$

$$WV = r\Delta t,$$

if r denotes the average retardation due to the resistance of the air.

If the trajectory is sufficiently flat for $\cos i$ to be replaced by unity, then, since i diminishes as t increases, the curve QPR being concave downward, (7) becomes

$$v \frac{di}{dt} = -g, \quad \text{or} \quad di = -\frac{g}{v} dt \quad (8),$$

where v denotes the mean velocity during a very small increment of time dt , during which the direction of motion of the shot changes through di radians.

If the inclination δ , or change of direction $d\delta$, be in degrees, then

$$\frac{\delta}{180} = \frac{i}{\pi} \quad \text{and} \quad d\delta = \frac{180}{\pi} di = \frac{180g}{\pi v} dt.$$

The formula

$$v \frac{di}{dt} = -g$$

may also be proved thus:—

The arc QR is ds feet and the curvature of this arc is di radians, hence the mean curvature per foot is $\frac{di}{ds}$ radians, which may be taken as the curvature at P.

Resolve along the normal at P, where the velocity is v f/s,

$$\frac{v^2}{\rho} = g \cos i,$$

where ρ is the radius of curvature and is equal to $-\frac{ds}{di}$; the negative sign is taken because i decreases as s increases.

Therefore

$$v^2 \frac{di}{ds} = -g \cos i,$$

$$v^2 \frac{di}{dt} \cdot \frac{dt}{ds} = -g \cos i,$$

giving

$$v \frac{di}{dt} = -g \cos i,$$

and when i is small,

$$di = -\frac{g}{v} dt,$$

as before.

Employing this formula with that previously proved

$$\frac{dt}{C} = \frac{dv}{Cr},$$

where C is the ballistic coefficient,

$$\frac{di}{C} = -\frac{g}{v} \frac{dv}{Cr} \quad (9),$$

hence

$$\int_0^i \frac{di}{C} = \sum_v \left(-\frac{g}{v} \frac{dv}{Cr} \right) = \int_v -\frac{g}{v} \frac{dv}{Cr},$$

$$\frac{i}{C} = \int_v \frac{g}{v} \cdot \frac{dv}{[A] v^n} = \frac{g}{[A]} \int_v \frac{dv}{v^{n+1}},$$

giving

$$\frac{i}{C} = I(V) - I(v) = \sum_v \left(\frac{g}{v} \cdot \frac{dv}{Cr} \right) \quad (10),$$

which is employed for calculating Table IV. If V and v both lie in the same region of velocity, only one integration is necessary, but more than one if these do not lie in the same region of velocity, see p. 44.

TABLE II.

Time t seconds between Velocity V and v f/s. $t = C(T_v - T_r)$.

$$\frac{t}{C} = \sum_v \frac{dv}{Cv} = \int_v^V \frac{dv}{[A]v^n} \text{ from p. 33,}$$

hence

$$\begin{aligned} \frac{t}{C} &= \frac{1}{[A]} \left\{ \frac{v^{-n+1}}{-n+1} \right\}_v^V \\ &= \frac{1}{[A](n-1)} \left\{ \frac{1}{v^{n-1}} - \frac{1}{V^{n-1}} \right\}, \end{aligned}$$

where V is the upper limit in each region of velocity. Thus in *Region I*, V is 5000 f/s and v is the variable velocity. Therefore

$$\frac{t}{C} = \frac{1}{(n-1)[A]} \frac{1}{v^{n-1}} - B_t,$$

where B_t is a constant for any one region of velocity, and considering *Region I* we have $4000 < v < 5000$ f/s.

$$[A] = [A_1] = [3.9790234],$$

$$n = n_1 = 1.45,$$

$$B_t = B_1 = \frac{1}{0.45 [3.9790234] (5000)^{0.45}}.$$

$$\log 0.45 = \bar{1}.6532125,$$

$$[A_1] = 3.9790234$$

$$3.6322359,$$

therefore

$$\log \frac{1}{0.45 [A_1]} = 2.3677641.$$

$$0.45 \log 5000 = 1.6645365,$$

$$\log B_1 = 0.7032276,$$

$$B_1 = 5.0493.$$

Hence in *Region I*, $4000 < v < 5000$,

$$\frac{t}{C} = [2.3677641] v^{-0.45} - 5.0493,$$

where v may have any value from 4000 to 5000 f/s, and so the value of $\frac{t}{C}$ is found for a fall of velocity of 5000 to any other velocity down to 4000 f/s.

Region II, $2600 < v < 4000$.

$$[A] = [A_2] = [3.1865702],$$

$$n = n_2 = 1.67,$$

$$\frac{t}{C} = \frac{1}{0.67 [3.1865702] v^{0.67}} - B_2 \text{ (a constant).}$$

To find B_2 , we must have the value of $\frac{t}{C}$ for $v = 4000$ f/s in Region I, the same as that for $v = 4000$ f/s in Region II, hence

$$\text{must be equal to } [2.3677641](4000)^{-0.45} - 5.0493$$

$$\frac{1}{0.67 [3.1865702](4000)^{0.67}} - B_2.$$

$$2.3677641,$$

$$0.45 \log 4000 = 1.620927$$

$$\text{Difference} = 0.7468371 = \log 5.5826,$$

and

$$5.5826 - 5.0493 = 0.5333.$$

Again,

$$-\log A_2 = 3.1865702,$$

$$\log 0.67 = 1.8260748$$

$$\text{Sum} = 3.0126450,$$

therefore

$$\log \frac{1}{0.67 [3.1865702]} = 2.9873550$$

$$0.67 \log 4000 = 2.4133802$$

$$\text{Difference} = 0.5739748,$$

therefore

$$\frac{1}{0.67 [3.1865702](4000)^{0.67}} = \text{antilog } 0.5739748 = 3.7495.$$

Hence

$$0.5333 = 3.7495 - B_2,$$

$$B_2 = 3.2162,$$

so that in Region II, $2600 < v < 4000$,

$$\frac{t}{C} = [2.9873550] v^{-0.67} - 3.2162.$$

Region III, $2000 < v < 2600$.

$$[A] = [A_3] = [3.7671157],$$

$$n = n_3 = 1.5,$$

and

$$\frac{t}{C} = \frac{1}{0.5 [3.7671157]^{1.5}} - B_3,$$

and B_3 must be found in similar fashion to B_2 , the value of $\frac{t}{C}$ for $v = 2600$ f/s in Region II being made equal to that in Region III for a like velocity.

Therefore

$$\frac{1}{0.5 [3.7671157](2600)^{0.5}} - B_3 = [2.9873550](2600)^{-0.67} - 3.2162,$$

from which

$$B_3 = 4.9175.$$

Also

$$\frac{1}{0.5 [3.7671157]} = [2.5329143],$$

hence in Region III, $2000 < v < 2600$ f/s,

$$\frac{t}{C} = [2.5339143] v^{-0.5} - 4.9175.$$

For the remaining five regions of velocity, similar calculations are made. The summary for all eight regions is as follows :—

Region.	v	$T_{5000} - T_v = T'_v$
I	5000-4000	$[2.3677641] v^{-0.45} - 5.0493$
II	4000-2600	$[2.9873550] v^{-0.67} - 3.2162$
III	2600-2000	$[2.5339143] v^{-0.5} - 4.9175$
IV	2000-1460	$[3.3201033] v^{-0.8} - 2.0505$
V	1460-1190	$[6.7193870] v^{-2} + 1.6373$
VI	1190-1040	$[16.8946576] v^{-3.45} + 3.9800$
VII	1040-840	$[6.9212593] v^{-2} - 0.9022$
VIII	below 840	$[3.3501470] v^{-0.6} - 28.4874$

In Table II, T_{5000} is given the value of 113, an arbitrary but convenient number, which ensures that all values of T_v are positive.

Therefore

$$T_v = T_{5000} - T'_v = 113 - T'_v.$$

Examples :—In Region VIII, put $v = 100$ f/s,

$$T'_{100} = [3.3501470] (100)^{-0.6} - 28.4874,$$

$$3.3501470$$

$$0.6 \log 100 = 1.2$$

$$\text{Difference} = 2.1501470 = \log 141.301.$$

herefore

$$T'_{100} = 141.301 - 28.4874 = 112.814,$$

$$T_{100} = 113 - 112.814 = 0.186,$$

and this is the actual value for T_{100} in Table II.

From the difference for 1 f/s in the formula for T'_v we have

$$T_{100} = 0.186$$

$$\Delta = 0.841$$

$$T_{101} = 1.027$$

$$\Delta = 0.828$$

$$T_{102} = 1.855, \text{ and so on,}$$

and these are the values of T_{101}, T_{102}, \dots in the table.

Again, in Region I, put $v = 4000$, then

$$T'_{4000} = [2 \cdot 367764] (4000)^{-0.45} - 5 \cdot 0493,$$

$$2 \cdot 367764$$

$$0 \cdot 45 \log 4000 = 1 \cdot 620927$$

$$\text{Difference} = 0 \cdot 746837 = \log 5 \cdot 58256.$$

Therefore

$$T_{4900} = 5 \cdot 58256 - 5 \cdot 0493 = 0 \cdot 53326 = 0 \cdot 5333,$$

Hence

$$T_{4000} = 113 - 0 \cdot 533 = 112 \cdot 467.$$

which is the actual value of T_{4000} found in the table.

So for any other value of v ; thus in Region II put $v = 2800$, then from the formula for T_v we get

$$T'_{2800} = 1 \cdot 546,$$

giving

$$T_{2800} = 113 - 1 \cdot 546 = 111 \cdot 454,$$

the value found in the table.

The Ballistic Table for T_v , as constructed by the integrals, presents the function as decreasing as the velocity increases. It is more convenient to have T_v increase with the velocity, and this can be done by direct subtraction from a constant; this can be done without in any way affecting a Gunnery problem, because the absolute value of T_v is never required, but only the difference such as $T_v - T_r$; hence the suitable value of 113 is given to the constant, from which are subtracted the various values of T_v .

Table II for T_r might be compiled by the finite difference employed by Bashforth for his Ballistic Tables.

From the formula

$$\frac{dt}{C} = \frac{dv}{pg} = \frac{dv}{Cv},$$

if 10 f/s intervals be taken for the drop of velocity over any interval, then writing

$$\frac{\Delta t}{C} = \frac{\Delta v}{pg} = \frac{10}{pg}, \quad \text{see (3),}$$

the increment Δt for any velocity is found as soon as the value pg is obtained from Table I for the required velocity; thus:—

v	\bar{v}	$\Delta T = \frac{10}{pg}$	T
2000			
	2005	0.01904	110.272*
2010			110.291
	2015	0.01890	
2020			110.310
	2025	0.01876	
2030			110.329
	2035	0.01862	
2040			110.347
	2045	0.01849	
2050			110.366

* This value of T_{2000} is quoted from Table II.

TABLE III.

Distance s in feet between velocity V and v f/s, $s = C(S_v - S_v)$.

$$\begin{aligned} \frac{s}{C} &= \sum_v \frac{v dv}{Cv} = \int_v^V \frac{v dv}{[A] v^n} = \frac{1}{[A]} \int_v^V \frac{dv}{v^{n-1}}, \\ \frac{1}{[A]} \left\{ \frac{v^{-n+2}}{-n+2} \right\}_v^V &= \frac{+1}{(n-2)[A] v^{n-2}} - \frac{1}{(n-2)[A] V^{n-2}}, \\ &= \frac{+1}{(n-2)[A] v^{n-2}} - B_s, \end{aligned}$$

where B_s may be taken as constant for any one region of velocity, and V is the upper limit in each region of velocity; thus in Region I V is 5000 f/s and v is the variable velocity.

Consider Region I, $4000 < v < 5000$

$$[A] = [A_1] = 3.9790234,$$

$$n-2 = -0.55,$$

$$-B_s = \frac{+1}{0.55 [3.9790234] (5000)^{-0.55}},$$

$$\log 0.55 = 1.7403627,$$

$$[A_1] = 3.9790234$$

$$3.7193861.$$

Therefore

$$\log \frac{1}{0.55 [A_1]} = 2.2806139,$$

$$0.55 \log 5000 = 2.0344335,$$

therefore

$$\log (-B_s) = 4.3150474 = \log 20656.06,$$

therefore

$$-B_s = 20656.06.$$

Hence, in Region I, $4000 < v < 5000$,

$$\frac{s}{C} = 20656.06 - [2.2806139] v^{0.55}.$$

Region II, $2600 < v < 4000$.

$$\frac{s}{C} = \frac{-1}{0.33 [3.1865702] v^{-0.33}} - B_s.$$

The value of s_{4000} obtained from this must equal that got from Region I of velocities, hence,

$$\frac{-1}{0.33 [3.1865702] (4000)^{-0.33}} - B_s = 20656.06 - [2.2806139] (4000)^{0.55},$$

$$\log \frac{1}{0.33 [3.1865702]} = 3.2949159,$$

$$\log (4000)^{0.55} = 1.1886798,$$

therefore

$$\log \frac{1}{0.33 [A_s] (4000)^{-0.33}} = [4.4835957] = \log 30450.58,$$

also

$$0.55 \log 4000 = 1.9811330$$

$$\text{add } 2.2806139$$

$$4.2617469 = \log 18270.35,$$

therefore

$$-30450.58 - B_s = 20656.06 - 18270.35, \quad -B_s = 32836.29,$$

and for Region II,

$$\frac{s}{C} = 32836.29 - [3.2949159] v^{0.33}.$$

For Region III, $2000 < v < 2600$.

$$\begin{aligned} \frac{s}{C} &= \frac{1}{0.5 [3.7671157] v^{-0.5}} - B_s \\ &= 23855.06 - [2.5339143] v^{0.5}, \end{aligned}$$

and so on.

The results in tabular form are :—

Region.	v	$S_{5000} - S_v = S'_v$
I	5000-4000	$20656.06 - [2.2806139] v^{0.55}$
II	4000-2600	$32836.29 - [3.2949159] v^{0.33}$
III	2600-2000	$23855.06 - [2.5339143] v^{0.5}$
IV	2000-1460	$46791.21 - [3.9221633] v^{0.2}$
V	1460-1190	$[7.0204170] v^{-1} + 3717.23$
VI	1190-1040	$[16.9826941] v^{-4.45} + 10545.79$
VII	1040-840	$[7.2222893] v^{-1} - 1891.20$
VIII	below 840	$67623.71 - [3.5262383] v^{0.4}$

In Table III, S_{5000} is given the value of 50000, and

$$S_v = S_{5000} - S'_v.$$

Examples :—In Region VIII, for $v < 840$ f/s, put

$$v = 100,$$

$$S'_{100} = 67623.71 - [3.5262383] (100)^{0.4},$$

$$= 46428.5,$$

therefore

$$S_{100} = 50000 - 46428.5 = 3571.5,$$

which is the value found in the Table.

So

$$S_{101} = 3656.1$$

$$\text{Difference for } 1 \text{ f/s} = 84.0$$

$$S_{102} = 3740.1, \text{ \&c.}$$

$$\begin{aligned} S_{2800} (\text{Region II}) &= 50000 - 32836.29 + [3.2949159] (2800)^{2.3} \\ &= 50000 - 5766.9 = 44233.1, \end{aligned}$$

and so on.

By means of this process Table III for S_v gives a function which increases as the velocity increases; and since in Gunnery the absolute value of S_v is not required, but only the difference such as $S_v - S_v$, the subtracting of S_v from the constant 50000 presents the table for S_v in the more convenient form.

In a similar manner to Bashforth's, the method of a finite difference of 10 f/s drop in the velocity over any interval could be employed for the construction of Table III for S_v thus:—

Taking the value of pg for any required velocity from Table I, also see (5).

v	\bar{v}	$\frac{10}{pg}$	$\Delta s = \frac{v \Delta v}{pg} = \frac{10v}{pg}$	S_v
2000				41435.7*
2010	2005	0.01904	38.180	41473.9
2020	2015	0.01890	38.084	41512.0
2030	2025	0.01876	37.990	41550.0
2040	2035	0.01862	37.897	41587.9
2050	2045	0.01849	37.804	41625.7

* This value of S_{2800} is quoted from Table III.

TABLE IV.—THE I (v) TABLE.

$$\frac{i}{C} = \frac{\phi^\circ - \theta^\circ}{37.3 C} = I(V) - I(v) \quad \text{or} \quad \tan \phi - \tan \theta = C [I(V) - I(v)].$$

From (10)

$$I(V) - I(v) = \frac{g}{[A]} \int_v^V \frac{dv}{v^{n+1}} = \frac{g}{n[A]} \frac{1}{v^n} - \frac{g}{n[A]} \frac{1}{V^n}.$$

In Region I of velocities, $4000 < v < 5000$,

$$n = n_1 = 1.45$$

and

$$[A] = [A_1] = [3.9790234],$$

$V = 5000$ f/s, v is the variable velocity; put

$$I'(v) = \frac{g}{n_1 [A_1] v^{1.45}} - B_1,$$

where B_1 is a constant, which disappears in $I(v) - I(v)$.

In Region I take $B_1 = 0$, therefore

$$I'(v) = \frac{g}{1.45 [3.9790234] v^{1.45}}.$$

For the computation of tables such as the double entry table, the values of $2I(v)$ are required; for this reason it is found to involve less labour to calculate $2I(v)$ at first, and to halve this for the table giving $\frac{i}{C} = I(V) - I(v)$.

For Region I of velocities,

$$2I'(v) = \frac{2g}{1.45 [3.9790234] v^{1.45}},$$

$$\log [A_1] = 3.9790234,$$

$$\log 1.45 = 0.1613680$$

$$\hline 2.1403914,$$

$$\log 2g = 1.8087615,$$

$$\log \frac{2g}{n_1 [A_1]} = 3.6683701,$$

therefore

$$2I'(v) = [3.6683701] v^{-1.45}.$$

Region II. of velocities, $2600 < v < 4000$,

$$[A_2] = [3.1865702],$$

$$n = n_2 = 1.67,$$

and

$$\frac{2g}{n_2 [A_2]} = [4.3994748].$$

hence for $v = 4000$ f/s

giving

$$\frac{2g}{n_1 [A_1] (4000)^{1.45}} = \frac{2g}{n_2 [A_2] (4000)^{1.67}} - B_5$$

$$0.027886 = 0.024212 - B_5$$

$$-B_5 = 0.003674,$$

hence

$$2I'(v) = [4.3994748] v^{-1.67} + 0.003674.$$

The value of $2I'(v)$ for each region of velocity is as follows:—

Region.	v	$2I'(v)$
I	5000-4000	$[3.6683701] v^{-1.45}$
II	4000-2600	$[4.3994748] v^{-1.67} + 0.003674$
III	2600-2000	$[3.8655545] v^{-1.5} - 0.001960$
IV	2000-1460	$[4.7766823] v^{-1.8} + 0.011713$
V	1460-1190	$[8.3520572] v^{-3} + 0.059897$
VI	1190-1040	$[18.6302559] v^{-6.45} + 0.131293$
VII	1040-840	$[8.5539295] v^{-3} - 0.038958$
VIII	below 840	$[4.7329398] v^{-1.9} - 0.567525$

From Region VIII, for $v = 500$,

$$\begin{aligned} 2I'_{500} &= [4.7329398] (500)^{-1.9} - 0.567525 \\ &= [0.4145878] - 0.567525 \\ &= 2.59769 - 0.567525 \\ &= 2.030165 \equiv 2.03017, \end{aligned}$$

$$I'_{500} = 1.015085,$$

$$I'_{501} = 1.01094,$$

$$I'_{502} = 1.00681,$$

$$I'_{503} = 1.00271,$$

and so on.

From Region I,

$$I'_{4000} = 0.013943.$$

Since differences of $I(v)$, such as $I(v) - I(u)$, only are required in practice, the value of I_{500} is taken in Table IV as zero, and for other velocities we have

$$I_{500} = 0,$$

$$I_{501} = 1.015085 - 1.01094 = 0.00415,$$

$$I_{502} = 1.015085 - 1.00681 = 0.00827,$$

$$I_{503} = 0.01238,$$

$$\dots \dots \dots$$

$$I_{4000} = 1.015085 - 0.013943 = 1.00114.$$

These are the values found in the Table IV.

The above process, put into the form of a formula, is

$$I_v = \frac{1}{2} [2 \cdot 03017 - 2I'(v)] = 1 \cdot 015085 - I'v$$

or

$$I_v = k - I'(v),$$

and by means of this Table IV the I function increases with the velocity.

By taking intervals in which the drop of velocity is 10 f/s, as was made by Bashforth in compiling his tables, the I_v table can be calculated thus: taking $\log g = 1 \cdot 50773$ or $g = 32 \cdot 19$.

v	\bar{v}	$\frac{\Delta t}{C} = \frac{10}{pg}$	$\frac{\Delta t}{C} = \frac{g}{v} \frac{10}{pg}$	I_v
2000				0.97505*
2010	2005	0.01904	0.000306	0.975356
2020	2015	0.01890	0.000302	0.975658
2030	2025	0.01876	0.000298	0.975956
2040	2035	0.01862	0.000295	0.97625
2050	2045	0.01849	0.000291	0.97654

* This value of I_{2000} is quoted from Table IV.

TABLE V FOR "A," THE ALTITUDE FUNCTION.

By definition (see Chapter IV, p. 64)

$$d\{A(v)\} = \frac{vI(v)dv}{Cr} = I_e dSc.$$

Therefore

$$2A(v) = \int_v^{\infty} -\frac{v}{[A]v^n} \left(\frac{2g}{n[A]v^n} - B_i \right) dv,$$

since

$$Cr = [A]v^n,$$

and

$$2I(v) = \frac{2g}{n[A]v^n} - B_i,$$

where B_i is a constant which has already been found for each of the eight regions of velocity

Therefore

$$\begin{aligned} 2A(v) &= \frac{2}{n[A]^2} \int_v^{\infty} \frac{dv}{v^{2n-1}} + \frac{B_i}{[A]} \int_v^{\infty} \frac{dv}{v^{n-1}} - B_a \\ &= \frac{2g}{2n(n-1)[A]^2 v^{2(n-1)}} - \frac{B_i}{(n-2)[A]v^{n-2}} - B_{a_2} \end{aligned}$$

Therefore

$$2A(v) = \frac{g}{n(n-1)[A]^2 v^{2(n-1)}} - \frac{B_i}{(n-2)[A]v^{n-2}} - B_{a_2}$$

where B_a is a constant to be found for each region of velocity.

The explanation of the altitude function $A(v)$ is left for a later chapter; the actual value or definition is stated above, and from such definition the compilation only of the A table can be proceeded with.

"A" Table for Region I, $4000 < v < 5000$.

For this region B_i is zero, and

$$n = n_1 = 1.45, \quad [A] = [A_1] = [3.9790234],$$

hence the term containing B_i disappears and

$$2A_v = \frac{g}{1.45 \times 0.45 [3.9790234]^2 v^{0.9}} - B_a.$$

Take $2A_{5000}$ equal to zero, then, since

$$\left. \begin{aligned} \log \frac{g}{1.45 \times 0.45 [A_1]^2} &= 5.7351042 \\ 0.9 \log 5000 &= 3.32907300 \end{aligned} \right\} \text{difference} = 2.4060312$$

therefore

$$\log B_a = 2.4060312,$$

$$B_a = 254.701,$$

and for Region I of velocities,

$$2A_v = [5.7351042]v^{-0.9} - 254.701.$$

"A" Table for Region II, $2600 < v < 4000$.

$$[A] = [A_2] = [3.1865702], \quad n = n_2 = 1.67,$$

$$2A_v = \frac{g}{n_2(n_2 - 1)[A_2]^2 v^{2(n_2 - 1)}} - \frac{B_i}{(n_2 - 2)[A_2]v^{n_2 - 2}} - B_u,$$

$$= \frac{g}{1.67 \times 0.67[A_2]^2 v^{1.34}} - \frac{B_i}{(-0.33)[A_2]v^{-0.33}} - B_u,$$

$$\left. \begin{aligned} \log 1.67 &= 0.2227165 \\ \log 0.67 &= 1.8260748 \\ 2 \log A_2 &= 5.3731404 \\ \hline &6.4219317 \end{aligned} \right\}$$

$$\left. \begin{aligned} \log g &= 1.5077315 \\ \text{Difference} &= 7.0857998 \end{aligned} \right\}.$$

Hence first term is $[7.0857998] v^{-1.34}$.

$$\left. \begin{aligned} \log 0.33[A_2] &= 4.7050841 \\ \log B_i &= \log 0.003674 = 3.5651392 \\ \log \frac{B_i}{0.33[A_2]} &= 0.8600551 \end{aligned} \right\}$$

therefore

Hence second term is $[0.8600551] v^{0.33}$, therefore

$$2A_v = [7.0857998] v^{-1.34} - [0.8600551] v^{0.33} + B_u.$$

To find B_u , the value of $2A_{4000}$ must be the same whether obtained from Region I or from Region II, hence, putting $v = 4000$,

$$[5.7351042](4000)^{-0.9} - 254.701 = [7.0857998](4000)^{-1.34} - [0.8600551](4000)^{0.33} + B_u.$$

$$\left. \begin{aligned} 0.9 \log 4000 &= 3.24185400 \\ 5.7351042 \\ \hline \log 311.351 &= 2.49325020 \end{aligned} \right\}$$

$$\begin{array}{r} 311.351 \\ 254.701 \\ \hline 56.650 \end{array}$$

$$\left. \begin{aligned} 7.0857998 \\ 1.34 \log 4000 &= 4.8267604 \\ \hline \log 181.568 &= 2.2590394 \\ 0.33 \log 4000 &= 1.1886798 \\ 0.8600551 \\ \hline \log 111.876 &= 2.0487349 \end{aligned} \right\}$$

$$\left. \begin{aligned} 181.568 \\ 111.876 \\ \hline 69.692 \end{aligned} \right\}$$

so that

$$56.650 = 69.692 - B_u,$$

$$B_u = 13.042,$$

and the law for Region II of velocities is

$$2A(v) = [7.0857998] v^{-1.34} - [0.8600551] v^{0.33} - 13.042.$$

The law for the other regions is found in a similar manner:—

Region.	v	$2A(v)$
I	5000-4000	$[5.7351042] v^{-0.93} - 254.701$
II	4000-2600	$[7.0857998] v^{-1.34} - [0.8600551] v^{0.33} - 13.042$
III	2600-2000	$[6.0984388] v^{-1} + [1.8261704] v^{0.5} - 303.331$
IV	2000-1460	$[7.7957556] v^{-1.6} - [1.9908314] v^{0.2} + 474.926$
V	1460-1190	$[14.7704142] v^{-4} + [5.7978221] v^{-1} + 35.262$
VI	1190-1040	$[35.2238835] v^{-10.93} + [16.1009356] v^{-4.45} + 546.710$
VII	1040-840	$[15.1741588] v^{-4} - [5.8128860] v^{-1} + 586.340$
VIII	below 840	$[7.7820568] v^{-1.2} + [3.2802233] v^{0.4} - 44114.15$

The value of $A(v)$ obtained from these laws decreases as the velocity increases. Table V is made out for a function A_v which increases as the velocity increases, and also $A_{500} = 0$.

The direct conversion cannot be carried out by the same simple method employed by I_v , because $A_v - A_v$ is a direct function, not of $I_v - I_v$ or ΔI , but of the absolute value of I_v at any point.

Let S'_v , I'_v denote absolute values as obtained from the integration formulæ on pp. 42, 45; let S_v , I_v denote the values got from Tables III and IV.

Let

$$d(A'_v) = I_v dS_v.$$

Now

$$I_v = k - I'_v,$$

where

$$k = 1.015085, \text{ see p. 46.}$$

Therefore

$$d(A'_v) = k d(S'_v) - I'_v d(S'_v).$$

Integrate

$$A'_{500} - A'_v = k(S'_{500} - S'_v) - \{A_{(500)} - A_{(v)}\},$$

where $2A_{(500)}$ and $2A_{(v)}$ are the values got direct from the integration formulæ given just above.

Put

$$A_v = A'_{500} - A'_v,$$

then

$$A_v = \frac{1}{2} [2k(S'_{500} - S'_v) - (2A_{(500)} - 2A_{(v)})],$$

in which S'_{500} , S'_v , $2A_{(500)}$, $2A_{(v)}$ are got direct from the tabular formulæ on pp. 42, 49, and $2k = 2.03017$, since $S'_{500} - S'_v = S_{500} - S_v$, this difference can also be got direct from Table III. The values of A_v found from this formula are those in Table V.

Example 1.—To calculate A_{500} for Table V:—

$$\begin{array}{rcl} S'_{500} & = & 27275 \cdot 31 \\ S'_{500} & = & 24222 \cdot 80 \\ \hline \Delta S' & = & 3052 \cdot 51 \end{array} \qquad \begin{array}{rcl} 2A_{(500)} & = & 13722 \cdot 16 \\ 2A_{(500)} & = & 8589 \cdot 07 \\ \hline \text{Difference} & = & 5133 \cdot 09 \end{array}$$

$$\begin{aligned} \log \Delta S' &= 3 \cdot 4846571 \\ \log 2k &= 0 \cdot 3075324 \\ \log (2k \cdot \Delta S') &= 3 \cdot 7921895 \\ 2k \cdot \Delta S' &= 6197 \cdot 11 \end{aligned}$$

$$2A_{(500)} - 2A_{(500)} = 5133 \cdot 09$$

$$2A_{500} = 1064 \cdot 02$$

$$A_{500} = 532 \cdot 01$$

Example 2.—To calculate A_{800} :—

$$\begin{array}{rcl} S'_{800} & = & 27275 \cdot 31 \\ S'_{800} & = & 18929 \cdot 84 \\ \hline \Delta S' & = & 8345 \cdot 47 \end{array} \qquad \begin{array}{rcl} 2A_{(800)} & = & 13722 \cdot 16 \\ 2A_{(800)} & = & 3397 \cdot 75 \\ \hline 2A_{(v)} & = & 10324 \cdot 41 \end{array}$$

$$\begin{aligned} \log \Delta S' &= 3 \cdot 9214508 \\ \log 2k &= 0 \cdot 3075324 \\ \log (2k \cdot \Delta S') &= 4 \cdot 2389832 \end{aligned}$$

$$\begin{aligned} 2k \cdot \Delta S' &= 16942 \cdot 72 \\ 2A_{(v)} &= 10324 \cdot 41 \end{aligned}$$

$$2A_p = 6618 \cdot 31$$

$$A_p = 3309 \cdot 16$$

Example 3.—To calculate A_{1200} :—

$$\begin{array}{rcl} S'_{1200} & = & 27275 \cdot 31 \\ S'_{1200} & = & 12451 \cdot 70 \\ \hline \Delta S' & = & 14823 \cdot 61 \end{array} \qquad \begin{array}{rcl} 2A_{(1200)} & = & 13722 \cdot 16 \\ 2A_{(1200)} & = & 842 \cdot 67 \\ \hline 2\Delta A_p & = & 12879 \cdot 49 \end{array}$$

$$\begin{aligned} \log \Delta S' &= 4 \cdot 1709539 \\ \log 2k &= 0 \cdot 3075324 \\ \log (2k \cdot \Delta S') &= 4 \cdot 4784863 \end{aligned}$$

$$\begin{aligned} 2k \Delta S' &= 30094 \cdot 45 \\ 2\Delta A_{(v)} &= 12879 \cdot 49 \end{aligned}$$

$$2A_p = 17214 \cdot 96$$

$$A_p = 8607 \cdot 48$$

To save labour in dealing with large numbers, a change is made at certain intervals and smaller numbers have in consequence to be dealt with. The intervals selected are at velocities of 800, 1000, 1600, and 2800; a slight smoothing is made before entry.

As an example, calculate A_{1200} :—

$$\begin{aligned} A_v &= \frac{1}{2} [2k (S'_{200} - S'_v) - \{2A_{(200)} - 2A_{(v)}\}], \\ &= \frac{1}{2} [2k (S'_{1000} - S'_v) - \{2A_{(1000)} - 2A_{(v)}\}], \\ &+ \frac{1}{2} [2k (S'_{200} - S'_{1000}) - \{2A_{(200)} - 2A_{(1000)}\}], \end{aligned}$$

which may be written

$$A_v = A'_v + A_{(1000)}$$

where

$$\begin{aligned} A'_v &= \frac{1}{2} [2k (S'_{1000} - S'_v) - \{2A_{(1000)} - 2A_{(v)}\}], \\ S_{1000} &= 14792.38 & 2A_{(1000)} &= 1429.72 \\ S'_{1200} &= 12451.68 & 2A_{(1200)} &= 842.67 \\ \Delta S' &= 2340.70 & 2\Delta A_{(v)} &= 587.05 \end{aligned}$$

$$\log \Delta S' = 3.3693458$$

$$\log 2k = 0.3075324$$

$$\log (2k \cdot \Delta S') = 3.6768782$$

$$\left. \begin{aligned} 2k \cdot \Delta S' &= 4752.02 \\ 2\Delta A_{(v)} &= 587.05 \end{aligned} \right\}$$

Therefore

$$2k \Delta S' - 2\Delta A_{(v)} = 2A'_v = 4164.97,$$

$$\left. \begin{aligned} A'_v &= 2082.49 \\ A_{1000} &= 6525.02 \end{aligned} \right\}$$

$$A_{1200} = 8607.51,$$

which is practically as found before.

Table V for A_v could be calculated by means of Tables I and IV, by taking intervals of 10 f/s drop of velocity, as was done by Mr. Hadcock, who calculated the altitude function from the formula

$$\Delta A = \frac{vI(v)}{F(v)} \Delta v = \frac{v \Delta S}{pg} I_v = I_v \Delta S_v$$

v	\bar{v}	$I(\bar{v})$	ΔS_v^*	$\Delta A = I_v \Delta S_v$	A_v
2000					12309.13†
2010	2005	0.97520	38.180	37.232	12346.362
2020	2015	0.97551	38.084	37.151	12383.513
2030	2025	0.97581	37.990	37.071	12420.584
2040	2035	0.97610	37.897	36.992	12457.576
2050	2045	0.97639	37.804	36.912	12494.478

* These values of ΔS_v are obtained from p. 43, in calculating S_v by finite differences method.

† This value of A_{2000} is quoted from Table V.

NOTE.—For calculation of the double-entry Table VIII, see p. 79.

CHAPTER III.

THE UNRESISTED MOTION OF A PROJECTILE.

IN ordinary problems of direct fire, the attraction of gravity is a force which is usually small in comparison with the resistance of the air, and may therefore be left out of account in a first approximation to the solution of these problems. In high-angle fire with low velocities, the reverse conditions hold good, and the force of gravity is much greater than the resistance of the air, and the latter may (as an approximation) be left out of account when compared with the force of gravity. As an example, take a projectile weighing 100 lbs. and 2 inches calibre, fired under standard conditions of air density, shape and steadiness; then from Table I, p. 149.

v	$R = \mu^2 \text{ lbs.}$	Weight in lbs.
400	$0.337 \times 36 = 12.1$	100
520	$0.512 \times 36 = 18.4$	100
800	$1.02 \times 36 = 36.7$	100
1000	$1.86 \times 36 = 67$	100
2000	$16.25 \times 36 = 585$	100
2400	$21.36 \times 36 = 770$	100
2800	$27.26 \times 36 = 980$	100

On the assumption that the resistance of the air may be disregarded for high-angle fire with low velocities, a fair approximation to the trajectory is obtained at short ranges, as with howitzer and mortar fire.

Supposing R , the resistance of the air, and therefore also r , the retardation it produces, to be zero,

$$\frac{d^2x}{dt^2} = 0 \quad (1),$$

$$\frac{d^2y}{dt^2} = -g \quad (2).$$

Integrating these equations with respect to t , supposing the shot is projected from the origin O with velocity V f/s at an elevation α ,

$$\frac{dx}{dt} = \text{a constant} = V \cos \alpha \quad (3),$$

$$\frac{dy}{dt} = \text{a constant} - gt = V \sin \alpha - gt \quad (4).$$

Integrating the equation (3) and (4) again with respect to t ,

$$x = Vt \cos \alpha \quad (5),$$

$$y = Vt \sin \alpha - \frac{1}{2}gt^2 \quad (6),$$

no constant of integration being required if the time of flight, t , is reckoned from the instant the shot leaves the point of projection O ; these are the equations employed in Chapter II, Part I.

The co-ordinates of the focus F are

$$h = NN' = \frac{V^2}{2g} \sin 2\alpha \quad \dots \dots \dots (13),$$

$$k - \frac{1}{4}p = \frac{V^2}{2g} \sin^2 \alpha - \frac{V^2}{2g} \cos^2 \alpha = -\frac{V^2}{2g} \cos 2\alpha;$$

and the height of the directrix HK is

$$OH = k + \frac{1}{4}p = \frac{V^2}{2g} \quad \dots \dots \dots (14);$$

and

$$AN' = k - y, \quad N'P = x - h,$$

so that

$$\frac{N'P^2}{AN'} = \text{latus rectum} = \frac{2V^2}{g} \cos^2 \alpha \quad \dots \dots \dots (15),$$

the property of a parabola.

Denoting by v the velocity at any point (x, y) of the parabolic trajectory,

$$\begin{aligned} v^2 &= \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \\ &= V^2 \cos^2 \alpha + (V \sin \alpha - gt)^2 \\ &= V^2 - 2g(Vt \sin \alpha - \frac{1}{2}gt^2) \\ &= V^2 - 2gy \\ v^2 &= 2g(OH - MP) = 2g \cdot PK \quad \dots \dots \dots (16). \end{aligned}$$

So that the velocity v is that which would be due to falling freely from the level of the directrix, the depth PK below the directrix being called the *head* or *impetus* of the velocity v .

Denoting by X the range in feet, and T the time of flight in seconds, over a horizontal line Ox through O , obtained by putting $y = 0$ in (6) and (8), then

$$T = \frac{2V \sin \alpha}{g} \quad \dots \dots \dots (17),$$

$$X = \frac{2V^2 \sin \alpha \cos \alpha}{g} = \frac{V^2 \sin 2\alpha}{g} = p \tan \alpha \quad \dots \dots \dots (18).$$

Thus for a given value of V , the range X is a maximum when

$$\sin 2\alpha = 1, \quad \text{or} \quad \alpha = 45^\circ.$$

Generally

$$\sin 2\alpha = \frac{gX}{V^2} \quad \dots \dots \dots (19),$$

giving the elevation α required for a range X ; or

$$V^2 = gX \operatorname{cosec} 2\alpha \quad \dots \dots \dots (20),$$

giving the initial velocity V required for a range X with elevation α , as in Chapter II, page 68 (Part I).

From Equation (8), since $y = 0$, when $x = 0$ or X ,

$$\begin{aligned} y &= x \tan \alpha \left(1 - \frac{x}{X}\right), \\ \tan \alpha &= \frac{y}{x \left(1 - \frac{x}{X}\right)} = \frac{y}{x} + \frac{y}{X - x}, \end{aligned}$$

giving

$$\tan \alpha = \tan \theta + \tan \phi \quad \dots \dots \dots (21),$$

where θ , ϕ are the angular elevations of the point P, as seen from O and R, the beginning and end of the range; this theorem is useful in determining the elevation required with a given range X, so as to clear an obstacle, a wall or rampart, of height y at a distance x from O (see fig. 1).

Denoting, as before, the whole time of flight over the range on a horizontal plane through O by T, and the time of flight from O to P by t ; denoting also the time of flight from P down to the ground again by t' , then

$$t + t' = T \quad (22)$$

and

$$V \sin \alpha = \frac{1}{2}gT;$$

so that (6) may be written

$$\begin{aligned} y &= \frac{1}{2}gTt - \frac{1}{2}gt^2 \\ &= \frac{1}{2}gt(T-t) = \frac{1}{2}gtt' \quad (23); \end{aligned}$$

Colonel Sladen's formula, useful in plotting approximately points on a trajectory in direct fire, even when the resistance of the air is taken into account, but where the vertical component of the resistance is insensible.

At the vertex A, $t = t' = \frac{1}{2}T$, and the height of the vertex

$$H = \frac{1}{8}gT^2 = 4T^2 = (2T)^2 \quad (24),$$

taking $g = 32$; hence the practical rule:—

"The square of twice the time of flight in seconds is the height of the vertex of the trajectory in feet."

Thus if the time of flight is 5 seconds, the height of the vertex is 100 feet; if

$$T = 0.1 \text{ sec.}, H = \frac{1}{25} \text{ foot, less than } \frac{1}{2} \text{ inch; } V = 3000 \text{ f/s.}$$

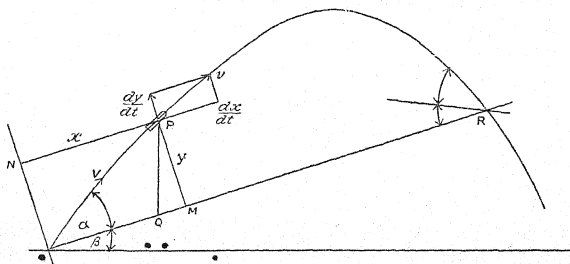
$$T = 60 \text{ secs.}, H = 14,400 \text{ feet.}$$

When firing up or down a slope Oe, at an inclination of β to the horizon, the equations of motion are

$$\frac{d^2x}{dt^2} = -g \sin \beta \quad (25);$$

$$\frac{d^2y}{dt^2} = -g \cos \beta \quad (26);$$

Fig. 2.



and, integrating twice,

$$\frac{dc}{dt} = V \cos \alpha - gt \sin \beta;$$

$$\frac{dy}{dt} = V \sin \alpha - gt \cos \beta;$$

$$x = Vt \cos \alpha - \frac{1}{2}gt^2 \sin \beta. \quad (27);$$

$$y = Vt \sin \alpha - \frac{1}{2}gt^2 \cos \beta. \quad (28);$$

and α now denotes the *tangent* elevation of the gun, the *quadrant* elevation being $\alpha + \beta$.

Then, with the preceding notation,

$$T = \frac{2V \sin \alpha}{g \cos \beta} \quad (29);$$

$$\begin{aligned} X &= \frac{2V^2 \sin \alpha \cos \alpha}{g \cos \beta} - \frac{2V^2 \sin^2 \alpha \sin \beta}{g \cos^2 \beta} \\ &= 4a \frac{\sin \alpha \cos (\alpha + \beta)}{\cos^2 \beta} \\ &= 2a \frac{\sin (2\alpha + \beta) - \sin \beta}{\cos^2 \beta} \quad (30), \end{aligned}$$

if a denotes $\frac{1}{2}V^2/g$, head or *impetus* of the velocity V .

Thus for given V or a , and a given slope β , the range X is a maximum when

$$\sin (2\alpha + \beta) = 1,$$

$$2\alpha + \beta = 90^\circ,$$

$$\alpha = 45^\circ - \frac{1}{2}\beta,$$

a direction which bisects the angle between the slope and the vertical.

Also, as before,

$$y = \frac{1}{2}gt^2 \cos \beta \quad (31),$$

so that the distance from the slope OR , measured vertically, is $\frac{1}{2}gt^2$.

The parabolic theory is sometimes useful in assigning limits within which the real trajectory in a resisting medium must lie; an example showing this is given in Part I, p. 71.

The area of the parabolic segment OPR (fig. 1), where the equation $x^2 = py$ represents the curve with origin at A , and the axis of y downwards is

$$2 \int_0^y x \, dy,$$

where the upper limit is the height of A above OR .

Therefore

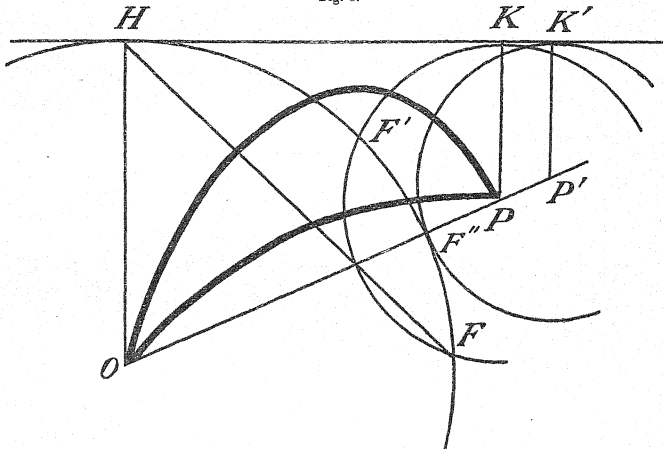
$$\begin{aligned} \text{area of segment} &= 2 \int_0^y p^{\frac{1}{2}} y^{\frac{1}{2}} \, dy \\ &= 2p^{\frac{1}{2}} y^{\frac{3}{2}} = \frac{2}{3}xy. \end{aligned}$$

So that the area of the parabolic segment is two-thirds that of the circumscribing rectangle, and therefore the mean height of the ordinates of the arc is two-thirds of the maximum ordinate; hence the average height of a projectile in a parabolic trajectory is two-thirds of the height of the vertex. Captain James M. Ingalls, U.S.A., has pointed out the

practical use of this result for the tenuity of the air at great altitudes in a long trajectory, as showing that a good approximation is obtained to the average density of the air traversed by the projectile at a height in the atmosphere of two-thirds of the estimated height of the vertex.

To determine geometrically the directions of projection from a point O , with given velocity due to the head OH , so as to strike a point P , describe a circle with centre P and radius PK , touching the horizontal line HK through H in K ; then if this circle cuts the circle with centre O and radius OH in F and F' , the required directions are perpendicular to HF and HF' , or bisect the angles HOF and HOF' ; and these directions, therefore, are equally inclined to the bisector of the angle POH , which is the direction of projection for maximum range on the plane OP ; and if the circles do not intersect, the point P is out of range.

Fig. 3.



These results follow because

$$\begin{aligned} FO &= OH, & FP &= PK; \\ F'O &= OH, & F'P &= PK; \end{aligned}$$

so that F and F' are the foci of the parabolas that can be drawn passing through O and P , and having the common directrix HK .

The lower parabola with the smaller angle of projection is that required for *direct fire*, and the upper parabola for *high angle or mortar fire*.

When the points F and F' coalesce in F'' the circles touch, and the point P is out of the range attainable from O in the direction OP , when it is beyond P' , where

$$OP' = OH + P'K' = P'K'',$$

and

$$K''K' = OH,$$

so that the locus of points just within range is the parabola whose focus is O and vertex H , and the space inside the paraboloid generated by the revolution of this parabola about its axis is the space which can be covered from O with the given velocity of projection; points outside this paraboloid being out of range from O .

Suppose, for instance, that OP is the trace of an inclined plane through O , this plane will cut the paraboloid in an ellipse with focus at O , and this ellipse will be the area covered on the inclined plane OP by a gun at O .

The section of the paraboloid made by a vertical plane PK will be a parabola; this will be, for instance, the area covered on a vertical wall PK by a fire engine at O , supposing OH is the greatest height to which the engine can send the jet; and to attain the boundary of the area, the jet must be aimed at points on the wall lying on the horizontal straight line at a height $2OH$, twice the *impetus* or *head* of the velocity.

The following treatment of a parabolic arc will be found useful in the next chapter on high angle fire.

Let OP , fig. 4, be an arc of a parabola, vertex at A , the direction of a projectile's trajectory at O is ϕ and at P it is θ .

Let the equation of the parabola of which OP is an arc be $x^2 = py$.

Any point on this is $x = \lambda p$, $y = \lambda^2 p$, where λ is the variable; at any point P ,

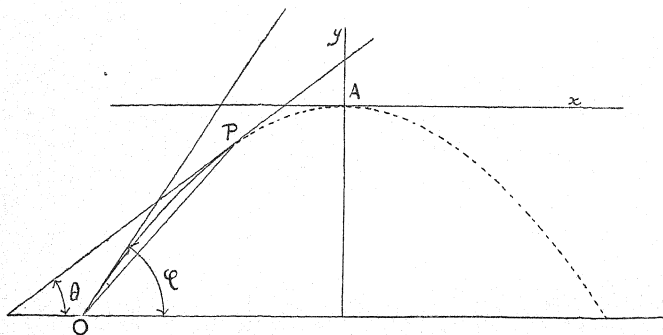
$$\frac{dy}{dx} = \tan \theta = \frac{2\lambda p}{p} = 2\lambda,$$

also

$$\frac{dx}{d\theta} = \frac{dx}{d\lambda} \cdot \frac{d\lambda}{d\theta} = \frac{p \cdot \sec^2 \theta}{2},$$

$$\frac{ds}{d\theta} = \frac{ds}{dx} \cdot \frac{dx}{d\theta} = \sec \theta \cdot \frac{p}{2} \sec^2 \theta = \frac{p}{2} \sec^3 \theta.$$

Fig. 4.



so that the mean value from ϕ to θ of the secant of the angle of direction is

$$\sec \eta = \frac{s}{x} = \frac{\int_{\phi}^{\theta} \frac{ds}{d\theta} d\theta}{\int_{\phi}^{\theta} \frac{dx}{d\theta} d\theta} = \frac{\int_{\phi}^{\theta} \sec^3 \theta d\theta}{\int_{\phi}^{\theta} \sec^2 \theta d\theta}.$$

Integrating by parts,

$$\int \sec^3 \theta d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \int \sec \theta d\theta,$$

and

$$\int \sec \theta \, d\theta = \int \frac{d\theta}{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}} = \int \frac{\sec^2 \frac{\theta}{2} \, d\theta}{1 - \tan^2 \frac{\theta}{2}},$$

put

$$x = \tan \frac{\theta}{2}, \quad dx = \frac{1}{2} \sec^2 \frac{\theta}{2} \, d\theta,$$

and

$$\int \sec \theta \, d\theta = \log \frac{1+x}{1-x} = \log \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} = \log (\sec \theta + \tan \theta).$$

Therefore

$$\sec \eta = \frac{i(\phi) - i(\theta)}{\tan \phi - \tan \theta},$$

where

$$i(\phi) = \frac{1}{2} \tan \phi \sec \phi + \frac{1}{2} \log (\sec \phi + \tan \phi).$$

This function is tabulated in Table VI; it is useful also in the calculation of a trajectory when the quadratic law of resistance is assumed.

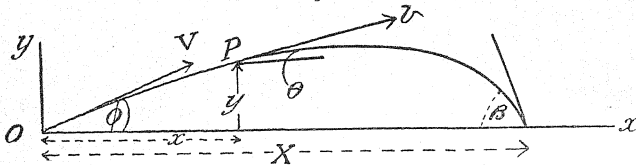
CHAPTER IV.

HIGH ANGLE FIRE.

WHEN the curvature of the trajectory becomes considerable, as in high angle and curved fire, the methods of Chapter I, Part II, for direct fire require modification. We proceed then to consider the equations of motion of a projectile in a resisting medium, when projected with given velocity in a given direction; and to show how these equations, where otherwise intractable, can be slightly modified so as to give tangible practical results.

The motion is referred to two co-ordinate axes, Ox and Oy , drawn horizontally and vertically in the plane of fire through O , the muzzle of the gun; the resistance of the air is taken to act in the opposite direction to the motion of the centre of gravity of the projectile, so that there is no cause tending to draw the shot out of its original plane of fire, and to cause drift or deviation: this subsidiary effect must be considered separately.

Fig. 1.



Let the velocity at O be V f/s and the direction of the shot be ϕ (in radians of circular measure); let x, y denote (in feet) the co-ordinates of the C.G. of the shot at P after a time of flight of t seconds, and θ the inclination (radians) at P , so that θ is the angle which the tangent to the trajectory at P makes with the horizontal Ox ; the velocity at P being v f/s, and the retardation r feet per second per second,

$$\frac{d^2x}{dt^2} = \frac{d(v \cos \theta)}{dt} = -r \cos \theta \quad \dots \dots \dots (1),$$

$$\frac{d^2y}{dt^2} = \frac{d(v \sin \theta)}{dt} = -g - r \sin \theta \quad \dots \dots \dots (2).$$

Eliminating r from (1) and (2)

$$\cos \theta d(v \sin \theta) - \sin \theta d(v \cos \theta) = -g \cos \theta dt,$$

multiply both sides by $\frac{\sec^2 \theta}{v}$,

$$\frac{1}{v \cos \theta} d(v \sin \theta) - \frac{\sin \theta}{v \cos^2 \theta} d(v \cos \theta) = -\frac{g dt}{v \cos \theta},$$

$$d\left(\frac{v \sin \theta}{v \cos \theta}\right) = -g \frac{dt}{v \cos \theta},$$

$$g dt = -v \frac{d\theta}{\cos \theta} \quad \dots \dots \dots (3).$$

This formula has already been proved in Chapter II, page 35, where it was shown that $v \frac{d\theta}{dt} = -g \cos \theta$; see also *Notes on Dynamics* (1908), by Sir G. Greenhill, F.R.S., pp. 123, 124.

To obtain dx , dy , $d(v \cos \theta)$, ds in terms of $d\theta$, substitute the value of dt from (3) in

$$\frac{dx}{dt} = v \cos \theta, \quad \frac{dy}{dt} = v \sin \theta, \quad \frac{ds}{dt} = v,$$

and in (1) where $\frac{d(v \cos \theta)}{dt} = -r \cos \theta$.

From (3)

$$g dt = -v \frac{d\theta}{\cos \theta} \quad (4),$$

and

$$g dx = -v^2 d\theta \quad (5),$$

$$g dy = -v^2 \tan \theta d\theta \quad (6),$$

$$g d(v \cos \theta) = +v \frac{F(v)}{C} d\theta \quad (7);$$

also

$$g d(\tan \theta) = g \sec^2 \theta d\theta \quad (8).$$

In (7) $\frac{F(v)}{C}$ has been substituted for r , and is obtained thus:—

$$\frac{r}{g} = \frac{R}{w} = \frac{n d^2 p}{w} = \frac{p}{C} \text{ (see p. 11);}$$

also

$$pg = Cr = K \left(\frac{v}{1000} \right)^3 = F(v),$$

so that $\frac{F(v)}{g} = p$ = resistance of the air in pounds to a 1-inch projectile moving with velocity v f/s, under standard conditions, also

$$\frac{F(v)}{C} = r.$$

Equations (4) to (8) have θ as the independent variable; to turn these so that the independent variable be the horizontal component of velocity, $v \cos \theta = v_1$, make use of (7), from which,

$$d\theta = \frac{Cg}{v_1^2 F(v_1)} dv_1 \quad (9),$$

then equations (4) to (8) become

$$dt = -\frac{C \sec \theta}{F(v_1 \sec \theta)} \cdot dv_1 \quad (10),$$

$$dx = -\frac{Cv_1 \sec \theta}{F(v_1 \sec \theta)} dv_1 \quad (11),$$

$$dy = -\frac{Cv_1 \sec \theta \tan \theta}{F(v_1 \sec \theta)} \cdot dv_1 \quad (12),$$

$$\theta = +\frac{Cg}{v_1 \sec \theta F(v_1 \sec \theta)} \cdot dv_1 \quad (13),$$

$$d(\tan \theta) = +\frac{Cg \sec \theta}{v_1 F(v_1 \sec \theta)} \cdot dv_1 \quad (14).$$

Equations (10) to (14) have $r_1 (= r \cos \theta)$ as the independent variable; and integrating these, supposing V_1 the initial value of v_1 and making V_1 the upper limit so as to cancel the negative sign,

$$t = C \int_{r_1}^{V_1} \frac{\sec \theta}{F(v_1 \sec \theta)} dv_1 \quad (15),$$

$$x = C \int_{r_1}^{V_1} \frac{v_1 \sec \theta}{F(v_1 \sec \theta)} dv_1 \quad (16),$$

$$y = C \int_{r_1}^{V_1} \frac{v_1 \sec \theta \tan \theta}{F(v_1 \sec \theta)} dv_1 \quad (17),$$

$$\phi - \theta = C \int_{r_1}^{V_1} \frac{g}{v_1 \sec \theta F(v_1 \sec \theta)} dv_1 \quad (18),$$

$$\tan \phi - \tan \theta = C \int_{r_1}^{V_1} \frac{g \sec \theta}{v_1 F(v_1 \sec \theta)} dv_1 \quad (19).$$

Equations (15) to (17) cannot be integrated as they stand, because the relation between θ and v_1 is not known.

But, as originally pointed out by Euler, these difficulties can be turned if we notice that in the ordinary trajectories in practice the quantities θ , $\cos \theta$, and $\sec \theta$ vary so slowly that they may be replaced by their mean values η , $\cos \eta$, and $\sec \eta$; especially if in the calculations the trajectory, when considerable, is divided up into arcs of small curvature (the curvature of an arc is defined as the angle between the tangents or normals at the ends of the arc).

In equations (15), (16), (18), (19) the mean angle η enters only in the form of $\sec \eta$ or $\cos \eta$, slowly varying quantities for moderate values of η , so that η need not be determined with great accuracy.

According to Didion (*Traité de Balistique*, p. 119), the mean value of $\sec \eta$ is obtained by supposing the arc from ϕ to θ a portion of a parabola, with a vertical axis, and that

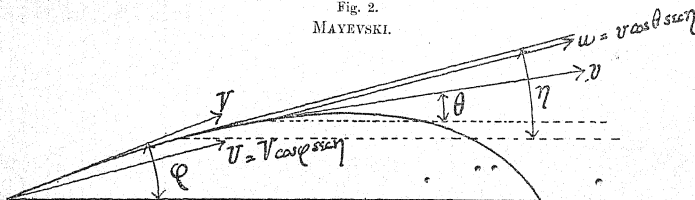
$$\sec \eta = \frac{s}{x} = \frac{\int_{\theta}^{\phi} \frac{ds}{d\theta} d\theta}{\int_{\theta}^{\phi} \frac{dx}{d\theta} d\theta} = \frac{i(\phi) - i(\theta)}{\tan \phi - \tan \theta} \quad (\text{see p. 59}) \quad (20),$$

where $i(\phi) = \frac{1}{2} \tan \phi \sec \phi + \frac{1}{2} \log (\sec \phi + \tan \phi)$, a function tabulated in Table VI. Replacing the variable angle θ by some mean angle η , and introducing the Mayevski pseudo-velocities U and u , defined by

$$\left. \begin{aligned} U &= V_1 \sec \eta = V \cos \phi \sec \eta, \\ u &= v_1 \sec \eta = r \cos \theta \sec \eta, \\ dr_1 &= \cos \eta du, \end{aligned} \right\} \quad (21).$$

so that

Fig. 2.
MAYEVSKI.



the equations (15), (16), (18), (19) may now be written (u being the independent variable)

$$t = C \int_u^v \frac{1}{F(u)} du \quad (22),$$

$$x = C \cos \eta \int_u^v \frac{u}{F(u)} du \quad (23),$$

$$\phi - \theta = C \cos \eta \int_u^v \frac{g}{uF(u)} du \quad (24),$$

$$\tan \phi - \tan \theta = C \sec \eta \int_u^v \frac{g}{uF(u)} du \quad (25).$$

But $F(u) = Cr$, see p. 61 of this chapter, and from Chapter II Cr is of the form $[A] u^n$, or, shortly, Au^n (see p. 32); hence, as in Chapter II for direct fire, these integrals are the same as those which gave the functions T , S , and I , but with the pseudo-velocity u as the argument instead of the real velocity r .

Equations (22) to (25) become

$$t = C \int_u^v \frac{1}{Au^n} du = T(U) - T(u) \quad (26),$$

$$\frac{x}{C} = \cos \eta \int_u^v \frac{1}{Au^{n-1}} du = \cos \eta \{S(U) - S(u)\} \quad (27),$$

$$\frac{\phi - \theta}{C} = \cos \eta \int_u^v \frac{g}{Au^{n+1}} du = \cos \eta \{I(U) - I(u)\} \quad (28),$$

$$\frac{\tan \phi - \tan \theta}{C} = \sec \eta \int_u^v \frac{g}{Au^{n+1}} du = \sec \eta \{I(U) - I(u)\} \quad (29).$$

If ϕ and θ are expressed in degrees, then

$$\frac{\phi^\circ - \theta^\circ}{C} = \frac{180}{\pi} \cos \eta \{I(U) - I(u)\} \quad (30).$$

It will be noticed that η cannot be exactly the same mean angle in equations (27), (28), and (29); thus it is obviously different in equations (28) and (29); but, when dealing with arcs of small curvature, the discrepancies due to using the same η throughout will be insensible.

Equations (26) to (29) are in the form employed by General Mayevski, who modified Siacci's original equations by introducing Euler's mean angle η ; the direction of the pseudo-velocity " u " is also different to Siacci's u , as will be seen when his method is demonstrated.

THE ALTITUDE FUNCTION $A(u)$.

Replacing $\tan \theta$ by $\frac{dy}{dx}$ in equation (29),

$$\tan \phi - \frac{dy}{dx} = C \sec \eta \{I(U) - I(u)\} \quad (31).$$

Integrate with respect to x over the arc considered,

$$x \tan \phi - y = C \sec \eta \left\{ x I(U) - \int_0^x I(u) dx \right\}.$$

But from equation (23)

$$\frac{dx}{du} = -C \cos \eta \frac{u}{F(u)},$$

the negative sign is taken because x increases as u decreases, and making U the upper limit,

$$x \tan \phi - y = Cx \sec \eta I(U) - C^2 \int_U^u \frac{uI(u)}{F(u)} du \quad \dots \quad (32).$$

In Siacci's notation

$$\int_U^u \frac{uI(u)}{F(u)} du = A(U) - A(u),$$

where $A(u)$ is called the *altitude function*.

Dividing (32) by x ,

$$\tan \phi - \frac{y}{x} = C \sec \eta I(U) - C^2 \frac{A(U) - A(u)}{x}.$$

But from (27),

$$x = C \cos \eta \{S(U) - S(u)\},$$

therefore

$$\frac{y}{x} = \tan \phi - C \sec \eta \left\{ I(U) - \frac{A(U) - A(u)}{S(U) - S(u)} \right\} \quad \dots \quad (33).$$

This is Mayevski's equation with the altitude function A , in which $\sec \eta$ and the initial and final pseudo-velocities U and u for any arc depend upon ϕ and θ , the initial and final inclinations to the horizontal of the tangents to the arc.

By means of Mayevski's method high-angle trajectories can be calculated with great exactness by breaking such trajectories into small arcs.

The equations are

$$C = f \frac{w}{\kappa \sigma \tau_0 d^2},$$

where τ_0 is the tenuity factor at the earth's surface, f is the factor which corrects for the mean height of the projectile above the ground in any particular arc of the trajectory,

$$\left. \begin{aligned} x \text{ (in feet)} &= C \cos \eta [S(U) - S(u)], \\ t \text{ (in seconds)} &= C [T(U) - T(u)], \\ \tan \theta &= \tan \phi - C \sec \eta [I(U) - I(u)], \\ \frac{y}{x} &= \tan \phi - C \sec \eta \left[I(U) - \frac{A(U) - A(u)}{S(U) - S(u)} \right] \end{aligned} \right\} \quad \dots \quad (33A)$$

U and u are the pseudo-velocities at the commencement and end of any arc, so that

$$\left. \begin{aligned} U &= V \cos \phi \sec \eta \\ u &= v \cos \theta \sec \eta \end{aligned} \right\} \quad \dots \quad (33B).$$

For very small arcs, no matter what the elevation be, these equations are accurate. It is, however, found that the work of calculating a high-angle trajectory by means of arcs in which the total change of direction is as small as 1° is much simplified, for the following reasons:—

1. The equations are strictly accurate.
2. The mean angle η may be taken as $\frac{\phi + \theta}{2}$ without error.

3. After obtaining three values of the altitude factor f , the succeeding values of f can be found by plotting a curve of f in conjunction with θ , thus obviating the necessity of calculating the arc more than once. The first three or four arcs are re-calculated until no change in the elements is obtained; a second re-calculation is always necessary.

4. A general table can be constructed for

$$\frac{\tan \phi - \tan \theta}{\sec \eta} = E, \text{ say,}$$

this simplifies the calculations; thus

Arc.	log E.
80°-79°	2 98222
79°-78°	2 94303
78°-77°	2 90723
&c.	

5. The altitude formula

$$\frac{y}{x} = \tan \phi - C \sec \eta \left[I(U) - \frac{A(U) - A(u)}{S(U) - S(u)} \right]$$

can be replaced with great exactness by

$$\frac{y}{x} = \tan \eta.$$

The example of a high-angle trajectory for the 18-pr. Q F., which is worked out below in tabular form, was calculated by Captain J. F. R. N. Maitland-Addison, R.A., and an explanation of his method now follows:—

*Operations for Calculating a High-Angle Trajectory by means of small arcs, employing
Mayer's Equations.*

Before commencing calculations a curve of f should be drawn for all the heights likely to be obtained, the arguments being the barometer and thermometer values at the ground. Range tables are compiled for barometer 30 inches, thermometer 60° F. See fig. 1, p. 7 of Chapter I, for a curve of f , selecting the middle one or else the table on p. 7.

1. $C_0 = \frac{w}{\kappa \sigma d^2}$, ignoring f .

2. Calculate the muzzle pseudo-velocity, U .

3. Calculate the remaining pseudo-velocity, u .

4. Calculate the horizontal distance = x .

5. Calculate the height, $y = x \tan \eta$.

6. Find f from the curve and the value got for y .

7. Include f in ballistic coefficient, and thus $C_1 = f \frac{w}{\kappa \sigma \cdot d^2}$.

8. Re-calculate arc as before.

Example.—It will be observed that for the arc 80°-79° the value of f obtained in the re-calculation of the arc is the same as that first obtained, so that the elements obtained in the second calculation will not be altered by a third calculation, which is therefore unnecessary.

9. Carry out the re-calculation for the first three or four arcs as above.

10. Plot the values of f thus obtained; values of f for other arcs can be obtained with (9263)

accuracy by extrapolation for a change of 1° of arcs, thus rendering a re-calculation of any arc unnecessary.

Example :—Arc $76^\circ-75^\circ$;

f by extrapolation = 1.444 ;

f as obtained from the calculated value of $y = 1.448$.

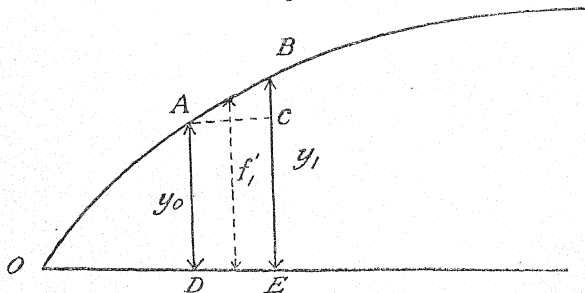
There is no accumulation of error in f , as this latter value is now plotted instead of the former and the curve extrapolated for the next arc.

The value of f for succeeding arcs is taken for a height at the half-way point of the arc, thus if

OA represents 1st arc,

AB „ 2nd „

Fig. 3.



then f_1 for the 2nd arc is that obtained for a value of $y = AD + \frac{BC}{2}$, and so on for other arcs.

Proceeding as above shown, the calculations are very quickly performed for any number of arcs.

11. The time of flight for each arc is then calculated.
12. For balloon firing the angles of sight to the target are given by

$$\theta = \tan^{-1} \frac{y}{x}.$$

13. The tangent elevations required are given by

$$\text{T.E.} = \phi - \theta.$$

14. The ranges along the lines of sight are given by

$$X (\text{feet}) = \sqrt{x^2 + y^2}.$$

TABLE OF $\frac{\tan \phi - \tan \theta}{\sec \eta} = E.$

Arc.	log cos η .	log E.	Arc.	log cos η .	log E.
$\phi \quad \theta$			$\phi \quad \theta$		
80-79	I·26063	2·98222	40-39	I·88741	2·35457
79-78	I·29966	2·94303	39-38	I·89354	2·34823
78-77	I·33534	2·90722	38-37	I·89947	2·34258
77-76	I·36819	2·87429	37-36	I·90518	2·33670
76-75	I·39860	2·84378	36-35	I·91069	2·33131
75-74	I·42690	2·81543	35-34	I·91599	2·32592
74-73	I·45334	2·78892	34-33	I·92111	2·32078
73-72	I·47814	2·76408	33-32	I·92603	2·31590
72-71	I·50148	2·74070	32-31	I·93077	2·31116
71-70	I·52350	2·71865	31-30	I·93532	2·30657
70-69	I·54433	2·69781	30-29	I·93970	2·30218
69-68	I·56408	2·67802	29-28	I·94390	2·29801
68-67	I·58284	2·65926	28-27	I·94793	2·29389
67-66	I·60070	2·64134	27-26	I·95179	2·29025
66-65	I·61773	2·62432	26-25	I·95549	2·28631
65-64	I·63398	2·60808	25-24	I·95902	2·28289
64-63	I·64953	2·59248	24-23	I·96240	2·27963
63-62	I·66441	2·57759	23-22	I·96562	2·27610
62-61	I·67866	2·56334	22-21	I·96868	2·27339
61-60	I·69234	2·54967	21-20	I·97159	2·27022
60-59	I·70547	2·53651	20-19	I·97435	2·26749
59-58	I·71809	2·52393	19-18	I·97696	2·26499
58-57	I·73022	2·51169	18-17	I·97942	2·26250
57-56	I·74189	2·50012	17-16	I·98174	2·26004
56-55	I·75313	2·48881	16-15	I·98391	2·25807
55-54	I·76395	2·47803	15-14	I·98594	2·25592
54-53	I·77439	2·46759	14-13	I·98783	2·25406
53-52	I·78445	2·45747	13-12	I·98958	2·25227
52-51	I·79415	2·44775	12-11	I·99119	2·25078
51-50	I·80351	2·43849	11-10	I·99267	2·24915
50-49	I·81254	2·42933	10-9	I·99410	2·24806
49-48	I·82126	2·42071	9-8	I·99520	2·24659
48-47	I·82968	2·41220	8-7	I·99627	2·24571
47-46	I·83781	2·40410	7-6	I·99720	2·24468
46-45	I·84566	2·39626	6-5	I·99800	2·24376
45-44	I·85324	2·38866	5-4	I·99866	2·24318
44-43	I·86056	2·38131	4-3	I·99919	2·24272
43-42	I·86763	2·37441	3-2	I·99959	2·24238
42-41	I·87446	2·36736	2-1	I·99985	2·24189
41-40	I·88105	2·36091	1-0	I·99998	2·24202

18. *md.* Q.F.

Angle of Projection $\phi = 80^\circ$; M.V. = 1590 f/s; $\tau_0 = 1$ and $\sigma\tau$ is Unity, giving $C_0 = (3 \cdot 3)^3$

	Are . . .	80°-79°	79°-78°	78°-77°	77°-76°	76°-75°	75°-74°	74°-73°	73°-72°
$C = \frac{m}{nd}$	$\log C_0$ $\log f$ $\log C$	0.23015 0.0 0.23015	0.23015 0.07882 0.30897	0.23015 0.11227 0.34242	0.23015 0.12611 0.34276	0.23015 0.14418 0.35872	0.23015 0.15058 0.37754	0.23015 0.15558 0.39173	0.23015 0.16058 0.40503
$U = V \cos \phi \sec \eta$ $\eta = \frac{\phi + \theta}{2}$	$\log V$ $\log \cos \phi$ $\log \sec \eta$ $\log U$	3.20140 1.23067 0.73687 1515	3.09108 1.23000 0.73064 1012	2.95172 1.17888 0.69181 865	2.80233 0.93209 0.48181 770	2.65783 0.78398 0.30140 686	2.51811 0.61380 0.27310 637	2.37813 0.44034 0.24646 549	
$I(\phi) = I(U) - C$	$\log E$ $\log C$ $\log A$ $\log I$ $I(U)$ $I(\phi)$	2.98222 0.23015 2.75207 0.95650 0.92828 1005	2.94808 0.23013 2.71807 0.94806 0.80184 949	2.90722 0.30857 2.59825 0.93671 0.75797 827	2.87169 0.31276 2.56153 0.92409 0.64735 746	2.83478 0.33877 2.53406 0.90245 0.53872 680	2.81543 0.40769 2.40774 0.87257 0.41722 626	2.78862 0.42188 2.36704 0.84663 0.29048 581	
$x = O \cos \eta$ [S(U) - S(ϕ)]	S(U) S(η) S(ϕ) $\log A$ (S) $\log S$ $\log \phi$ $\log \eta$ $\log \frac{\phi}{x}$	3.9370 3.9370 2.7852 3.44489 3.29663 2.39663 2.33517 861	3.8505 3.84457 1.193 3.40130 2.90665 2.70693 512	3.7606 3.7121 0.852 0.9897 1.33534 2.58034 385	3.6831 2.9727 0.603 0.94276 1.36819 2.40170 310	3.6031 2.9753 0.378 0.9857 1.38840 2.41814 262	3.5283 2.9519 0.308 0.9769 1.42600 2.32413 211	3.4557 2.9222 0.235 0.9507 1.45834 2.24629 181	
$y = x \tan \eta$	$\log x$ $\log \tan \eta$ $\log y$ $\log \phi + \frac{\theta}{2}$ $y = x \tan \eta$	2.93217 0.73203 0.73203 2.93217 1.068	2.70658 0.69154 0.40147 0.40112 5981	2.55017 0.65424 0.37158 0.37158 5819	2.40170 0.61375 0.34345 0.34345 5681	2.41814 0.61865 0.31485 0.31485 5411	2.32413 0.57501 0.27614 0.27614 5169	2.24629 0.52840 0.23666 0.23666 4907	
$v = w \sec \theta \cos \eta$	$\log w$ $\log \sec \theta$ $\log v$	3.04415 1.25063 0.71940 3.76418 1057	2.74718 0.25066 0.48512 2.95891 501	2.61008 0.34308 0.04731 0.34308 739	2.47818 0.43354 0.04731 0.43354 721	2.3851 0.50700 0.28311 0.50700 688	2.29657 0.42680 0.24606 0.42680 607	2.21848 0.34331 0.20026 0.34331 528	

* These values of f are obtained by extrapolation, making use of figures given by previous ones.

Calculation for Time of Flight.

Arc.	80°-79°.	79°-78°.	78°-77°.	77°-76°.
U	1515	1012	865	770
u	1107	940	830	747
T(U)	109·083	105·757	102·754	99·968
T(u)	107·006	104·461	101·796	99·205
$T(U) - T(u)$	2·077	1·296	0·958	0·763
$\log [T(U) - T(u)]$	0·31744	0·11261	1·98137	1·88252
$\log C$	0·25872	0·30897	0·34242	0·36877
$\log t$	0·57616	0·42158	0·32379	0·25129
t	3·77	2·64	2·11	1·78

Calculation of the Angle of Sight, Tangent Elevation, and Range along the Line of Sight.

Arc.	80°-79°.	78°-77°.	76°-75°.	73°-72°.
$\log y$	3·67403	3·95347	4·05354	4·11879
$\log x$	2·94201	3·24895	3·37144	3·46090
$\log \tan \theta$	0·73202	0·70452	0·68210	0·65789
θ	79° 30'	78° 50'	78° 16'	77° 36'
$\phi - \theta$	0° 30'	1° 10'	1° 44'	2° 24'
$(x^2 + y^2)^{1/2}$	4802	9158	11554	13460
Also t	3·77	8·52	11·82	15·05
Remaining velocity	1057	799	658	528

18-PR. Q.F. Q.E. = 80°.

TABULATED RESULTS (TABLE I).

Range in yards along line of sight.	Angle of sight, θ .	Height in feet.	Time of flight.	Remaining velocity, f/s.
1601	79 30	4721	3·77	1057
3053	78 50	8984	8·52	799
3851	78 16	11312	11·82	658
4487	77 36	13146	15·05	528

Plotting these ranges and angles of sight and other functions with range, the following results are obtained :—

TABLE II FOR 18-PR. Q.E. = 80°.

Range in yards along line of sight.	Angle of sight.	Height in feet.	Time of flight.	Remaining velocity.	Tangent elevation.
0	80 0	—	—	1590	0 0
500	79 52	1480	1·01	1382	0 8
1000	79 43	2975	2·18	1216	0 17
2000	79 20	5880	4·95	976	0 40
3000	78 52	8815	8·32	809	1 8

For the construction of a complete range table for firing at balloons or such like targets it is sufficient to calculate trajectories in arcs for $\phi = 80^\circ, 60^\circ, 40^\circ, 20^\circ, 0^\circ$, and then all the intermediate elements can be found by plotting.

Thus Tables I and II for $\phi = 80^\circ$ are also calculated for $\phi = 60^\circ, 40^\circ, 20^\circ$, and 0° , and then the final range table is obtained by selecting the elements for any particular range as argument from each of the five tables of Table II; plotting the elements and reading for even angles of sight. For other ranges and angles of sight the elements can be obtained by further plotting until the intervals are sufficiently close for interpolating by eye.

SIACCI'S METHOD.

Starting with the general equations of motion (4), (5), (7), and (8),

$$\left. \begin{aligned} g \, dt &= -v \frac{d\theta}{\cos \theta} \quad \dots \dots \dots (4), \\ g \, dx &= -v^2 d\theta \quad \dots \dots \dots (5), \\ g \, d(v \cos \theta) &= v \frac{F(v)}{C} d\theta \quad \dots \dots \dots (7), \\ g \, d(\tan \theta) &= g \sec^2 \theta \, d\theta \quad \dots \dots \dots (8). \end{aligned} \right\}$$

Writing v_1 for $v \cos \theta$ then as before from (7), $d\theta = \frac{Cg}{vF(v)} dv_1$, substitute in (4), (5), and (8), then

$$dt = -C \frac{dv_1}{\cos \theta F(v)} \quad \dots \dots \dots (34),$$

$$dx = -C \frac{v}{F(v)} dv_1 \quad \dots \dots \dots (35),$$

$$d(\tan \theta) = Cg \frac{\sec^2 \theta}{vF(v)} dv_1 \quad \dots \dots \dots (36).$$

Now $F(v) = Cr = [A]v^n$, or writing A in short for $[A]$, $F(v) = Av^n$, hence

$$F(v) = A(v_1 \sec \theta)^n = Av_1^n \sec^n \theta$$

The equations now become

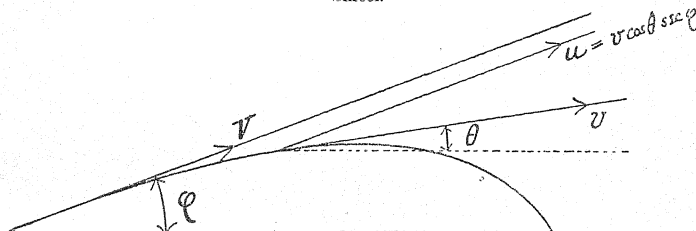
$$dt = -C \frac{\cos^{n-1} \theta}{Av_1^n} dv_1 \quad (37),$$

$$dx = -C \frac{\cos^{n-1} \theta}{Av_1^{n-1}} dv_1 \quad (38),$$

$$d(\tan \theta) = Cg \frac{\cos^{n-1} \theta}{Av_1^{n+1}} dv_1 \quad (39).$$

These three equations are not integrable, unless $\cos \theta$ is made equal to unity as is done in direct fire, or unless some approximate value is given to $\cos^{n-1} \theta$ in high-angle fire.

Fig. 4.
SIACCI.



Siacci's approximation is

$$\cos^{n-1} \theta = \cos^{n-2} \phi.$$

Siacci also makes use of a pseudo-velocity (see fig. 4) u , defined thus:—

$$\left. \begin{aligned} v \cos \theta \sec \phi &= v_1 \sec \phi = u, \\ V_1 \sec \phi &= V \cos \phi \sec \phi = V. \end{aligned} \right\}$$

It is seen from this that $v \cos \theta = u \cos \phi$, and u for any are or for the whole trajectory is the component of v (the velocity at any point) parallel to the line of departure.

Converting the three general equations of motion so that the independent variable is Siacci's pseudo-velocity u , as defined above, and making use of the approximation $\cos^{n-1} \theta = \cos^{n-2} \phi$, then, since $v_1 \sec \phi = u$, we have

$$dt = -C \frac{\cos^{n-2} \phi}{A(v_1 \sec \phi)^n} \cdot \sec^n \phi \cdot \frac{d(v_1 \sec \phi)}{\sec \phi} = -\frac{C}{A \cos \phi} \cdot \frac{du}{u^n} \quad (40),$$

$$dx = -C \frac{\cos^{n-2} \phi}{A(v_1 \sec \phi)^{n-1}} \cdot \sec^{n-1} \phi \cdot \frac{d(v_1 \sec \phi)}{\sec \phi} = -\frac{C}{A} \cdot \frac{du}{u^{n-1}} \quad (41),$$

$$d(\tan \theta) = \frac{Cg}{A} \cdot \frac{\cos^{n-2} \phi}{(v_1 \sec \phi)^{n+1}} \cdot \sec^{n+1} \phi \cdot \frac{d(v_1 \sec \phi)}{\sec \phi} = \frac{Cg}{A \cos^2 \phi} \cdot \frac{du}{u^{n+1}} \quad (42),$$

from which

$$t = \frac{C}{(n-1)A \cos \phi} \left(\frac{1}{u^{n-1}} - \frac{1}{V^{n-1}} \right) = \frac{C}{\cos \phi} \{T(V) - T(u)\} \quad (43),$$

$$x = \frac{C}{(n-2)A} \left(\frac{1}{u^{n-2}} - \frac{1}{V^{n-2}} \right) = C \{S(V) - S(u)\} \quad (44),$$

$$\tan \phi - \tan \theta = \frac{C}{\cos^2 \phi} \{I(V) - I(u)\} \quad (45).$$

In this last equation put $\tan \theta = \frac{dy}{dx}$, integrate as before, then

$$\frac{y}{x} = \tan \phi - \frac{C}{\cos^2 \phi} \left\{ I(V) - \frac{A(V) - A(u)}{S(V) - S(u)} \right\} \quad (46).$$

In this, the ballistic coefficient C is defined by Siacci thus,

$$C = \frac{1}{\beta} \frac{w}{\kappa \sigma \left(\frac{\tau_0}{f} \right) d^2} = \frac{1}{\beta} \frac{w}{nd^2},$$

where κ , σ , τ_0 , f , w , d have the usual meaning as given in Chapter I, but β is a factor which must be found to correct for various values of

$$\beta = \frac{\cos^{n-2} \phi}{\cos^{n-1} \theta} \quad (47).$$

The various values of β are shown in tabulated form below; these have been calculated by Captain J. F. R. N. Maitland-Addison, R.A.

SIACCI'S FORMULAS FOR CALCULATING IN ARCS SUMMED UP.

$$\left. \begin{aligned} C &= f \frac{w}{\beta \cdot \kappa \sigma \cdot \tau_0 \cdot d^2} \\ x &= C [S(V) - S(u)] \\ t &= C \sec \phi [T(V) - T(u)] \\ \tan \theta &= \tan \phi - \frac{C}{\cos^2 \phi} [I(V) - I(u)] \\ \frac{y}{x} &= \tan \phi - \frac{C}{\cos^2 \phi} \left[I(V) - \frac{A(V) - A(u)}{S(V) - S(u)} \right] \\ V &= V \cos \phi \sec \phi = V \\ u &= v \cos \theta \sec \phi \end{aligned} \right\} \quad (48).$$

TABLE OF FUNCTION β .

Degrees.	Range in yards.														
	1000	2000	3000	4000	5000	6000	7000	8000	9000	10,000	11,000	12,000	13,000	14,000	15,000
6	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	0.94	0.99	0.99	0.99	0.99	0.97	0.99
7	1.00	1.00	1.00	1.00	0.96	0.97	0.99	0.99	0.94	0.97	0.97	0.97	0.99	0.95	0.97
8	1.00	1.00	1.00	1.00	0.97	0.97	0.99	0.98	0.93	0.95	0.95	0.96	0.98	0.93	0.95
9	1.00	1.00	1.00	1.00	0.96	0.97	0.99	0.98	0.92	0.93	0.93	0.94	0.98	0.92	0.95
10	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.90	0.91	0.95	0.98	0.91	0.93
11	1.00	1.00	1.00	1.00	0.98	0.98	0.99	0.96	0.92	0.90	0.87	0.94	0.97	0.90	0.92
12	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.92	0.97	0.88	0.91
13	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
14	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
15	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
16	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
17	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
18	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
19	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
20	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
21	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
22	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
23	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
24	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
25	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
26	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
27	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
28	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
29	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
30	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
31	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
32	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
33	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
34	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
35	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
36	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
37	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
38	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
39	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
40	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
41	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
42	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
43	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91
44	1.00	1.00	1.00	1.00	0.96	0.97	0.98	0.95	0.90	0.88	0.84	0.90	0.95	0.88	0.91
45	1.00	1.00	1.00	1.00	0.97	0.98	0.99	0.96	0.91	0.89	0.85	0.92	0.97	0.88	0.91

(9263)

K

TABLE OF FUNCTION $\bar{\beta}$ FOR QUADRATIC LAW.

V = 0 to 800 f.s.

For all ranges within the Quadratic Law.

Degrees.	$\bar{\beta}$.	Degrees.	$\bar{\beta}$.	Degrees.	$\bar{\beta}$.	Degrees.	$\bar{\beta}$.
1	1.00	16	1.02	31	1.07	46	1.19
2	1.00	17	1.02	32	1.07	47	1.20
3	1.00	18	1.02	33	1.08	48	1.21
4	1.00	19	1.02	34	1.09	49	1.23
5	1.00	20	1.03	35	1.09	50	1.24
6	1.00	21	1.03	36	1.10	51	1.26
7	1.00	22	1.03	37	1.11	52	1.28
8	1.00	23	1.03	38	1.11	53	1.30
9	1.00	24	1.04	39	1.12	54	1.32
10	1.00	25	1.04	40	1.13	55	1.34
11	1.01	26	1.05	41	1.14	56	1.36
12	1.01	27	1.05	42	1.15	57	1.38
13	1.01	28	1.05	43	1.16	58	1.40
14	1.01	29	1.06	44	1.17	59	1.42
15	1.01	30	1.06	45	1.18	60	1.45

Captain J. F. R. N. Maitland-Addison, R.A., points out that for very SMALL arcs

$$\bar{\beta} = \frac{\cos^{n-2} \phi}{\cos^{n-1} \theta} = \frac{\cos^{n-2} \phi}{\cos^{n-1} \phi} = \sec \phi.$$

It is still more accurate, however, when working in very SMALL arcs, to put

$$\bar{\beta} = \sec \eta, \quad \text{where} \quad \eta = \frac{\phi + \theta}{2},$$

and then the formulas are of the same form as Mayevski's but the pseudo-velocities V and u are, of course, different from Mayevski's both in magnitude and *direction*.

HIGH-ANGLE TRAJECTORY IN ONE ARC BY THE USE OF SLAGG'S β FUNCTION.

Strictly speaking, all high-angle trajectories should be calculated in a series of arcs, the extent of any arc depending on its inclination with the horizontal. For this purpose Mayevski's equations can be used with advantage.

But with a very fair approximation the calculations for range, &c., can be made *in one arc*, care being taken to give to the compensating factor β its proper value.

In (48), the direction of the pseudo-velocity u is always parallel to that of V , and considering the trajectory in one arc, $y = 0$ at the striking point when this is on the same level as the firing point, so that

$$0 = \tan \phi - \frac{C}{\cos^2 \phi} \left[I(V) - \frac{A(V) - A(u)}{S(V) - S(u)} \right],$$

which gives

$$\sin 2\phi = Cu \quad \dots \dots \dots (49),$$

where

$$C = f \frac{w}{\beta \kappa \sigma \tau_0 d^2},$$

$$a = 2 \left[I(V) - \frac{A(V) - A(u)}{S(V) - S(u)} \right].$$

The following example has been worked out by Captain J. F. R. N. Maitland-Addison, R.A.:-

Example.

Find the range of a projectile fired from a 9.2-inch high-angle gun, with a muzzle velocity of 2065 f/s at an angle of 30° , the weight of the projectile being 290 lbs.

It is reasonable to assume that, over such a curved trajectory, the projectile will not have the same steadiness as it would have in direct fire. Taking a 10 per cent. of unsteadiness gives $\sigma = 1.1$, the first step is to determine a first approximation of the range, with

$$\tau = 1.00 \text{ for } \begin{cases} \text{barometer of 30 inches,} \\ \text{thermometer of } 60^\circ \text{ F.,} \\ \text{air } \frac{2}{3} \text{rds saturated.} \end{cases}$$

$$\sigma = 1.1, \quad \kappa = 1.0, \quad \tau = 1.0, \quad \beta = 1.0.$$

$$w = 290, \quad \log w = 2.46240$$

$$d = 9.2, \quad 2 \log d = 1.92758$$

$$\hline 0.53482$$

$$n = \kappa \sigma \tau = 1.1, \quad \log n = 0.04139$$

$$\log C = 0.49343$$

$$\sin 2\phi = C \cdot a.$$

$$\phi = 30, \quad \log \sin 2\phi = 1.93753$$

$$\log C = 0.49343$$

$$\log a = 1.44410$$

$$\hline a = 0.27804$$

Looking out Table VIII, for a velocity of 2065 f/s, we see that for

$$\frac{R}{C} = 4100, \quad u = 0.27533, \quad \frac{R}{C} = 4200, \quad a = 0.28745.$$

Hence, for $a = 0.27804$, the value of

$$\frac{R}{C} \text{ is } = 4100 + \frac{a-1}{1.212} 100 = 4122.4.$$

$$\log \frac{R}{C} = 3.61515$$

$$\log C = 0.49343$$

$$\log R = 4.10858$$

$$R = 12840 \text{ yards.}$$

Knowing this, it now remains to determine the values of f and β .

As an approximation, the height of the trajectory can be determined from the time of flight, and therefore the remaining pseudo-velocity must first be ascertained from

$$X = C \{S_v - S_u\}.$$

$$R = 12,840 \text{ yards,}$$

$$\log X = 4.58570$$

$$X = 38,520 \text{ feet,}$$

$$\log C = 0.49343$$

$$\log \frac{X}{C} = 4.09227$$

$$\frac{X}{C} = 12367.2$$

$$S_v = 41682.2$$

$$S_u = 29315.0$$

$$u = 730 \text{ f/s,}$$

and then

$$t = C \sec \phi \{T_v - T_u\}.$$

$$T_{2065} = 110.393$$

$$T_{730} = 98.617$$

$$\Delta T = 1.07100$$

$$\log \sec \phi = 0.06247$$

$$\log C = 0.49343$$

$$\log t = 1.62690$$

$$t = 42.35 \text{ seconds.}$$

Then, approximately, $y = 4t^2 = 7174$ feet, and the mean height $= \frac{2}{3}y = 4782$ feet. From (fig. 1) p. 7, $f = 1.16$ approximately. From the β function table it can be seen that the value of β corresponding to a range of 12,840 yards and an angle of projection of 30° is about 0.8. It is evident that this value, combined with the value of $f = 1.16$, when inserted in the ballistic coefficient, will materially increase the range, so that before proceeding to make a re-calculation, it is advisable to take different values for β and f to those above. On looking in the β table, it is seen that at the point in question β increases rapidly with the range. Hence we will take a value of say 0.9, corresponding to a range of about 14,500 yards, and since the height of the trajectory will be greater than 7174 feet, we will take $f = 1.2$: and now the ballistic coefficient must be re-constructed and the range calculated *de novo*.

As a second approximation

$$\begin{aligned}\log \frac{w}{m d^2} &= 0.49343 \\ f = 1.2, \log f &= 0.07918 \\ &\underline{0.57261} \\ \beta = 0.9, \log \beta &= 1.95424 \\ \log C &= 0.61837 \\ \log \sin 2\phi &= 1.93753 \\ \log C &= 0.61837 \\ &\underline{0.31916} \\ \log u &= 1.31916 \\ u &= 0.20853\end{aligned}$$

From Table VIII, the value of $\frac{R}{C}$ corresponding to this value of " u " is

$$\begin{aligned}\frac{R}{C} &= 3500 \\ \log \frac{R}{C} &= 3.54407 \\ \log C &= 0.61837 \\ &\underline{0.92570} \\ \log R &= 4.16244 \\ R &= 14,536 \text{ yards.}\end{aligned}$$

It will be seen that the value of $\beta = 0.9$ was a good prediction, and a further approximation is not necessary.

Actually at practice, the mean of 5 rounds fired under the conditions in this example was 14,755 yards, or about 200 yards more than the calculated range, so that the estimation of the steadiness as 1.1 was rather too large, but the error involved cannot be said to be large.

The remaining pseudo-velocity at the vertex can be calculated from :—

$$\tan \phi - \tan \theta = \frac{C}{\cos^2 \phi} \{I_V - I_u\}$$

and since $\tan \theta = 0$, at the vertex

$$\sin 2\phi = 2C \{I_V - I_u\}$$

$$I_u = I_V - \frac{\sin 2\phi}{2C}$$

$$= I_V - \frac{u}{2}$$

and

$$u = 1034 \text{ f/s.}$$

The range to the vertex is 8204 yards.

The height to the vertex is given by

$$\frac{y}{x} = \tan \phi - \frac{C}{\cos^2 \phi} \left\{ I_V - \frac{A_V - A_u}{S_V - S_u} \right\}$$

where x is the range (horizontal) to the vertex. In this case $y = 8923$ feet, so that the mean height would be 5948 feet, giving f as about 1.2; the prediction made for f was therefore good. Only experience will show how to make such predictions; and until such be acquired, several approximations must be made until no further alterations in the calculations take place.

DIRECT FIRE.

In direct fire the complete trajectory is calculated as one arc, the angle of projection ϕ is small, β may be taken as unity, also the tenuity of the air throughout is considered to be the same as at the firing point, so that $f = 1$; the value of C is therefore

$$C = \frac{w}{\kappa \sigma \tau_0 d^2}.$$

Suppose X feet denote the range in feet on a horizontal plane, obtained with an initial velocity V f/s, elevation ϕ° , and that v denotes the actual velocity at any point of the trajectory, then the pseudo-velocity u is replaced by v .

Let ω denote the angle of descent, which on a horizontal plane is the same as the angle of arrival, then the formulas for a direct-fire trajectory become (see (48)) :—

$$\left. \begin{aligned} C &= \frac{w}{\kappa \sigma \tau_0 d^2} \\ X &= C \{S(V) - S(v)\} \\ T &= C \sec \phi \{T(V) - T(v)\} \\ \tan \theta &= \tan \phi - C \sec^2 \phi \{I(V) - I(v)\} \\ \frac{y}{x} &= \tan \phi - C \sec^2 \phi \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\} \end{aligned} \right\} \dots \dots \dots (50).$$

But when ϕ is very small, as for flat trajectories, or when the projectile is considered as moving horizontally between screens, such as in the experiments carried out for finding the resistance of the air (Chapter I) for the compilation of the Ballistic Tables (Chapter II), then the 3rd and 4th equations in equation (50) become

$$T = C \{T(V) - T(v)\} \dots \dots \dots (51)$$

$$\tan \theta - \tan \phi = -C \{I(V) - I(v)\} \dots \dots \dots (52).$$

At the striking point of the trajectory $y = 0$, $\theta = -\omega$, and v becomes the striking velocity; from (50), putting $y = 0$, then

$$\tan \phi = C \sec^2 \phi \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\},$$

or

$$\sin 2\phi = 2C \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\},$$

$$\sin 2\phi = C\alpha \dots \dots \dots (53),$$

where

$$\alpha = 2 \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\} \dots \dots \dots (54).$$

Also from (50), when $\theta = -\omega$,

$$\begin{aligned} \tan \omega &= \tan \phi + C \sec^2 \phi \{I(V) - I(v)\} \\ &= -C \sec^2 \phi \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\} + C \sec^2 \phi \{I(V) - I(v)\} \\ &= C \sec^2 \phi \left\{ \frac{A(V) - A(v)}{S(V) - S(v)} - I(v) \right\}. \end{aligned}$$

For flat trajectories, $\frac{\cos \phi}{\cos \omega}$ is approximately unity, and the last equation becomes

$$\sin 2\omega = 2C \left\{ \frac{A(V) - A(v)}{S(V) - S(v)} - I(v) \right\} \dots \dots \dots (55).$$

Let v_0 denote the velocity of the projectile at the vertex of the trajectory, where it moves horizontally for an instant and $\theta = 0$, then from (50)

$$\tan \phi = C \sec^2 \phi \{I(V) - I(v_0)\}$$

or

$$\sin 2\phi = 2C \{I(V) - I(v_0)\} \quad (56).$$

From (53) and (56)

$$I(v_0) = \frac{A(V) - A(v)}{S(V) - S(v)} \quad (57),$$

therefore from (55)

$$\sin 2\omega = 2C \{I(v_0) - I(v)\} \quad (58).$$

Then (52), putting $\theta = -\omega$,

$$\tan \omega = -\tan \phi + C \{I(V) - I(v)\} \quad (59).$$

From (52) the direction of the projectile to the horizontal can be found at any point of the trajectory when the remaining velocity v is known and *vice versa*.

In equations (53), (54), (55), (57), (58), (59) the v refers to the striking velocity at the end of the trajectory.

SUMMARY OF FORMULAS FOR DIRECT FIRE WITH FLAT TRAJECTORIES.

$$C = \frac{W}{K\sigma\tau g t^2},$$

$$X = C \{S(V) - S(v)\},$$

$$T = C \{T(V) - T(v)\},$$

$$\tan \theta = \tan \phi - C \{I(V) - I(v)\},$$

$$\sin 2\phi = Ca,$$

$$*a = 2 \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\};$$

also

$$\sin 2\phi = 2C \{I(V) - I(v_0)\},$$

$$I(v_0) = \frac{A(V) - A(v)}{S(V) - S(v)},$$

$$\sin 2\omega = 2C \left\{ \frac{A(V) - A(v)}{S(V) - S(v)} - I(v) \right\} = 2C \{I(v_0) - I(v)\},$$

$$\tan \omega = -\tan \phi + C \{I(V) - I(v)\}.$$

* The double-entry Table VIII gives the values of $a = 2 \left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\}$ in terms of V and the reduced range $\frac{R}{C}$ where R is in yards, and $\frac{R}{C} = \frac{3X}{C} = S(V) - S(v)$; hence by giving to V different values, v is calculated for different ranges, supposing $C = 1$; by means of Tables IV, V, a series of values of a can be obtained for these ranges direct.

Example 1.—A 9·2-inch gun has a muzzle velocity $V = 2600$ f/s; $C = 5$; find the value of a for $V = 2600$, range $R = 5000$ yards.

The reduced range $\frac{R}{C} = 1000$ yards = 3000 feet, therefore

$$3000 = S_{2600} - S_v,$$

hence

$$v = 1785 \cdot 7 \text{ f/s},$$

$$a = 2 \left\{ I_{2600} - \frac{A(2600) - A(1785 \cdot 7)}{8000} \right\} = 0 \cdot 01845.$$

This is the figure found in Table VIII for values of $V = 2600$ f/s, $\frac{R}{C} = 1000$ yards.

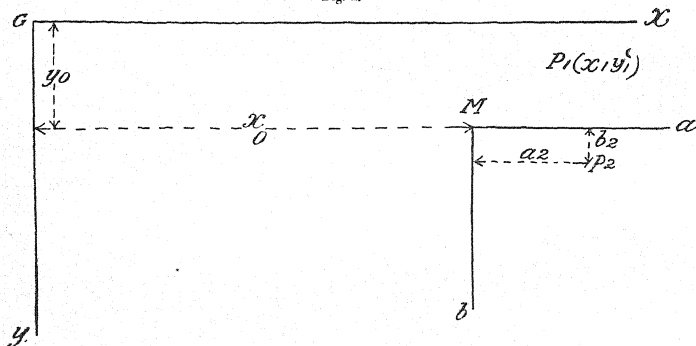
CHAPTER V.

ACCURACY OF FIRE.

IN Part I, Chapter III, examples were given for finding 50 per cent. zones from the results of firing a number of rounds; the practical use to be made of the Table of Probability factors was also shown, not only when the mean point of impact is the centre of the target, but also when it is not so situated.

In this chapter the theoretical basis of the rules employed will be given. Take as co-ordinate axes the line drawn from the gun G to the centre of the target for axis of x , and for axis of y the horizontal line through the gun at right angles to the x axis.

Fig. 1.



The abscissæ of the points of impact, P_1, P_2, \dots , give the ranges actually obtained, and the arithmetic mean of the abscissæ, or the average abscissa, yields the abscissa of the centre of impact.

Similarly, the arithmetic mean or average of the ordinates gives the ordinate of the centre of impact.

Let M be the centre of impact and let the co-ordinates of the n points of impact $P_1, P_2 \dots P_n$ be

$$x_1, x_2, x_3 \dots x_n,$$

$$y_1, y_2, y_3 \dots y_n.$$

Let (x_0, y_0) represent the co-ordinates of M , then

$$x_0 = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x}{n},$$

$$y_0 = \frac{y_1 + y_2 + \dots + y_n}{n} = \frac{\sum y}{n}.$$

Hence the position of the centre of impact is determined.

The choice of co-ordinate axes is quite arbitrary. It may be convenient sometimes to choose an origin of co-ordinates on the target itself; this is frequently done, and is, of course, done when the target is vertical. Occasionally, however, it is useful to put the successive ranges in evidence as has been done above, so that the ordinate of the centre of impact gives the mean range of the gun as fired.

Now transfer the origin to the centre of impact without altering the directions of the axes.

Let

$$a_1, a_2, a_3 \dots a_n,$$

$$b_1, b_2, b_3 \dots b_n$$

denote respectively the abscissae and ordinates of the points of impact referred to the new axes.

Since the centre of impact is now at the origin,

$$0 = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n} = \frac{\Sigma a}{n},$$

$$0 = \frac{b_1 + b_2 + b_3 + \dots + b_n}{n} = \frac{\Sigma b}{n},$$

and

$$\Sigma a = \Sigma b = 0.$$

The numbers $a_1, a_2 \dots a_n$ are called the *longitudinal deviations* of the points of impact, and the numbers $b_1, b_2 \dots b_n$ the *horizontal or lateral deviations* of the points of impact.

Observe that *deviations* have always reference to the centre of impact.

The result reached indicates that the algebraic sum of the horizontal or longitudinal deviations is zero, so that the sum of the positive deviations (in either direction) is equal to the sum of the negative deviations, numerical value being alone attended to.

When the position of the centre of impact on a horizontal plane is known, then the angle of descent (assumed to be the same for all the shots) determines the position of the centre of impact and of all the points of impact upon a vertical target, z denoting the vertical axis through the centre of the target.

Suppose any one shot strike the horizontal target at a distance l_1 yards from the vertical one and beyond it, then this vertical target will be struck at a height $l_1 \tan \beta$ yards above the ground, so that (neglecting ricochets) the centre of impact is at a height

$$z = \frac{l_1 \tan \beta + l_2 \tan \beta + \dots + l_n \tan \beta}{n} = \frac{l_1 + l_2 + \dots + l_n \tan \beta}{n} = l \tan \beta.$$

The centre of impact has an important property connected with what is known as the "theory of least squares."

The sum of the squares of the longitudinal (or horizontal) deviations with reference to the centre of impact is a minimum; that is, less than if a point, other than the centre of impact, were taken as origin of the co-ordinate axes with reference to which the deviations are measured.

This can easily be proved, because

$$a_1 = x_1 - x_0; \quad a_2 = x_2 - x_0; \quad a_3 = x_3 - x_0 \text{ and so on,}$$

therefore

$$a_1^2 + a_2^2 + \dots + a_n^2 = (x_1 - x_0)^2 + (x_2 - x_0)^2 + \dots + (x_n - x_0)^2,$$

$$\Sigma a^2 = \Sigma x^2 - 2x_0 \Sigma x + nx_0^2.$$

But

$$\Sigma x = nx_0,$$

therefore

$$\Sigma a^2 = \Sigma x^2 - nx_0^2,$$

showing that Σa^2 is always less than Σx^2 , the defect being nx_0^2 , an essentially positive quantity, unless $x_0 = 0$, when, obviously, there must be equality.

Hence Σx^2 is a minimum when the origin is the centre of impact.

It follows that the sum of the squares of the absolute deviations has the minimum value

$$\Sigma a^2 + \Sigma b^2,$$

when the deviations are taken with respect to the centre of impact.

Certain definitions are now necessary in order that we may connect the dispersion of the points of impact with the accuracy and precision of the weapon.

The *mean longitudinal deviation* is the arithmetical mean of the absolute values of the longitudinal deviations. By absolute value is meant numerical value with abstraction of algebraic sign.

This is calculated either by dividing the sum of the absolute values by the number of shots or by dividing the sum of the values of the positive deviations by half the number of shots.

With abstraction of sign, the expression is

$$\frac{\Sigma a}{n}.$$

The *mean longitudinal quadratic deviation*, as found by theory, is (see p. 93)

$$\sqrt{\frac{\Sigma a^2}{n-1}},$$

which, when n is not very small, so that $n-1$ may be replaced by n , is practically the square root of the arithmetic mean of the squares of the longitudinal deviations.

The *probable longitudinal deviation* is that, with respect to which the probabilities of obtaining greater and less deviations are equal; that is to say, in the results of a large number of shots of the same series, half of the longitudinal deviations should be less than the probable deviation and the other half greater; and the probability of obtaining a deviation less than the probable deviation from any particular shot would be one-half. The deviation of a single shot is evidently just as likely to exceed as to fall short of the probable deviation.

The same definitions apply, *mutatis mutandis*, to horizontal, vertical, and absolute deviations.

Similar definitions are employed with regard to "errors" in the theory of observation. The following notation will be used:—

$$\text{The mean longitudinal deviation : } \epsilon(x) = \frac{\Sigma(a)}{n};$$

$$\text{,, ,, horizontal deviation : } \epsilon(y) = \frac{\Sigma(b)}{n};$$

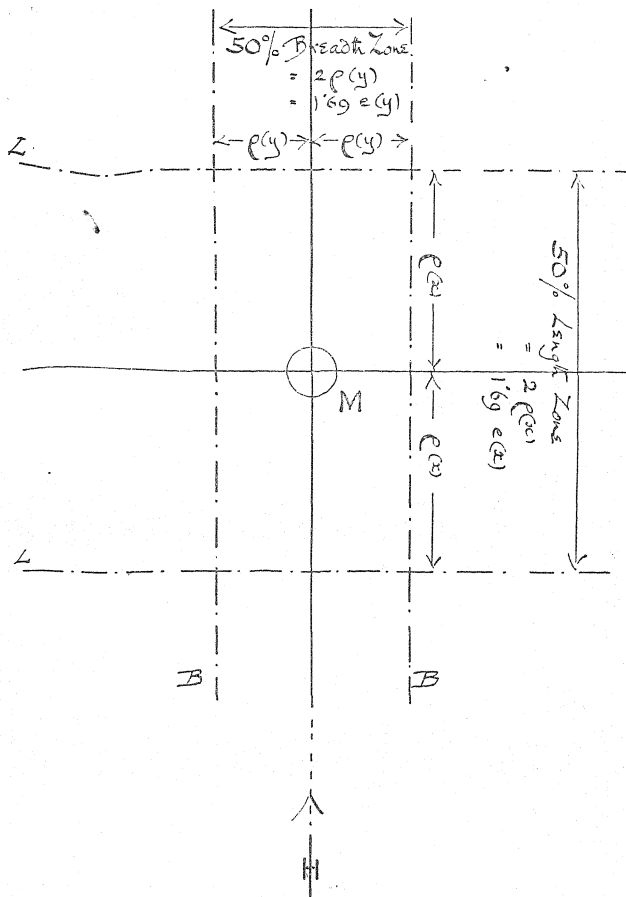
$$\text{,, ,, longitudinal quadratic deviation : } E(x) = \sqrt{\frac{\Sigma(a^2)}{(n-1)}};$$

$$\text{,, ,, horizontal quadratic deviation : } E(y) = \sqrt{\frac{\Sigma(b^2)}{(n-1)}}.$$

$$\text{The probable longitudinal deviation : } \rho(x);$$

$$\text{,, ,, horizontal deviation : } \rho(y).$$

Fig. 2.



On page 94, from the results established in the Theory of Probabilities, in the case when n is large:—

$$\rho = 0.6745E;$$

$$E = 1.4826\rho;$$

$$\rho = 0.8453\epsilon;$$

$$\epsilon = 1.1829\rho;$$

$$\frac{E}{\epsilon} = \sqrt{\frac{\pi}{2}}; \quad E = 1.2533\epsilon;$$

$$\epsilon = 0.7978E;$$

where all the letters may refer to either x or y . Of the three quantities e , E , and ρ , the probable deviation ρ is usually chosen as a means of comparison of different guns or different series of shots with the same gun.

From the results of a series of shots both e and E may be calculated by measurements connected with the group of impacts, and from either or both of these quantities ρ may be deduced by multiplication by a numerical coefficient. The calculation of e being more simple than that of E , ρ is deduced with greater facility from e than from E ; but unless the number of shots is very great, the calculation of ρ from E has a greater guarantee of accuracy than from e .

Suppose that lines are drawn parallel to the line joining the gun G with the centre of impact M , and distant ρ_y to the right and left of M ; also through M , and on either side of it at a distance ρ_x , draw lines at right angles to MG .

Then, looking to the definition of ρ_x , in the zone LL , of length $2\rho_x$ and of indefinite width, 50 % of the shots (the number being large) will probably fall; this is termed the 50 % length zone.

Hence the 50 % length zone has a length

$$2\rho_x = 1.69e_x = 1.349E_x.$$

So in the breadth zone BB of indefinite length but of width $2\rho_x$, 50 % of the shots will probably fall; this is called the 50 % breadth zone, its width is

$$2\rho_y = 1.69e_y = 1.349E_y.$$

The 50 % height zone is similarly constructed, its height on a vertical target is

$$2\rho_z = 2\rho_x \tan \beta,$$

where β is the angle of descent for the range, and the mean point of impact on the vertical target is in the middle of this 50 % height zone.

If the 50 % breadth and length zones be superposed we obtain a rectangle which must contain 50 % of 50 % or 25 % of the total number of hits. This is called a 25 % rectangle.

In a similar manner there is a 25 % rectangle on a vertical target derived from the 50 % breadth and height zones.

The relative accuracy of different guns at different ranges is frequently estimated by the dimensions of this rectangle.

Example 1.—From data obtained at Sandy Hook with a M.L. rifled mortar at a mean range of 3357 yards the following values of x and y were obtained, the origin being on the horizontal target at the shortest range ("Handbook of Problems of Direct Fire," by Captain James M. Ingalls):—

No. of round.	Range.	x .	y .	a .	b .
		yards.	yards.		
178	3264	0	4	- 93.11	-4.67
179	3348	84	16	- 9.11	+7.33
180	3296	82	9	- 61.11	+0.33
181	3427	163	12	+ 69.89	+3.33
182	3473	209	0	+115.89	-8.67
183	3318	54	6	- 39.11	-2.67
184	3320	56	10	- 37.11	+1.33
185	3408	144	12	+ 50.89	+3.33
186	3360	96	9	+ 2.89	+0.33

Here

$$\Sigma x = 838, \quad \Sigma y = 78.$$

Therefore

$$X_0 = \frac{1}{9} \Sigma x = 93.11 \quad \text{and} \quad Y_0 = \frac{1}{9} \Sigma y = 8.67,$$

giving the position of the centre of impact.

Since

$$a_1 = x_1 - X_0, \text{ \&c., and } b_1 = y_1 - Y_0, \text{ \&c.,}$$

we calculate the a and b columns which give the co-ordinates of the points of impact referred to the centre of impact as origin.

The sum of the absolute values of the deviations a is 479.11, and that of the deviations b is 31.99.

Hence

$$\epsilon(x) = \frac{479.11}{9} = 53.23; \quad \epsilon(y) = \frac{31.99}{9} = 3.55;$$

and from the numerical formulas

$$\rho(x) = 0.845\epsilon(x) = 44.98 \text{ yards}; \quad 2\rho(x) = 1.69\epsilon(x) = 89.96 \text{ yards};$$

$$\rho(y) = 0.845\epsilon(y) = 2.99 \text{ yards}; \quad 2\rho(y) = 1.69\epsilon(y) = 5.99 \text{ yards};$$

giving the probable longitudinal and horizontal deviations and the width of the 50 % length and breadth zones as computed from the mean deviations.

Also

$$\Sigma a^2 = 36306.9; \quad \text{hence} \quad E(x) = \sqrt{\frac{\Sigma a^2}{8}} = 67.37;$$

$$\Sigma b^2 = 182; \quad E(y) = \sqrt{\frac{\Sigma b^2}{8}} = 4.77.$$

Therefore

$$\rho(x) = 0.6745E(x) = 45.44 \text{ yards} \quad \text{and} \quad 2\rho(x) = 1.349E(x) = 90.88 \text{ yards};$$

$$\rho(y) = 0.6745E(y) = 3.215 \text{ yards} \quad \text{and} \quad 2\rho(y) = 1.349E(y) = 6.43 \text{ yards};$$

the similar results computed from the mean quadratic deviations, and it will be seen that they differ but slightly from those obtained from the mean deviations.

A 25 % rectangle made by the overlapping of the 50 % zones is 90.88 yards by 6.43 yards.

The diagrams given herewith are extracted from Krupp's *Artillerie*, a handbook issued at Düsseldorf, 1902; they will serve as further exercises for determining 50 % zones from practice.

Fig. a.

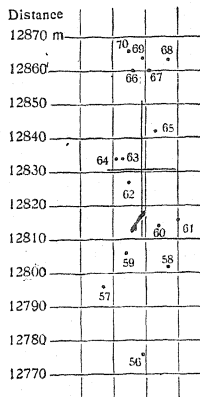
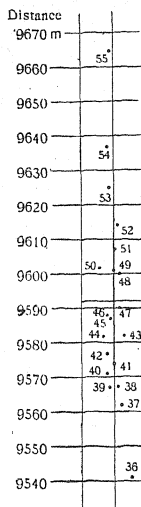
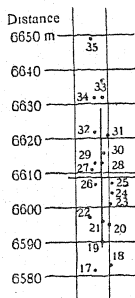


Fig. b.

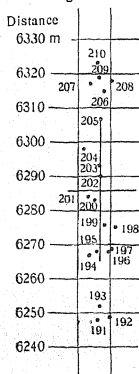
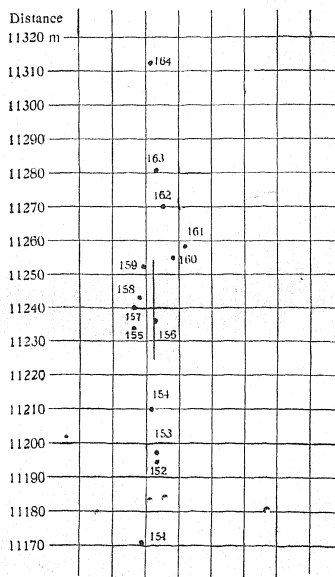


Fig. c.



The percentage of hits in other zones, which are symmetrical about the centre of impact in the direction of either axis, may be determined, and also the width of zone that may be expected to include a given percentage of hits.

To do this, we require a table of probability factors deduced from theoretical considerations, explained on p. 94.

TABLE OF PROBABILITY FACTORS.

The following gives the proportional width of other zones (containing a different percentage of hits) to one of 50 % as unity.

Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.
1	0.02	21	0.40	41	0.80	61	1.27	81	1.94
2	0.04	22	0.41	42	0.82	62	1.30	82	1.98
3	0.06	23	0.43	43	0.84	63	1.33	83	2.03
4	0.07	24	0.45	44	0.86	64	1.36	84	2.08
5	0.09	25	0.47	45	0.89	65	1.39	85	2.13
6	0.11	26	0.49	46	0.91	66	1.42	86	2.18
7	0.13	27	0.51	47	0.93	67	1.45	87	2.24
8	0.15	28	0.53	48	0.95	68	1.48	88	2.30
9	0.17	29	0.55	49	0.98	69	1.51	89	2.37
10	0.18	30	0.57	50	1.00	70	1.54	90	2.44
11	0.20	31	0.59	51	1.02	71	1.57	91	2.52
12	0.22	32	0.61	52	1.04	72	1.60	92	2.60
13	0.24	33	0.63	53	1.07	73	1.64	93	2.69
14	0.26	34	0.65	54	1.09	74	1.67	94	2.78
15	0.28	35	0.67	55	1.12	75	1.71	95	2.91
16	0.30	36	0.70	56	1.14	76	1.74	96	3.04
17	0.32	37	0.72	57	1.17	77	1.78	97	3.32
18	0.34	38	0.74	58	1.19	78	1.82	98	3.45
19	0.36	39	0.76	59	1.22	79	1.86	99	3.82
20	0.38	40	0.78	60	1.25	80	1.90	100	Infinite*

* As a factor of 4 gives more than 99 % of the rounds fired, it may be taken for practical purposes to give the total of 100 %.

In the first column will be found numbers representing the percentages of hits that may be expected in the zones; the corresponding factors represent the multiples that the widths of the zones are of the width of the 50 % zone.

To find the width of the length zone that will contain 75 % of the hits, we enter the table at the number 75 in the column headed "Per cent.," and find the corresponding factor to be 1.71. We deduce, therefore, that the width of the required zone is 1.71 times the width of the 50 % length zone.

Also to find the percentage of hits that will be included in breadth zone 1.25 times the width of the 50 % breadth zone, we enter the table at the number 1.25 in the column headed "Factor," and find the corresponding percentage to be 60. We conclude that 60 % of hits will be found in the given breadth zone.

Intermediate results can be obtained from the table by interpolation.

Rectangles containing a given percentage of hits can be obtained, and conversely we can determine the percentage of hits that will be found in any given rectangle which is symmetrical about the centre of impact.

Suppose a rectangle to be obtained by superposition of a length zone of $p\%$ and a breadth zone of $q\%$, then the rectangle will contain

$$p\% \text{ of } q\%, \text{ or } \frac{pq}{100}\% \text{ of the hits.}$$

For the design of a rectangle to contain $R\%$ of hits we have the relation

$$\frac{pq}{100} = R$$

for the determination of p and q . The equation has an infinite number of solutions, so that we can design an infinite number of rectangles containing the given percentage R of hits. We may give q any value we please, and thence determine p from the equation

$$q = \frac{100R}{p}.$$

We look out p and $\frac{100R}{p}$ in the column of the table headed "Per cent.," and thence find the widths of the length and breadth zones, which, by superposition, give an $R\%$ rectangle. These widths are the longitudinal and horizontal sides of the rectangle.

The 25% rectangle already met with is thus only one of an infinite number of 25% rectangles. For its design we excluded 50% of hits for horizontal deviations and 50% for longitudinal deviations.

It is frequently desired, as in this case, to exclude the same number of hits for horizontal as for longitudinal deviations, and then the determination of the rectangle rests upon the equation

$$q^2 = 100R,$$

or

$$q = 10\sqrt{R}.$$

An example will make the subject clearer.

Example 2.—Find a rectangle containing 50% of hits such that the same number of hits may be excluded for horizontal as for longitudinal deviations. Here $R = 50$, and if q be the percentage of hits in the breadth and length zones which, by superposition, give the rectangle

$$q = p = 10\sqrt{50} = 70.7;$$

entering the table we find, by interpolation, the factor 1.56 , so that the widths of the zones are 1.56 times the widths of the corresponding 50% zones. Hence the sides of the rectangle are,

$$1.56 \times 2\rho_x = 3.12\rho_x,$$

and

$$1.56 \times 2\rho_y = 3.12\rho_y.$$

A study of the table shows that a zone four times the width of the 50% zone practically contains the whole of the hits. This zone is termed the "*enveloping zone*." By superposition of the enveloping breadth and length zones we obtain the *enveloping rectangle*, which may be shown to comprise 98.6% (practically all) of the hits.

It is obvious that in many cases the horizontal deviations will not be of so much importance as those in the longitudinal direction, and that it will be useful to calculate rectangles which give relatively small importance to the horizontal deviations. In the extreme case of a gun which shoots practically perfectly as to line we need only consider the length zones which are the extreme cases of the rectangles.

Example 3.2—The numbers of hits excluded for horizontal and longitudinal deviations respectively being in the ratio of 2 to 3, determine the dimensions of the 50 % rectangle.

Let the longitudinal zone be one of p % and the width zone be one of q %.

Then

$$100 - q = \frac{2}{3}(100 - p),$$

or

$$3q = 2p + 100 \quad (1).$$

Also p % of q % gives the 50 % rectangle, therefore

$$\frac{p}{100} \times \frac{q}{100} = \frac{50}{100},$$

from (1) and (2)

$$pq = 5000 \quad (2),$$

$$p = 65.14,$$

$$q = 76.76.$$

From the table the factors are found to be 1.40 and 1.77.

Hence the sides of the rectangle are

$$1.40 \times 2\rho(x) = 2.80\rho(x)$$

$$1.77 \times 2\rho(y) = 3.54\rho(y)$$

The actual number of hits obtained upon a given target depends upon the position of the mean point of impact relative to the target. For examples showing the method of calculating the actual number of hits, see Part I, pp. 96-109.

The co-ordinates of the centre of impact, M (fig. 1), have been denoted by X_0 , Y_0 . If the target is horizontal and the origin at the firing point, X_0 is the arithmetic mean of the several ranges actually obtained; only when the number of rounds is increased indefinitely does X_0 represent the exact range appertaining to the gun as laid.

The probable deviation of a single point of impact has been denoted by $\rho(x)$; this also is deduced from the rounds fired, and is only exact when the number of rounds increases without limit. The probable deviation of the centre of impact, deduced from a series of n rounds, from the true centre of impact is found by dividing the probable deviation of a single shot, deduced from the series, by the square root of the number of shots.

Thus if $\rho(x)$ be the probable deviation in range of a single point of impact,

$$\frac{\rho(x)}{\sqrt{n}}$$

is the probable deviation in range of the centre of impact of a group of n shots.

As an example take the data of Example 1. From 9 rounds a mean range of 3,357 yards was obtained, and the probable deviation in range of a single shot was found to be 45.44 yards. The range that might be expected to be obtained from a single shot would be denoted by

$$3357 \pm 45.44 \text{ yards};$$

and it is an even chance that if another 9 shots were fired their arithmetic mean would fall within the limits :—

$$3357 \pm \frac{45.44}{\sqrt{9}},$$

or

$$3357 \pm 15.15 \text{ yards.}$$

(9263)

M

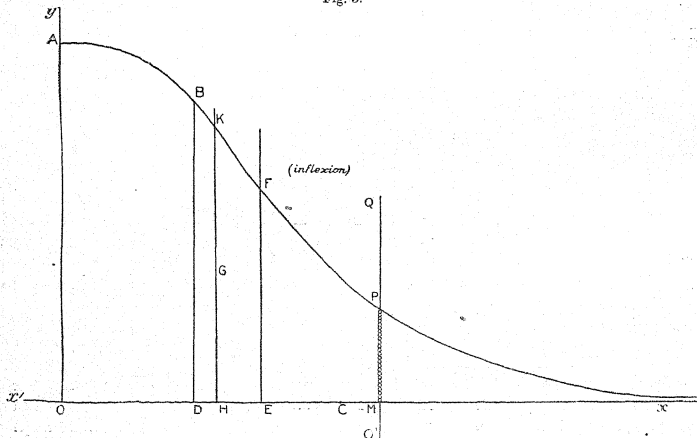
PROBABILITY OF FIRE

According to the theory of Probability, a certain curve, called the *error curve*, of the shape in fig. 3, can be drawn representing graphically by its area the percentage ($\%$) of shots which in the long run can be expected on a target of given dimensions.

Suppose, for instance, that $x'Ox$ in fig. 3 is drawn along the line of mean direction so that O is at the point of mean impact, and that in a very large series of practice all the shot which struck on the line QQ' are arranged in contact along the ordinate MP; if this is done with all the shot, they will be found arranged in a certain area, bounded by the straight line $x'Ox$ and the error curve $x'Ax$.

This curve of error can be realised experimentally by an instrument* invented by

Fig. 3.



Mr. Francis Galton, which he calls the *Quincunx*, from the Latin word describing the arrangement of trees in an orchard (see Part I, p. 96).

A charge of small shot is allowed to pour through the funnel at the top; the shot knock against pins arranged like trees, and are scattered thereby in an arbitrary manner; but it is found that the shot always group themselves in the stall at the bottom in a manner which imitates closely the profile of the error curve.

The error curve (fig. 3) is redrawn here to a larger scale and as accurately as possible, the right-hand half alone being given, and in accordance with abstruse theoretical principles, the error curve can be best represented by an equation of the form

$$y = ae^{-\frac{x^2}{c^2}} \text{ or } a \exp\left(-\frac{x^2}{c^2}\right).$$

So that when $x = 0$, $y = a = OA$, and " a " represents the number of shots (out of a large number) which struck on the line OA.

The curve $y = ae^{-\frac{x^2}{c^2}}$ is symmetrical to the right and left of QA, where O is the mean point of impact, thus stating that plus and minus deviations are equally likely to occur. Again,

as x increases the ordinate y decreases, which is the same as stating that small deviations (from 0) are more frequent than large ones. The curve rapidly approaches the axis xOx , on either side of OA , which includes the statement that large deviations are rare, and, beyond a certain limit, practically do not occur.

All the shots fall within the area $xOaAx'$. Denote this area by $2A$, then

$$\begin{aligned} 2A &= a \int_{-\infty}^{\infty} e^{-\frac{x^2}{c^2}} dx = 2a \int_0^{\infty} e^{-\frac{x^2}{c^2}} dx \\ &= 2ac \int_0^{\infty} e^{-t^2} dt \quad \text{where } t = \frac{x}{c}. \end{aligned}$$

But $\int_0^{\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$, a well-known definite integral.

Hence

$$2A = ac\sqrt{\pi}.$$

But $a \int_{-\infty}^{\infty} e^{-\frac{x^2}{c^2}} dx$ is the probability of a shot falling anywhere along xOx from $x = -\infty$ to $x = +\infty$, hence it is a certainty, and $2A = 1$, giving

$$a\sqrt{\pi} = \frac{1}{c};$$

denoting $\frac{1}{c}$ by h , then

$$\frac{1}{c} = h = a\sqrt{\pi}.$$

The probability curve can now be written

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}.$$

The total area of this curve is $2A = 1$, and twice the area OMPA therefore represents the probability that the error will be within $\pm x$. Denote the area OMPA by P , then

$$P = \frac{2h}{\sqrt{\pi}} \int_0^x e^{-h^2 x^2} dx.$$

Putting $hx = t$,

$$P = \frac{2}{\sqrt{\pi}} \int_0^{t(=hx)} e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \int_0^{t(=hx)} \exp(-t^2) dt.$$

This integral must be evaluated by approximate numerical computation.

Integrating by parts, and putting

$$\int_0^t e^{-t^2} dt = \int_0^x e^{-t^2} dt - \int_t^{\infty} e^{-t^2} dt.$$

But

$$\int_0^{\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$$

and

$$\int_0^t e^{-t^2} dt = \frac{\sqrt{\pi}}{2} - \frac{e^{-t^2}}{2t} \left\{ 1 - \frac{1}{2t^2} + \frac{1.3}{(2t^2)^2} - \frac{1.3.5}{(2t^2)^3} + \dots \right\},$$

so that

$$P = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt = 1 - \frac{e^{-t^2}}{t\sqrt{\pi}} \left\{ 1 - \frac{1}{2t^2} + \frac{1.3}{(2t^2)^2} - \frac{1.3.5}{(2t^2)^3} + \dots \right\}.$$

The tabulation of P for various values of t is as follows :—

$t = hx.$	$P = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt.$									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0.0	0.0000	0.0118	0.0226	0.0338	0.0451	0.0564	0.0676	0.0789	0.0901	0.1013
0.1	.1125	.1236	.1348	.1459	.1569	.1680	.1790	.1900	.2009	.2118
0.2	.2227	.2335	.2443	.2550	.2657	.2763	.2869	.2974	.3079	.3183
0.3	.3286	.3389	.3491	.3593	.3694	.3794	.3893	.3992	.4090	.4187
0.4	.4284	.4380	.4475	.4569	.4662	.4755	.4847	.4937	.5027	.5117
0.5	0.5205	0.5292	0.5379	0.5465	0.5549	0.5633	0.5716	0.5798	0.5879	0.5959
0.6	.6039	.6117	.6194	.6270	.6346	.6420	.6494	.6566	.6638	.6708
0.7	.6778	.6847	.6914	.6981	.7047	.7112	.7175	.7238	.7300	.7361
0.8	.7421	.7480	.7538	.7595	.7651	.7707	.7761	.7814	.7867	.7918
0.9	.7969	.8019	.8068	.8116	.8163	.8209	.8254	.8299	.8342	.8385
1.0	0.8427	0.8468	0.8508	0.8548	0.8586	0.8624	0.8661	0.8698	0.8733	0.8768
1.1	.8802	.8835	.8868	.8900	.8931	.8961	.8991	.9020	.9048	.9076
1.2	.9103	.9130	.9155	.9181	.9205	.9229	.9252	.9275	.9297	.9319
1.3	.9340	.9361	.9381	.9400	.9419	.9438	.9456	.9473	.9490	.9507
1.4	.9523	.9539	.9554	.9569	.9583	.9597	.9611	.9624	.9637	.9649
1.5	0.9661	0.9673	0.9684	0.9695	0.9706	0.9716	0.9726	0.9736	0.9745	0.9755
1.6	.9763	.9772	.9780	.9788	.9796	.9804	.9811	.9818	.9825	.9832
1.7	.9838	.9844	.9850	.9856	.9861	.9867	.9872	.9877	.9882	.9886
1.8	.9891	.9895	.9899	.9903	.9907	.9911	.9915	.9918	.9922	.9925
1.9	.9928	.9931	.9934	.9937	.9939	.9942	.9944	.9947	.9949	.9951
2.0	0.9953	0.9955	0.9957	0.9959	0.9961	0.9963	0.9964	0.9966	0.9967	0.9969
2.1	.9970	.9972	.9973	.9974	.9975	.9976	.9977	.9979	.9980	.9980
2.2	.9981	.9982	.9983	.9984	.9985	.9985	.9986	.9987	.9987	.9988
2.3	.9989	.9989	.9990	.9990	.9991	.9991	.9992	.9992	.9992	.9993
2.4	.9993	.9993	.9994	.9994	.9994	.9995	.9995	.9995	.9995	.9996
2.5	0.9996	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
2.6	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9999
∞	1.0000									

And by interpolation we have—

$hx = t.$	P.	$hx = t.$	P.
0.04434	0.05	0.5342	0.55
0.08888	0.10	0.5951	0.60
0.1337	0.15	0.6609	0.65
0.1791	0.20	0.7329	0.70
0.2253	0.25	0.8135	0.75
0.2724	0.30	0.9062	0.80
0.3208	0.35	1.0179	0.85
0.3708	0.40	1.1631	0.90
0.4227	0.45	1.3859	0.95
0.4769	0.50	1.8215	0.99

In the long run, the mean arithmetic error ϵ , is the abscissa OH of the C.G. of the area $OA\epsilon = A_{(x)}$, and the equation to AKP is $y = \frac{h}{\sqrt{\pi}} \exp(-h^2 x^2)$, so that

$$\begin{aligned}\epsilon \cdot A_{(x)} &= \int_0^\infty x \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} dx \\ &= \frac{h}{\sqrt{\pi}} \left(-\frac{1}{2h^2} \right) [e^{-h^2 x^2}]_0^\infty;\end{aligned}$$

$$\epsilon \cdot \frac{1}{2} = + \frac{1}{2h\sqrt{\pi}}, \quad \text{since} \quad A_{(x)} = \frac{1}{2}.$$

Therefore

$$\epsilon = \frac{1}{h\sqrt{\pi}}.$$

Now, let E denote the mean quadratic error, so that E is the radius of gyration of the area AO ϵ about OA y .

$$\begin{aligned}E^2 \cdot A_{(x)} &= \int_0^\infty x^2 \cdot y dx \\ &= \frac{h}{\sqrt{\pi}} \int_0^\infty x^2 e^{-h^2 x^2} dx \\ &= \frac{h}{\sqrt{\pi}} \cdot \left(-\frac{1}{2h^2} \right) \int_0^\infty x \frac{d(e^{-h^2 x^2})}{dx} dx.\end{aligned}$$

$$\text{Since } A(x) = \frac{1}{2},$$

$$\begin{aligned}(E)^2 \cdot \frac{1}{2} &= \frac{h}{\sqrt{\pi}} \left(-\frac{1}{2h^2} \right) [xe^{-h^2 x^2}]_0^\infty + \frac{h}{\sqrt{\pi}} \cdot \frac{1}{2h^2} \int_0^\infty e^{-h^2 x^2} dx \\ &= -\frac{1}{2h\sqrt{\pi}} \left[\frac{x}{1 + (hx)^2} + \frac{(hx)^4}{2!} + \dots \right]_0^\infty + \frac{h}{\sqrt{\pi}} \cdot \frac{1}{2h^2} \cdot \frac{\sqrt{\pi}}{2h} \\ &= \frac{1}{4h^2},\end{aligned}$$

therefore

$$(E)^2 = \frac{1}{2h^2} \quad \text{and} \quad E = \frac{1}{h\sqrt{2}};$$

so that the point F on the curve ABFP is a point of inflexion, as is seen by putting

$$\frac{d^2 y}{dx^2} = 0, \quad \text{where} \quad y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2};$$

so that

$$\frac{E}{\epsilon} = \sqrt{\frac{\pi}{2}},$$

and

$$h = \frac{t}{x} = \frac{1}{\epsilon\sqrt{\pi}} = \frac{1}{E\sqrt{2}}.$$

The abscissa ρ = OD of the ordinate BD, which cuts the area AO ϵ in half, is called the *probable error*, because in the long run half the shots have a greater error and the other half a less error than ρ . The line BD and the parallel symmetrical line B'D' cut out the middle half of the whole area A'B'A ϵ of the error curve, so that the probability is that half the total

number of shots fall within the zone DD' and the other half fall outside DD'; hence the zone D'D will probably catch 50 % of the shots, and of the remainder 25 % fall in the zone D'x' and 25 % in the zone Dx.

The zone D'D is called the 50 % zone.

To find the length D'D = 2ρ .

The value of $t (= h\pi)$ corresponding to $P = 0.5$ is .4769, hence

$$h \cdot \rho = .4769.$$

But

$$h = \frac{1}{\epsilon \sqrt{\pi}} = \frac{1}{E \sqrt{2}},$$

hence

$$\rho = .4769 \sqrt{\pi} \cdot \epsilon = 0.8453\epsilon,$$

$$2\rho = 1.6906\epsilon,$$

also

$$\rho = .4769 \sqrt{2}E = 0.6745E,$$

$$2\rho = 1.359E.$$

So that a 50 % zone breadth is equal to the mean arithmetical error obtained from the analysis of all available practice, multiplied by 1.69.

Other zones are found in a similar manner, thus the 25 % zone:—

When $P = 0.25$, then $t = h\pi = 0.2253$, and the 25 % zone is

$$2x = 0.2253 \times \frac{2}{h},$$

$$25 \% \text{ zone} = .2253 \times 2 \sqrt{\pi} \epsilon = 0.9024\epsilon,$$

also

$$\frac{\text{breadth of } 25 \% \text{ zone}}{\text{breadth of } 50 \% \text{ zone}} = \frac{.2253}{.4769} = 0.4725,$$

similarly

$$\frac{\text{breadth of } 75 \% \text{ zone}}{\text{breadth of } 50 \% \text{ zone}} = \frac{.8135}{.4769} = 1.7058,$$

and so on for any zone.

To determine the % of hits to be expected in a zone bounded by any ordinate MP and its symmetrical ordinate MP', the ratio of the breadth MM' of this zone to D'D, the breadth of the 50 % zone, is calculated, and called the *probability factor*, and a *Table of Probability Factors* is calculated, giving the % which the MPP'M' of the error curve bears to the whole area and the corresponding probability factor.

When the ordinates PM and P'M' which limit the zone occupied by the target are not symmetrical with respect to the line of mean impact AO, the % of hits to be expected on each part AOMP and AOMP' must be calculated separately, and these % are added or subtracted according as PM and P'M' are on opposite sides of AO or on the same side.

Thus, if the number of hits on a zone bounded by PM and P'M' is less than what should be expected, the inference is that the gun is not laid properly so as to bring the line of mean impact AO midway between PM and P'M'.

The probability table, on p. 87, has the factor 0.47 opposite the 25 % and 1.7 is opposite the 75 %; the factor for other percentages is worked out as shown above. It is understood that the centre of impact is at the centre of the target.

To four places of decimals, the factors for the following percentages are now given:—

Per cent.	Factor.	Per cent.	Factor.
5	0.0930	55	1.1201
10	0.1863	60	1.2479
15	0.2804	65	1.3857
20	0.3756	70	1.5368
25	0.4725	75	1.7058
30	0.5713	80	1.9002
35	0.6727	85	2.1344
40	0.7775	90	2.4389
45	0.8863	95	2.9061
50	1	99	3.8195

If the target fired at is limited by two dimensions, say, length and breadth, or breadth and height, it is treated as the overlapping of two such unlimited zones, for which the separate % of hits is calculated, and the product of these gives the required percentage.

Modern range tables contain three columns, giving at each range the size of the 50 % zone for errors in range, direction, and vertical deviation; and now the probability factor enables us to calculate the % of hits to be expected on a zone of given depth or length in range, or of breadth in direction, or of given vertical height; thence we infer the number of shots required to make an assigned number of hits, and can decide whether the object is worth the ammunition to be expended.

The theory of Probability is also useful in the design of match targets, and in comparing the results of competitive artillery practice carried out under different conditions.

In designing a vertical target for rifle shooting, the breadth and height may be taken as four times that of the 50 % zones, as more than 99 % of the shots should now be caught by the target, if the rifle is properly aimed.

The overlapping of the two 50 % zones will give a 25 % rectangle, which may be taken as appropriate for the *bull's-eye*; two 70.7 % zones will enclose a 50 % rectangle, which will serve as the boundary of the *inner*; while two 86.6 % zones will enclose a 75 % rectangle, appropriate for the *maggie*, the space between this and the enveloping rectangle being the *outer*.

On a circular target the radius of the bull's-eye, centre, inner, and outer would be obtained by the revolution of the error curve round *Oy*, and determining the radius of the cylinder which cuts out 25 %, 50 %, 75 %, and 99 % of the total volume enclosed by the surface generated.

Then if r is the radius of the circle, with centre at the point of mean impact on the target which catches 100 P % of the shots,

$$P = \frac{V(r)}{V(\infty)}.$$

The equation of the probability curve is

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}, \quad \text{or} \quad \frac{h}{\sqrt{\pi}} \exp(-h^2 x^2),$$

therefore

$$V(r) = \frac{h}{\sqrt{\pi}} \int_0^r e^{-h^2 x^2} \cdot 2\pi x \cdot dx = \frac{h}{\sqrt{\pi}} \cdot 2\pi \left(-\frac{1}{2h^2} \right) [e^{-h^2 x^2}]_0^r,$$

$$V(\infty) = \frac{h}{\sqrt{\pi}} \int_0^\infty e^{-h^2 x^2} \cdot 2\pi x \cdot dx = \frac{h}{\sqrt{\pi}} \cdot 2\pi \left(-\frac{1}{2h^2} \right) [e^{-h^2 x^2}]_0^\infty,$$

therefore

$$P = 1 - e^{-h^2 \rho^2}$$

and

$$\frac{1}{h} = \epsilon \sqrt{\pi} = E \sqrt{2},$$

where ϵ is the mean arithmetical error, and E is the mean quadratic error.

If ρ represent the radius of the probable deviation, $h\rho = .4769$ (see p. 94), then

$$rh = \sqrt{\left(\log_e \frac{1}{1-P}\right)},$$

and

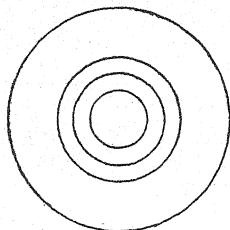
$$\begin{aligned} \frac{r}{\rho} &= \frac{1}{.4769} \sqrt{\left(\log_e \frac{1}{1-P}\right)}, \\ &= 2.097 \sqrt{2.302} \sqrt{\left(\log_{10} \frac{1}{1-P}\right)}, \\ &= 3.181 \sqrt{\left(\log_{10} \frac{1}{1-P}\right)}, \end{aligned}$$

so that the radii of the circles for the various $\%$ are calculated as follows:—

P.	$\frac{1}{1-P}$	$\log_{10} \frac{1}{1-P}$	$\frac{r}{\rho}$
0.25	1.3333	0.1249	1.13
0.50	2	0.3010	1.75
0.75	4	0.6020	2.5
0.99	100	2.000	4.5

Thus with a rifle at 500 yards range the probable deviation ρ might be about 4 inches, thus making the radii of the 25, 50, 75, and 99 % circles about 4.5, 7, 10, and 18 inches, as shown in fig. 4, drawn to a scale of $\frac{1}{10}$. An expert marksman should bring the centre of impact very close to the centre of the target, and then 99 % of the shots should be on the target, and 25, 25, 25, and 24 % in each compartment.

Fig. 4.



If four marks are scored for a bull's-eye and 3, 2, 1 marks for the other compartments, the probable score for 100 shots would be

$$25 \times 4 + 25 \times 3 + 25 \times 2 + 24 = 249.$$

The same thing will hold when the two errors, lateral and vertical, $\rho(x)$ and $\rho(y)$, are not equal; and now the circular curves must be replaced by similar ellipses.

DETERMINATION OF THE MEAN AND PROBABLE ERRORS OF FUNCTIONS OF INDEPENDENT OBSERVED QUANTITIES.

Let ϵ_e denote the probable error in yards at a given range due to the layer, and ϵ_g the probable error in yards at the same range due to the gun, these probable errors are independent of each other, and then

$$E^2 = \epsilon_e^2 + \epsilon_g^2$$

where E denotes the probable error due to the combined probable errors of the layer and the gun (see Chauvenet's *Spherical and Practical Astronomy*, Vol. II, p. 497).

Abnormal or Doubtful Rounds in Analysis.

In the analysis of a number of rounds of a series, one or more rounds may appear to differ abnormally from the rest, the question whether these rounds are due to causes of so exceptional a character that they are not samples of what may be expected and ought therefore to be rejected, should be decided by reference to some criterion which is well established.

In Chauvenet's *Spherical and Practical Astronomy*, Vol. II, pp. 558-566, such a criterion is explained.

Let x denote the error of a doubtful round so that x represents the difference between the range of the round and the mean arithmetic range.

Let r denote the probable longitudinal quadratic deviation or the mean quadratic error of all the rounds, so that (pp. 94)

$$r = \frac{0.4769}{h} = 0.6745 \sqrt{\frac{\sum (a^2)}{n-1}},$$

where $\sum (a^2)$ denote the sum of the squares of all the errors from the mean range, n is the number of rounds.

In the following table n denotes the number of rounds fired, and if $\frac{x}{r}$ is less than the number opposite n , the round should *not* be rejected.

If the ratio $\frac{x}{r}$ be greater than the number against n , then the round may be considered doubtful. The neighbouring means, that is to say, the mean longitudinal errors of the groups of rounds fired at the same elevations immediately above and below, may in some cases remove the doubt; in other cases there may be some equally good extraneous evidence. It must be clearly understood that only one doubtful round at a time can ever be cast out by this method.

The Value of $\frac{x}{r}$ corresponding to Different Values of n .

n .	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	2.05	2.27	2.44	2.57	2.67	2.76	2.84
1	2.91	2.96	3.02	3.07	3.12	3.16	3.19	3.22	3.26	3.29
2	3.32	3.35	3.38	3.41	3.43	3.45	3.47	3.49	3.51	3.53
3	3.55	3.57	3.58	3.60	3.62	3.64	3.65	3.67	3.68	3.69
4	3.71	3.72	3.73	3.74	3.75	3.77	3.78	3.79	3.80	3.81
5	3.82	3.83	3.84	3.85	3.86	3.87	3.88	3.88	3.89	3.90
6	3.91	3.92	3.93	3.94	3.95	3.95	3.96	3.97	3.97	3.98
7	3.99	3.99	4.00	4.01	4.02	4.02	4.03	4.04	4.05	4.05
8	4.06	4.06	4.06	4.07	4.07	4.08	4.09	4.09	4.10	4.11
9	4.11	4.12	4.13	4.14	4.14	4.15	4.15	4.15	4.16	4.16

If $n = 100$, $\frac{x}{r} = 4.16$; $n = 200$, $\frac{x}{r} = 4.48$; $n = 500$, $\frac{x}{r} = 4.90$.

Example 1:—

Round.	Range in yards.	Error from mean range, a yards.
1	4011	+ 11
2	4025	+ 25
3	4015	+ 15
4	4050	+ 50
5	4045	+ 45
6	3700	- 300
7	4038	+ 38
8	4008	+ 8
9	4008	+ 8
10	4032	+ 32
11	4025	+ 25
12	4008	+ 8
13	4010	+ 10
14	4025	+ 25

The mean range is 4000 yards for 14 rounds,

$$\Sigma(a^2) = 90,000 + 9506 = 99,506,$$

$$\sqrt{\frac{\Sigma(a^2)}{n-1}} = \sqrt{\frac{99,506}{13}} = 87.34.$$

$$r = 0.6745 \times 87.34.$$

The doubtful round is round 6.

For $n = 14$ the factor from the table gives 3.12 , and the criterion is whether

$$\frac{x}{r} \text{ is greater or not than } 3.12,$$

that is, if

$$x \text{ „ „ „ „ } 3.12r,$$

$$x \text{ is } 4000 - 3700 = 300,$$

$$3.12r = 3.12 \times 0.6745 \times 87.34 = 146.0,$$

hence x is greater than $3.12r$, and round 6 may be rejected as abnormal.

Example 2.—Take the data of the example on p. 84; consider round 182. Here $n = 9$ and 2.84 is the factor from the table,

$$\sqrt{\frac{\Sigma(a^2)}{n-1}} = \sqrt{\frac{36,306.6}{8}} = \sqrt{4538.3} = 67.38;$$

the criterion for non-rejection is that $x = a - 115.89$ should be less than

$$.6745 \times 67.38 \times 2.84 = 129.0.$$

hence $\frac{x}{r}$ is less than 2.84 , and round 182 must not be rejected.

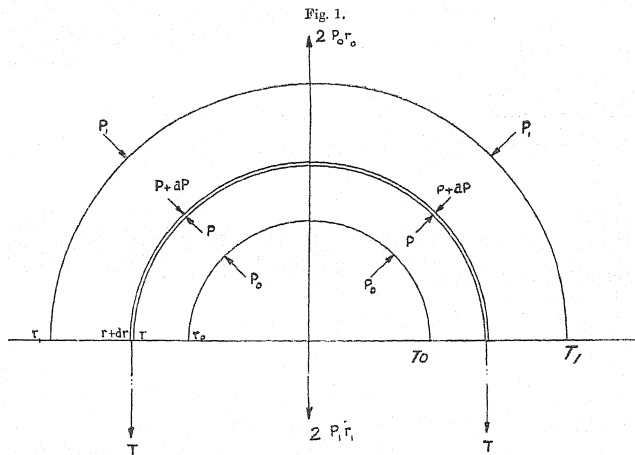
CHAPTER VI.

GUN CONSTRUCTION.

CONSIDERING an inch length of the bore of a gun, the hydrostatic thrust of the interior pressure of P_0 tons/in.² acting normally to the diametral plane $r_0 r_0 r_0 r_1$ is

$$\int_0^\pi P_0 r_0 d\theta \sin \theta = 2P_0 r_0.$$

The thrust of the exterior pressure of P_1 tons/in., normal to the plane $r_1 r_0 r_0 r_1$ is $2P_1 r_1$, and



acts in the opposite direction to $2P_0 r_0$. The difference, $2P_0 r_0 - 2P_1 r_1$, is the resultant tension acting on the metal across the two sections $r_0 r_0$.

Let T represent the tensional stress in tons/in.² at any point, the total stress is $2 \int_{r_0}^{r_1} T dr$, so that

$$\int_{r_0}^{r_1} T dr = P_0 r_0 - P_1 r_1 \dots \dots \dots (1).$$

At any point r in the section $r_0 r_1$, let the radial pressure be P and the circumferential tension T ; at $(r + dr)$ let these become $P + dP$ and $T + dT$; then the mean tension across the small area dr is

$$\frac{T + T + dT}{2} = T + \frac{dT}{2},$$

and

$$2P = 2(P + dP)(r + dr) + 2\left(T + \frac{dT}{2}\right)dr,$$

(9263)

N 2

from which

$$(P + T) dr = -r dP$$

$$P + T = -r \frac{dP}{dr},$$

$$T = -\frac{d}{dr}(Pr) \quad \dots \dots \dots (2).$$

The stresses to which the metal of a gun are subjected by the pressure of the powder gases, or by the shrinkage of the tubes in built-up guns, or the tension of the wire in wire-wound guns, must be examined before the principles of gun construction can be understood. A metal subjected to a stress will take a permanent set as soon as the stress reaches the elastic limit (in tons per square inch) of the metal.

For gun steel, the elastic limit of extension and of compression is practically the same.

When a piece of metal is pulled, as, for instance, a test piece of steel in a testing machine, it is found that the *extension*, measured by the ratio of the *elongation* to the original length, is proportional to the *tension*, which we shall measure in tons per square inch of cross-section.

Thus doubling the tension doubles the extension; and so on in proportion, provided the elastic limit is not exceeded.

This experimental law is called "Hooke's Law," and it is the axiomatic foundation of the Mathematical Theory of Elasticity. Expressed in an algebraical form, if a pull of P tons in a bar, K in.² in cross-section, stretches the length from L to $L + l$, then the tension $\frac{P}{K}$ tons/in.², and the extension $\frac{l}{L}$, are, by "Hooke's Law," connected by the relation

$$\frac{\frac{P}{K}}{\frac{l}{L}} = E, \text{ a constant} \quad \dots \dots \dots (3);$$

where E denotes a number of tons/in.², called Young's *modulus* of elasticity of the material; thus for steel E is about 12,500 tons/in.², and the extension,

$$\frac{l}{L} = \frac{P}{K} \cdot \frac{1}{E} = A \cdot \frac{P}{K}.$$

In this case the metal is subject to a single tension, and a certain amount of lateral contraction takes place; experiment shows that, for steel, the strain at right angles to the direction of a stress within the elastic limit of the metal is equal to one-third of the strain in the direction of the stress.

Consider a small brick-shaped piece of the metal of a gun, the dimensions defined as follows:—

- (i) By two adjacent concentric cylinders of radii r and $r + dr$;
- (ii) By two consecutive radial planes at an angle $d\theta$;
- (iii) By two transverse planes at distances x and $x + dx$ from one end.

This gives

$$\begin{aligned} \text{Width of lower part of the element} & \dots \dots \dots = r d\theta, \\ \text{Depth of element} & \dots \dots \dots = dr, \\ \text{Thickness of element} & \dots \dots \dots = dx. \end{aligned}$$

Let the element be acted upon, normally to the surface, by

- T , a circumferential stress;
- P , a radial stress;
- , a longitudinal stress, which is therefore parallel to the axis of the gun.

The piece of metal will be slightly altered in its dimensions, suppose the width $r d\theta$ becomes $(r+u) d\theta$, due to the increase u of the radius r of the circumferential fibre, so that the fibre is stretched from a length $2\pi r$ to a length $2\pi (r+u)$.

The circumferential extension is

$$e_t = \frac{2\pi (r+u) - 2\pi r}{2\pi r} = \frac{u}{r}.$$

The depth dr becomes $d(r+u)$, so that the radial extension is

$$e_p = \frac{d(r+u) - dr}{dr} = \frac{du}{dr}.$$

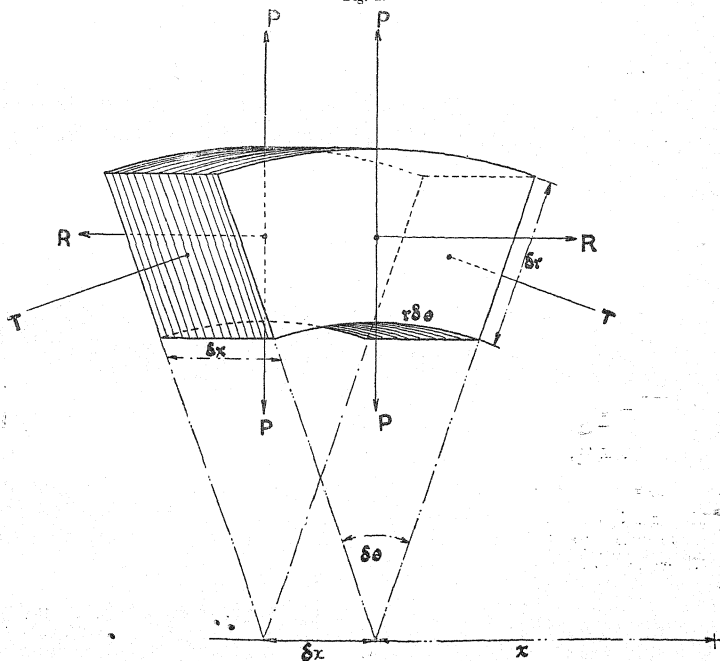
Let the longitudinal extension be denoted by

$$e_r = \frac{dl}{l}.$$

From Hooke's law, and considering the metal as homogeneous,

$$\left. \begin{aligned} e_t &= AT - B(P+R) \\ e_p &= AP - B(T+R) \\ e_r &= AR - B(P+T) \end{aligned} \right\} \dots \dots \dots (4)$$

Fig. 2.



where $A = \frac{1}{E}$ and $\frac{B}{A} = \sigma$ (Poisson's ratio), which is equal to $\frac{1}{3}$ for steel, so that

$$Ee_t = T - \sigma(P + R),$$

$$Ee_p = P - \sigma(T + R),$$

$$Ee_r = R - \sigma(P + T).$$

The interior and exterior radial pressures P act towards each other, and not as shown in the diagram, hence the sign of P must be changed in the last three equations, so that

$$\left. \begin{aligned} Ee_t &= T + \sigma(P - R) & (a) \\ Ee_p &= -P - \sigma(T + R) & (b) \\ Ee_r &= R + \sigma(P - T) & (c) \end{aligned} \right\} \dots \dots \dots (5).$$

The values of e_t , e_p , e_r may be positive or negative: if positive, the strain is positive and represents an elongation; if negative, the strain represents a contraction.

It is usually assumed that R is constant, the constant value of R being taken as equal to the total longitudinal thrust $p_0 \pi r_0^2$ tons, divided by the area of the cross-section of the material of the gun, or, in considering the *initial stresses* in a state of repose, R may be put equal to zero. On the assumption that any plane cross-section at right angles to the axis always remains plane under the stresses in the material, then e_r is constant or zero; and from the equation $E.e_r = R + \sigma(P - T)$ the result follows that

$$T - P = \text{constant} = 2b, \text{ say} \dots \dots \dots (6).$$

Also from (2)

$$P + T = -r \frac{dP}{dr} \dots \dots \dots (7),$$

hence, by subtraction of (6) from (7), and employing the small letters p and t ,

$$2p = -r \frac{dp}{dr} - 2b,$$

$$2(p + b) = -r \frac{dp}{dr},$$

$$\frac{dp}{p + b} = -2 \cdot \frac{dr}{r},$$

$$\log_e(p + b) = \log_e ar^{-2},$$

where a is a constant of integration.

$$p + b = \frac{a}{r^2},$$

$$p = \frac{a}{r^2} - b \dots \dots \dots (8),$$

therefore, from (6),

$$t = \frac{a}{r^2} + b \dots \dots \dots (9),$$

where a and b are constants whose values depend upon the data in any particular case.

Suppose for a single tube that the internal (radial) pressure p_0 be known, as also the external (radial) pressure p_1 , then

$$\left. \begin{aligned} p_0 &= \frac{a}{r_0^2} - b \\ p_1 &= \frac{a}{r_1^2} - b \end{aligned} \right\} \dots \dots \dots (10),$$

from which

$$\left. \begin{aligned} a &= \frac{r_1^2 r_0^2}{r_1^2 - r_0^2} (\rho_0 - p_1) \\ b &= \frac{\rho_0 r_0^2 - p_1 r_1^2}{r_1^2 - r_0^2} \end{aligned} \right\} \dots \dots \dots (11),$$

and in the interior of the tube, the radial pressure p at a point distant r from the axis of the tube is

$$p = \frac{a}{r^2} - b,$$

and the circumferential tensions are

$$\left. \begin{aligned} t_0 &= \frac{a}{r_0^2} + b \\ t_1 &= \frac{a}{r_1^2} + b \\ t &= \frac{a}{r^2} + b \end{aligned} \right\} \dots \dots \dots (12),$$

When $p_1 = 0$, then

$$\left. \begin{aligned} p &= p_0 \frac{r_0^2}{r^2} \cdot \frac{r_1^2 - r^2}{r_1^2 - r_0^2} \\ t &= p_0 \frac{r_0^2}{r^2} \cdot \frac{r_1^2 + r^2}{r_1^2 - r_0^2} \end{aligned} \right\} \dots \dots \dots (13),$$

also $\frac{a}{r_1^2} = b$ from (10); from (10), (12)

$$t_0 + p_0 = \frac{2a}{r_0^2}$$

and

$$t_0 - p_0 = 2b = \frac{2a}{r_1^2}$$

so that

$$\frac{r_1}{r_0} = \sqrt{\frac{t_0 + p_0}{t_0 - p_0}}.$$

From this it is seen that no thickness is sufficient to stand an internal pressure p_0 greater than t_0 if the exterior of the tube is unsupported, but this drawback is overcome by exterior reinforcing hoops shrunk on to an assigned initial tension.

As in Part I of Text-Book Gunnery, the different kinds of stresses set up in a gun are distinguished by symbols. These are now reproduced.

Powder stresses are those set up by the radial pressures of the powder gas throughout the mass of the gun. The symbols for these are P , T .

The *initial stresses* are those set up by the shrinking of one tube or hoop over another, or by the winding on of wire at considerable tension over a tube. The symbols for these are (p) , (t) .

The *firing stresses* are those which are created in the made up-gun when the powder charge is fired; it is therefore made up of both the powder and the initial stresses. The symbols for these are p , t .

So that firing stress = powder stress + initial stress.

Summing up, the symbols are

Powder stresses	P , T .
Firing	„	p , t .
Initial	„	(p) , (t) .

In any cross-section of a gun, distances are denoted by $r_0, r_1, r_2 \dots r_n$, where each is the radius in inches of the internal radius of the 1st (or inner tube) of the 2nd, 3rd ... n^{th} hoops respectively, so that r_0 denotes the radius of the surface exposed to the powder pressure, whilst r_3 would denote the common cylindrical surface which is the exterior surface of the 3rd hoop and the interior surface of the 4th hoop.

Powder Stress.

In dealing with powder stresses, all consideration of the initial stresses in the completed gun is left alone, the problem then amounts to this:—

A pressure P_0 is applied to the interior of the inner tube, suppose the gun consist of an inner tube and three hoops, what will be the radial pressure and circumferential tension produced at any part of the gun, due to the powder pressure P_0 ?

In dealing with this, the whole gun is considered as one homogeneous cylinder of internal radius r_0 and external radius r_3 , as though the surfaces of separation between any two hoops did not exist. This holds good, no matter how many hoops the gun may have.

Initial and Firing Stresses.

In these there is not a continuity of circumferential stress at the surfaces of separation between any two hoops, and the hoop tension can (and often does) change suddenly. It is therefore necessary to use different symbols for each surface between two hoops, as follows, for initial stresses:—

$(t_0), (t_1), (t_2), (t_3), (t_4)$ for the inner surface of each hoop,

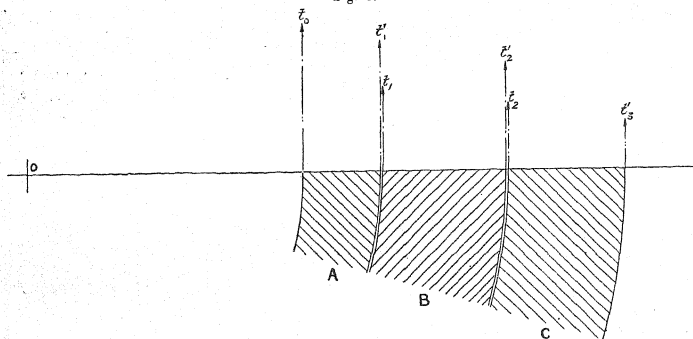
$(t'_1), (t'_2), (t'_3), (t'_4)$ for the outer surface of each hoop.

So, in the firing stresses:—

t_0, t_1, t_2, t_3 for the inner surface of each hoop,

t'_1, t'_2, t'_3 for the outer surface of each hoop.

Fig. 3.



So that

t_0 and t'_1 apply to tube A,

t_1 and t'_2 apply to tube B,

t_2 and t'_3 apply to tube C,

t_3 is outside C, and therefore does not enter with a gun of three tubes only.

Radially the pressure is the same at the surface of separation, so that there is no need of suffixed symbols for the radial stress.

Firing Stresses p, t .

Suppose the values r_0, r_1, r_2, r_3 , &c., at any section of the bore or chamber of a gun are known, we have two limits assigned, viz. :—

$$\begin{array}{l} \text{Carbon steel} \left\{ \begin{array}{l} \text{Tension of material for inner tube is 15 tons/in.}^2 \text{ (maximum permissible),} \\ \text{Tension of material for outer tubes is 18 tons/in.}^2 \text{ (maximum permissible),} \end{array} \right. \\ \text{Nickel Steel} \left\{ \begin{array}{l} \text{Tension of material for inner tube is 20 tons/in.}^2 \text{ (maximum permissible),} \\ \text{Tension of material for outer tubes is 24 tons/in.}^2 \text{ (maximum permissible),} \end{array} \right. \end{array}$$

then to find the firing stresses at any part of the section in the various hoops, we start with the outer hoop of the gun, and working inwards are enabled to find the maximum pressure in the bore; this maximum pressure is our maximum powder pressure, and knowing this the various powder stresses are found.

Deducting the powder stresses from the firing stresses gives the initial stresses, and then the amount of shrinkage is calculated which will produce the required initial stresses.

The maximum allowable pressure p_0 inside the gun is theoretical. As a matter of precaution no gun is allowed to be subjected to the full amount calculated, the charges being so arranged that the pressure shall not exceed a normal chamber pressure varying from $\frac{2}{3}$ to $\frac{1}{2}$ of this p_0 , so that under ordinary conditions the elastic limit of no part of the material may be reached, or permanent extension take place. Thus if it is found that $p_0 = 20$ tons/in.², the working pressure would vary from $20 \times \frac{2}{3}$ to $20 \times \frac{1}{2}$, i.e., from $13 \cdot 3$ to 10 tons/in.², as the case may be.

This working pressure, or normal chamber pressure, is that pressure which should not be exceeded by the ordinary service charge, and in the case of cordite the temperature of the charge is fixed at 80°F . It might be called the specification pressure.

The actual pressure which a charge does produce, as ascertained by means of the crusher gauge and coppers, is frequently below the working pressure, and is dependent on such things as wear of the gun, temperature of charge, &c.

Gunmakers Formula.

Consider any tube, the $(n+1)$ th, of a built-up gun; t_n and p_{n+1} are probably known, then from the fundamental formulas (8) and (9),

$$t_n = \frac{a}{r_n^2} + b,$$

$$p_{n+1} = \frac{a}{r_{n+1}^2} - b,$$

$$p_n = \frac{a}{r_n^2} - b,$$

whence

$$p_n - p_{n+1} = \frac{a}{r_n^2 r_{n+1}^2} (r_{n+1}^2 - r_n^2),$$

$$t_n + p_{n+1} = \frac{a}{r_n^2 r_{n+1}^2} (r_{n+1}^2 + r_n^2),$$

therefore

$$\frac{p_n - p_{n+1}}{t_n + p_{n+1}} = \frac{r_{n+1}^2 - r_n^2}{r_{n+1}^2 + r_n^2},$$

$$p_n = \frac{r_{n+1}^2 - r_n^2}{r_{n+1}^2 + r_n^2} (t_n + p_{n+1}) + p_{n+1}.$$

This formula is of great importance, and will be found very useful; it is generally called the gunmaker's formula.

A complete example is worked out in Part I, p. 163; in this example t_0 is taken as 15 and $t_1 = t_2 = 18$ tons/in.²: also

$$\begin{aligned} r_0 &= 4, & r_0^2 &= 16, \\ r_1 &= 5.6, & r_1^2 &= 31.36, \\ r_2 &= 8.7, & r_2^2 &= 75.69, \\ r_3 &= 11.8, & r_3^2 &= 139.24. \end{aligned}$$

The firing stresses are found by working from the outside tube first and making use of the gunmaker's formula; thus

$$\begin{aligned} p_3 &= 0, \\ p_2 &= \frac{r_3^2 - r_2^2}{r_3^2 + r_2^2} (t_2 + p_3) + p_3 = 5.32, \\ p_1 &= \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2} (t_1 + p_2) + p_2 = 15.0, \\ p_0 &= \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2} (t_0 + p_1) + p_1 = 24.74. \end{aligned}$$

The hoop tensions are found at once from formula

$$t'_n - p_n = t_{n-1} - p_{n-1} \quad \dots \quad (15).$$

Since in any one hoop (the n^{th})

$$t'_n = \frac{a}{r_n^2} + b; \quad p_n = \frac{a}{r_n^2} - b. \quad \text{Therefore} \quad t'_n - p_n = 2b.$$

$$t'_{n-1} = \frac{a}{r_{n-1}^2} + b; \quad p_{n-1} = \frac{a}{r_{n-1}^2} - b. \quad \text{Therefore} \quad t'_{n-1} - p_{n-1} = 2b.$$

Therefore

$$t'_n - p_n = t'_{n-1} - p_{n-1}.$$

Applying this,

$$\text{Jacket} \begin{cases} t'_3 = (t_2 - p_2) + p_3 = 18 - 5.32 + 0 = 12.68, \\ t_2 = 18, \end{cases}$$

$$\text{Breech-piece} \begin{cases} t'_2 = (t_1 - p_1) + p_2 = 18 - 15 + 5.32 = 8.32, \\ t_1 = 18, \end{cases}$$

$$\text{Tube} \begin{cases} t'_1 = (t_0 - p_0) + p_1 = 15 - 24.74 + 15 = 5.26, \\ t_0 = 15. \end{cases}$$

Powder Stress: P, T.

The maximum allowable pressure has been found to be 24.74 tons/in.², this therefore is the maximum powder pressure in the interior of the bore.

Therefore

$$P_0 = 24.74.$$

To obtain the powder stresses at other parts of the gun, we treat the whole gun as one homogeneous mass of internal radius r_0 and external radius r_s .

$$P_0 = 24.74 = \frac{a}{r_0^2} - b,$$

$$P_s = 0 = \frac{a}{r_s^2} - b,$$

at any point

$$P = \frac{a}{r^2} - b,$$

$$T = \frac{a}{r^2} + b,$$

and a, b are the same constants throughout tube, breech-piece, and jacket.

Solving the equations, we have

$$a = \frac{r_0^2 r_s^2}{r_s^2 - r_0^2} p_0; \quad b = \frac{r_0^2}{r_s^2 - r_0^2} p_0.$$

Therefore

$$P = \frac{r_0^2}{r_s^2 - r_0^2} \left(\frac{r_s^2}{r^2} - 1 \right) = \frac{r_0^2}{r^2} \cdot \frac{r_s^2 - r^2}{r_s^2 - r_0^2} p_0$$

$$T = \frac{r_0^2}{r_s^2 - r_0^2} \left(\frac{r_s^2}{r^2} + 1 \right) = \frac{r_0^2}{r^2} \cdot \frac{r_s^2 + r^2}{r_s^2 - r_0^2} p_0$$

Therefore

$$P_0 = 24.74,$$

$$P_1 = \frac{16}{31.36} \cdot \frac{139.24 - 31.36}{139.24 - 16} 24.74 = 11.1,$$

$$P_2 = \frac{16}{75.69} \cdot \frac{139.24 - 75.69}{123.24} 24.74 = 2.7,$$

$$P_3 = 0.$$

$$T_0 = \frac{16}{16} \cdot \frac{139.24 + 16}{139.24 - 16} \cdot 24.74 = 31.2,$$

$$T_1 = (T_0 - P_0) + P_1 = 17.56,$$

$$T_2 = (T_0 - P_0) + P_2 = 9.16,$$

$$T_3 = (T_0 - P_0) + P_3 = 6.46.$$

Initial Stress: (p), (t).

These are got by subtracting powder from firing stresses.

$$(p_0) = p_0 - P_0 = 24.74 - 24.74 = 0,$$

$$(p_1) = p_1 - P_1 = 15 - 11.1 = 3.9,$$

$$(p_2) = p_2 - P_2 = 5.32 - 2.7 = 2.62,$$

$$(p_3) = p_3 - P_3 = 0 - 0 = 0,$$

$$\text{Tube} \begin{cases} (t_0) = t_0 - T_0 = 15 - 31.2 = -16.2, \\ (t_1) = t_1 - T_1 = 5.26 - 17.56 = -12.30, \end{cases}$$

$$\text{Breech-piece} \begin{cases} (t_1) = t_1 - T_1 = 18 - 17.56 = 0.44, \\ (t_2) = t_2 - T_2 = 8.32 - 9.16 = -0.84, \end{cases}$$

$$\text{Jacket} \begin{cases} (t_2) = t_2 - T_2 = 18 - 9.16 = 8.84, \\ (t_3) = t_3 - T_3 = 12.68 - 6.46 = 6.22. \end{cases}$$

Figs. 8 and 9, facing p. 165 of Part I, 1907, show the various stresses for this 6-inch gun to one place of decimals. A similar procedure will apply for a gun built up of four or more parts.

With a 6-inch gun built up with carbon steel and of the dimensions just given, suppose a working chamber pressure of 16 tons/in.² is produced, and it is required to know the firing stresses.

The initial stresses already found hold good, the powder stresses are

$$P_0 = 16,$$

$$P_1 = 11 \cdot 1 \frac{16}{24 \cdot 74} = 7 \cdot 178,$$

$$P_2 = 2 \cdot 7 \frac{16}{24 \cdot 74} = 1 \cdot 746,$$

$$P_3 = 0,$$

$$T_0 = 31 \cdot 2 \frac{16}{24 \cdot 74} = 20 \cdot 18,$$

$$T_1 = 17 \cdot 56 \frac{16}{24 \cdot 74} = 11 \cdot 36,$$

$$T_2 = 9 \cdot 16 \frac{16}{24 \cdot 74} = 5 \cdot 924,$$

$$T_3 = 6 \cdot 46 \frac{16}{21 \cdot 74} = 4 \cdot 178.$$

Combining these powder stresses with the initial stresses already found, gives the required firing stresses when the chamber pressure is that produced by the service charge.

$$p_0 = 16,$$

$$p_1 = 11 \cdot 078,$$

$$p_2 = 4 \cdot 366,$$

$$p_3 = 0,$$

$$t_0 = 3 \cdot 98,$$

$$t'_1 = - 0 \cdot 94,$$

$$t_1 = 11 \cdot 80,$$

$$t'_2 = 5 \cdot 084,$$

$$t_2 = 14 \cdot 764,$$

$$t'_3 = 10 \cdot 398.$$

Case of a Cracked Tube.

If the A tube is cracked through longitudinally on both sides, its tensile strength is nil and the gun will not stand the same strain as before.

Thus in the 6-inch gun worked out, from the firing radial pressures,

$$p_1 = 15; \quad p_0 = 24 \cdot 74;$$

but with the A tube cracked p_0 will not be so large: the two outer tubes being undamaged we get as before

$$p_1 = 15.$$

Although the tensile strength of the A tube is nil, yet it serves to diminish the surface over which the powder pressure acts, and therefore

$$p_0 = p_1 \frac{r_1}{r_0} \dots \dots \dots (16).$$

This may be seen also from considering the equilibrium of the A tube; the forces acting on this tube are radial pressures,

$$p_1 = 15 \text{ tons/in.}^2 \text{ over a surface } 2\pi r_1, \text{ acting inwards,}$$

$$p_0 \text{ over a surface } 2\pi r_0, \text{ acting outwards,}$$

therefore

$$(2\pi r_1) p_1 = (2\pi r_0) p_0$$

$$p_0 = p_1 \frac{r_1}{r_0},$$

$$= 15 \cdot \frac{5 \cdot 6}{4} = 21 \cdot 0.$$

So that our maximum allowable pressure has been reduced from 24.74 to 21.0 tons/in.², due to the A tube being cracked.

Liners.

In the case of liners, no strength is accredited; for, being put in without shrinkage, they are taken as so much packing, and their effect as regards calculation of strength might be ignored but for the fact that they distribute the strain to a larger area. Of course, here, as in the case of shrinkage friction, with reference to longitudinal strength, any circumferential strength derived from the liner will be in addition to that calculated for.

Supposing the gun to have been designed for, and constructed originally with, a liner, then if p_0 represents the internal pressure on the liner, and p that transmitted to the interior of the A tube, and r_0 and r_1 the respective radii, the formula is simply

$$p_0 = p \frac{r}{r_0} \dots \dots \dots (17),$$

the liner acting as if cracked or segmental.

The Longitudinal Tension in the Gun.

Practically it is usual to take the longitudinal tension as uniform across a cross-section and as due to the powder pressure in the bore, treated as a closed vessel, closed at one end by the breech-screw, and at the other by the shot.

Thus supposing a breech-screw to gear into an A tube of which the internal radius is r_0 and the external radius of the gun is r_2 , taking P_0 as the powder pressure, the average value R of the longitudinal tension will be found as follows:—The pressure multiplied by the sectional area of the chamber is resisted by a cross-section of the metal subjected to longitudinal tension (according to the design of the gun); for equilibrium, these must be equal, therefore in this case

$$P_0 \pi r_0^2 = R \pi (r_2^2 - r_0^2)$$

$$R = P_0 \frac{r_0^2}{r_2^2 - r_0^2} \text{ tons/in.}^2 \dots \dots \dots (18).$$

In all steel guns of modern construction the breech-screw gears into the layer of metal above the A tube; in smaller guns direct; in heavier pieces of the most recent construction

by means of a steel bush; the inner tube is thus relieved of longitudinal stress at the breech. For a gun consisting of a tube and jacket only, the formula would then become

$$R = P_0 \frac{r_0^2}{r_2^2 - r_1^2}.$$

Considering the longitudinal strength of the 6-inch B.L. gun,

$$P_0 = \frac{r_2^2 - r_1^2}{r_0^2} R;$$

with the given numerical values and putting $R = R_1 = 18$ gives $P_0 = 121$, and dividing this by a normal pressure of 17 tons, we get a longitudinal factor of safety = 7.1.

In a heavy modern wire-built gun the longitudinal stress is taken up by the breech-piece and the jacket, but the jacket may not be taking its full share of the work, owing to the necessary connections between the two; hence a large factor of safety is obtained by considering the breech-piece only; let r and r' denote its internal and external radii, then

$$R = P_0 \frac{r_0^2}{r'^2 - r^2}.$$

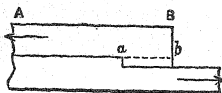
Practically the longitudinal strength is considered separately from the circumferential, and is specially provided for by shoulder, of which the *resistance to shearing* constitutes the longitudinal strength as calculated. No account is taken of frictional grip due to shrinkage, for it is considered as extremely probable that at the critical moment this becomes loosened by the elasticity of the different layers asserting itself more rapidly towards the interior as soon as the highest pressure has passed, while there is still a considerable longitudinal stress.

It is considered inadvisable to rely upon shrinkage in any way for longitudinal strength, and, consequently, any strength in this direction derived from the frictional grip will be in addition to the calculated strength.

The strain sustained by a shoulder is taken as a purely shearing one, and the strength of a shoulder is consequently dependent on its length; shearing strength, like resistance to tension, being directly proportional to the extent of surface where separation would take place. The strength is also here taken to be the same (in tons per square inch), which, if not strictly true, is rather in favour of the shearing strength.

The calculation, therefore, of length of shoulder for a given hoop is simple. For if AB is the supporting hoop of internal and external radii r_1, r_2 and length of shoulder $ab (= l$ say), it

Fig. 4.



is only necessary to make the cylindrical area of $ab =$ the annular sectional area of the hoop, or

$$\pi (r_2^2 - r_1^2) = 2\pi r_1 l,$$

whence

$$l = \frac{r_2^2 - r_1^2}{2r_1}.$$

The actual longitudinal strength of this arrangement would appear to be

$$\pi (r_2^2 - r_1^2) T,$$

or

$$2\pi r_1 l T,$$

T being the resistance to rupture by tension or shearing in tons per square inch of material where separation would take place.

Wire Gun Construction.

In a built-up gun, as seen from fig. 8, p. 165, of Part I, the curves of circumferential firing tensions show that the resistance offered by any tube when the gun is fired is unevenly distributed in the metal, the inner taking an undue share of resistance.

Great economy of material would be effected if we could make all the circumferential fibres take up a full uniform tension on firing; but to secure this condition only approximately, the number of layers of metal would have to be largely increased, and the cost, complication, and time of manufacture of a gun would be enormous.

But by adopting Mr. J. A. Longridge's plan of strengthening the tube by steel wire, wound round with appropriately varying tension, we are able to make the curve of circumferential firing tension a straight line for a given powder pressure, and now all parts of the wire coil are equally strained under the interior pressure, and take an equal share in the resistance.

For full theoretical investigation of this subject, see Mr. Longridge's "Treatise on the Application of Wire to the Construction of Ordnance" (1884), and a paper of 1887, "Further Investigations regarding Wire Gun Construction," also a Work by Lieutenant G. Moch, "Les Canons à Fils d'Acier."

By winding on many layers of steel wire on an A tube, we can get every layer of the wire coil to do its full share of resistance when the gun is fired, and so greatly strengthen the gun, provided each layer of wire is wound on with a properly adjusted tension.

A steel hoop shrunk over an A tube must not be subjected to a firing tension of over 18 to 24 tons/in.², whereas steel wire can safely be subjected to a tension of 50 tons/in.², therefore we have not only a much strengthened gun by using steel wire, but there is a considerable saving in the weight of metal to be used.

In a gun which has steel hoops shrunk on, the curve of radial pressure for each hoop is given by

$$p = \frac{a}{r^2} - b,$$

and the circumferential tension by

$$t = \frac{a}{r^2} + b,$$

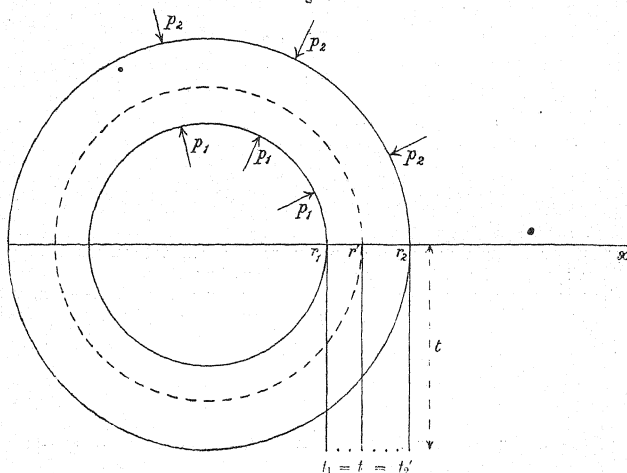
where a and b are constants to be determined for each hoop or tube.

These equations do not hold good for the firing stresses in a wire coil which has been wound round on an A tube; because the winding tension of each layer of wire has been adjusted so that the firing tension may be the same throughout the coil.

The jacket is required for the protection of the wire from damage and to provide the necessary longitudinal strength; it is fitted over the wire without any appreciable shrinkage.

When the gun is at rest the jacket will be free from stress, but when the gun is fired we may suppose the stress in it to be the powder stress only, on the assumption that the gun behaves as if homogeneous.

Fig. 5.



In fig. 5 let r_1 and r_2 be the internal and external radii of the wire coil, and let a powder pressure p_0 be applied so that the radial firing pressures at r_1 , r_2 be p_1 , p_2 respectively, and the tension throughout $= t$, so that

$$t_1 = t_2' = t.$$

To find the radial pressure p at any point r in the wire coil.

Consider the equilibrium of the portion of the wire between r_1 and r ; we have

$$p_1 r_1 = p r + t (r - r_1). \quad (19).$$

If the portion of wire coil between r and r_2 be considered, we have

$$p r = p_2 r_2 + t (r_2 - r),$$

and for the whole coil from r_1 to r_2 ,

$$p_1 r_1 = p_2 r_2 + t (r_2 - r_1).$$

From these last two equations we have, by subtraction,

$$p_1 r_1 - p r = t (r - r_1),$$

or

$$p_1 r_1 = p r + t (r - r_1),$$

which is equation (19) over again.

By writing x for r and y for p ,

$$p_1 r_1 = x y + t (x - r_1),$$

or

$$x (y + t) = (p_1 + t) r_1 = \text{const.} = A,$$

and moving the axis of x parallel to itself to a distance t , this reduces to

$$x y = \text{const.}, \text{ a rectangular hyperbola.}$$

From equation (19), $(p+t)r = p_1r_1 + t_1r_1 = \text{a constant, say } A$, therefore

$$p+t = \frac{A}{r} \quad \dots \dots \dots (20).$$

This gives a convenient equation to find the radial pressure at any point in the wire coil; the curve of these radial pressures is a rectangular hyperbola.

In the A tube and jacket only, the equations

$$p = \frac{a}{r^2} - b,$$

$$t = \frac{a}{r^2} + b,$$

hold good to find the stress at any point.

In wire guns, the limits which are usually accepted are as follows:—

MAXIMUM firing tensions to which the metal may be subjected, unless instructions to the contrary are given.

	Inner A tube.	Wire Coil.
Carbon steel	15 tons/in. ²	—
Nickel "	20 " "	—
Wire	—	40 tons/in. ²

Maximum compression of any tube not to exceed 26 tons/in.².

The radius of bore to be measured from the axis of piece to the surface of the land.

In calculating the strength of wire guns, no allowance is to be made for any shrinkage that may be given to the jacket.

In wire guns, at present, the wire is wound on to the A tube after the inner A tube has been fitted into it, and so at first the A tube is not fully compressed, which gives an advantage in the first life of the gun, but on boring out for lining then the full compression takes place, and the advantage referred to is lost.

Similarly in the 6-inch Q.F., Mark II, where the 1-B tube is shrunk on to the A tube under the wire, the metal under the wire coil should be treated as homogeneous, because in boring out for lining the A tube will be compressed, and the advantage of the shrinkage lost.

The slight shrinkage between the inner A and A tubes being neglected, it follows that the limits of tension (18 and 24 tons/in.²) do not enter into the calculations for various stresses in the built-up gun. Also, the limit of 40 tons/in.² for the firing tension of the wire must be considered as a guide only, the 40 tons/in.² may be exceeded in some cases up to about 50 tons/in.²

The gunmaker fixes the diameter of the chamber, the thickness of the metal in the various layers and the length of travel of the shot, &c.; in this experience is the best guide. These points being settled on he can now find, from the limits assigned, what p_0 , the maximum allowable powder-pressure in the bore, and also the corresponding circumferential tension in the wire coil in firing. Should the latter be excessive, or p_0 too small, then he can manipulate the radii until the desired result is attained.

The various radii having been fixed, the following examples show all the calculations of the stresses.

The dimensions across the 12" B.L., XI., across middle of chamber are:—

$$r_0 = 8.425 \text{ inches,}$$

$$r_1 = 11.74 \quad ,,$$

$$r_2 = 15.31 \quad ,,$$

$$r_3 = 20.11 \quad ,,$$

$$r_4 = 23.9 \quad ,,$$

so that

$$r_1 - r_0 = 3.315, \text{ the thickness of inner A tube,}$$

$$r_2 - r_1 = 3.57 \quad ,, \quad \text{A tube,}$$

$$r_3 - r_2 = 4.80 \quad ,, \quad \text{wire coil.}$$

$$r_4 - r_3 = 3.79 \quad ,, \quad \text{jacket.}$$

The limits are

$$t_0 = 20,$$

$$(t_0) = -26,$$

hence, since firing tension = powder tension + initial tension,

$$t_0 = T_0 + (t_0).$$

Therefore

$$T_0 = 20 - (-26) = 46.$$

Treating the whole gun (inner A, tube, wire coil and jacket) as one homogeneous mass for the powder stress,

$$P_4 = 0 = \frac{a}{r_4^2} - b,$$

$$T_0 = 46 = \frac{a}{r_0^2} + b,$$

and at any distance r , the radial pressure

$$P = \frac{a}{r^2} - b.$$

From the first two equations

$$a = \frac{r_4^2 r_0^2}{r_4^2 + r_0^2} \cdot T_0; \quad b = \frac{r_0^2}{r_4^2 + r_0^2} \cdot T_0,$$

so that

$$P = \frac{r_0^2}{r^2} \cdot \frac{r_4^2 - r^2}{r_4^2 + r_0^2} \cdot T_0,$$

from which

$$P_0 = \frac{r_4^2 - r_0^2}{r_4^2 + r_0^2} \cdot 46 = 35.84,$$

$$P_1 = \frac{r_0^2}{r_1^2} \cdot \frac{r_4^2 - r_1^2}{r_4^2 + r_0^2} \cdot 46 = 15.99,$$

so

$$P_2 = 7.32,$$

$$P_3 = 2.1,$$

$$P_4 = 0.$$

Again

$$T_4 - P_4 = 2b = T_3 - P_3 = T_2 - P_2 = T_1 - P_1 = T_0 - P_0.$$

Therefore

$$T_4 = (T_0 - P_0) + P_4 = 10 \cdot 16 + 0 = 10 \cdot 16,$$

$$T_3 = 10 \cdot 16 + P_3 = 12 \cdot 26,$$

$$T_2 = 10 \cdot 16 + P_2 = 17 \cdot 48,$$

$$T_1 = 26 \cdot 15,$$

$$T_0 = 46.$$

This completes the powder stresses.

Firing stress, p , t .

In the A tube (which includes the inner A tube, both being regarded as homogeneous).

$$p_0 = \frac{r_2^2 - r_0^2}{r_2^2 + r_0^2} (t_0 + p_2) + p_2 \text{ (gunmaker's formula),}$$

from which, since $p_0 = 35 \cdot 84 = P_0$ and $t_0 = 20$, $p_2 = 16 \cdot 4$, so

$$p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2} (t_0 + p_1) + p_1, \text{ and } p_1 = 22 \cdot 3,$$

also

$$t'_1 = t_1 = (t_0 - p_0) + p_1 = -15 \cdot 8 + 22 \cdot 3 = 6 \cdot 5,$$

$$t'_2 = (t_0 - p_0) + p_2 = -15 \cdot 8 + 16 \cdot 4 = 0 \cdot 6.$$

In the wire coil each layer of wire is wound on, so that on firing the tension is the same this gives that $t_2 = t'_3 = t$ throughout.

Then, from considering the equilibrium of the whole coil,

$$p_2 r_2 = p_3 r_3 + t (r_3 - r_2),$$

$$t = \frac{p_2 r_2 - p_3 r_3}{r_3 - r_2} = 43 \cdot 47.$$

Therefore

$$t_3 = t'_3 = t = 43 \cdot 47.$$

In the jacket there is no initial stress, hence

$$t_3 = T_3 = 12 \cdot 26; \quad p_3 = P_3 = 2 \cdot 1,$$

$$t'_4 = T'_4 = 10 \cdot 2; \quad p_4 = P_4 = 0.$$

This completes the firing stresses.

Initial stresses (p), (t).

Inner A tube	$\begin{cases} (p_0) = p_0 - P_0 = 0 & : (t_0) = t_0 - T_0 = -26. \\ (p_1) = p_1 - P_1 = 6 \cdot 31 & : (t'_1) = t'_1 - T_1 = -19 \cdot 65. \end{cases}$
A tube	$\begin{cases} (p_1) = 6 \cdot 31 & : (t_1) = t_1 - T_1 = -19 \cdot 65. \\ (p_2) = p_2 - P_2 = 9 \cdot 08 & : (t'_2) = t'_2 - T_2 = -16 \cdot 88. \end{cases}$
Wire coil	$\begin{cases} (p_2) = 9 \cdot 08 & : (t_2) = 26. \\ (p_3) = 0 & : (t'_3) = 31 \cdot 21. \end{cases}$
Jacket	$\begin{cases} (p_3) = 0 & : (t_3) = 0. \\ (p_4) = 0 & : (t'_4) = 0. \end{cases}$

The powder and initial stresses above tabulated for the 12 inches across the chamber are shown on fig. 6. the firing stresses are on fig. 7.

In the 12-inch B.L. Mark XI, at a distance of 35 inches from the seat of the shot, the dimensions are:—

$r_0 = 6$	inches
$r_1 = 9.29$	„
$r_2 = 11.83$	„
$r_3 = 14.95$	„
$r_4 = 17.95$	„

The various stresses areas follow:—

With limits $t_0 = 20$, $(t_0) = -26$,

$P_0 = 36.76$	$p_0 = 36.76$	$(p_0) = 0$
$P_1 = 12.64$	$p_1 = 20.2$	$(p_1) = 7.56$
$P_2 = 6.02$	$p_2 = 15.66$	$(p_2) = 9.64$
$P_3 = 2.04$	$p_3 = 2.04$	$(p_3) = 0$
$P_4 = 0$	$p_4 = 0$	$(p_4) = 0$
$T_0 = 46$	$t_0 = 20$	$(t_0) = -26$
$T_1 = 21.9$	$t'_1 = 3.44$	$(t'_1) = -18.46$
	$t_1 = 3.44$	$(t_1) = -18.46$
$T_2 = 15.26$	$t'_2 = -1.10$	$(t'_2) = -16.36$
$T_3 = 11.28$	$t_2 = t'_3 = 49.68$	$(t_2) = 34.42$
	$t_3 = 11.28$	$(t'_3) = 38.40$
$T'_4 = 9.24$	$t'_4 = 9.24$	$(t_3) = 0$
		$(t'_4) = 0$

At 105.6 inches from the seat of the shot in the 12-inch B.L. XI,

$r_0 = 6$	inches
$r_1 = 8.81$	„
$r_2 = 11.23$	„
$r_3 = 14.35$	„
$r_4 = 16.55$	„

and with limits $t_0 = 20$, $(t_0) = -26$,

$P_0 = 35.2$	$p_0 = 35.2$	$(p_0) = 0$
$P_1 = 13.4$	$p_1 = 20.4$	$(p_1) = 7.0$
$P_2 = 6.2$	$p_2 = 15.4$	$(p_2) = 9.2$
$P_3 = 1.63$	$p_3 = 1.63$	$(p_3) = 0$
$P_4 = 0$	$p_4 = 0$	$(p_4) = 0$
$T_0 = 46$	$t_0 = 20$	$(t_0) = -26$
$T_1 = 24.2$	$t'_1 = t_1 = 5.2$	$(t'_1) = t_1 = -19$
$T_2 = 17.0$	$t'_2 = 0.2$	$(t'_2) = -16.8$
$T_3 = 12.53$	$t_2 = t'_3 = 47.7$	$(t_2) = 30.7$
	$t_3 = 12.53$	$(t'_3) = 35.17$
$T'_4 = 10.8$	$t'_4 = 10.8$	$(t_3) = 0$
		$(t'_4) = 0$

FIGURE 6.

POWDER STRESSES.

ORDNANCE B.L. 12 IN. MK XI.

INITIAL STRESSES.

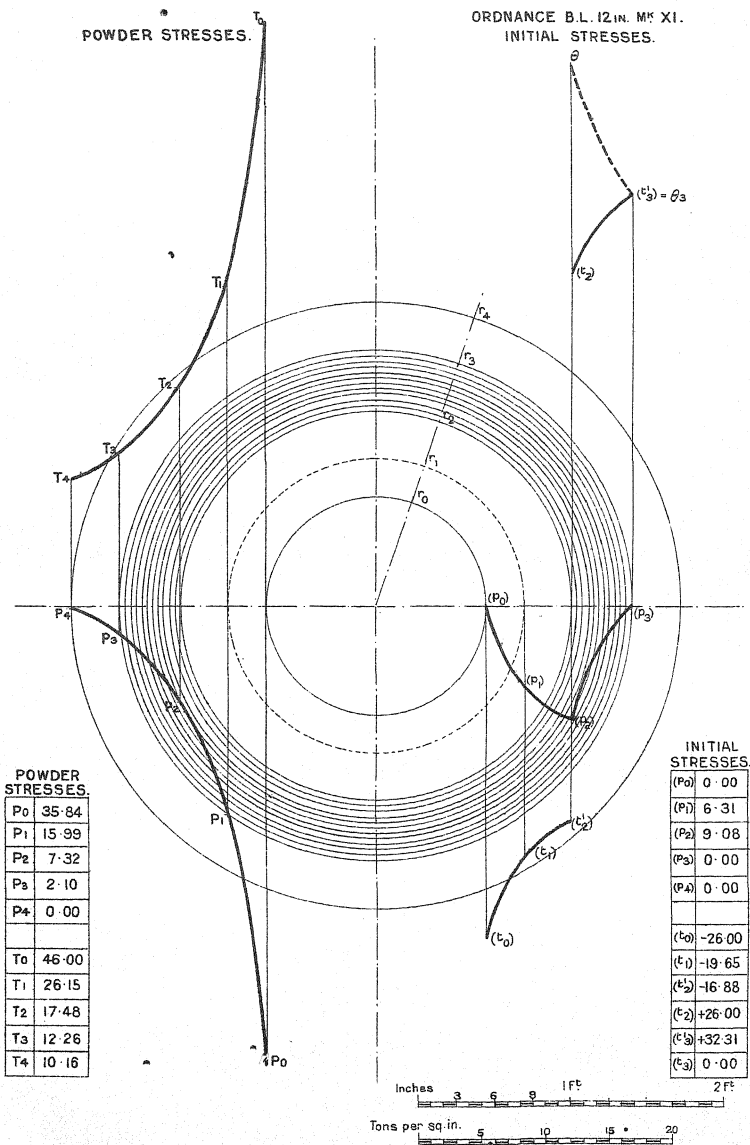
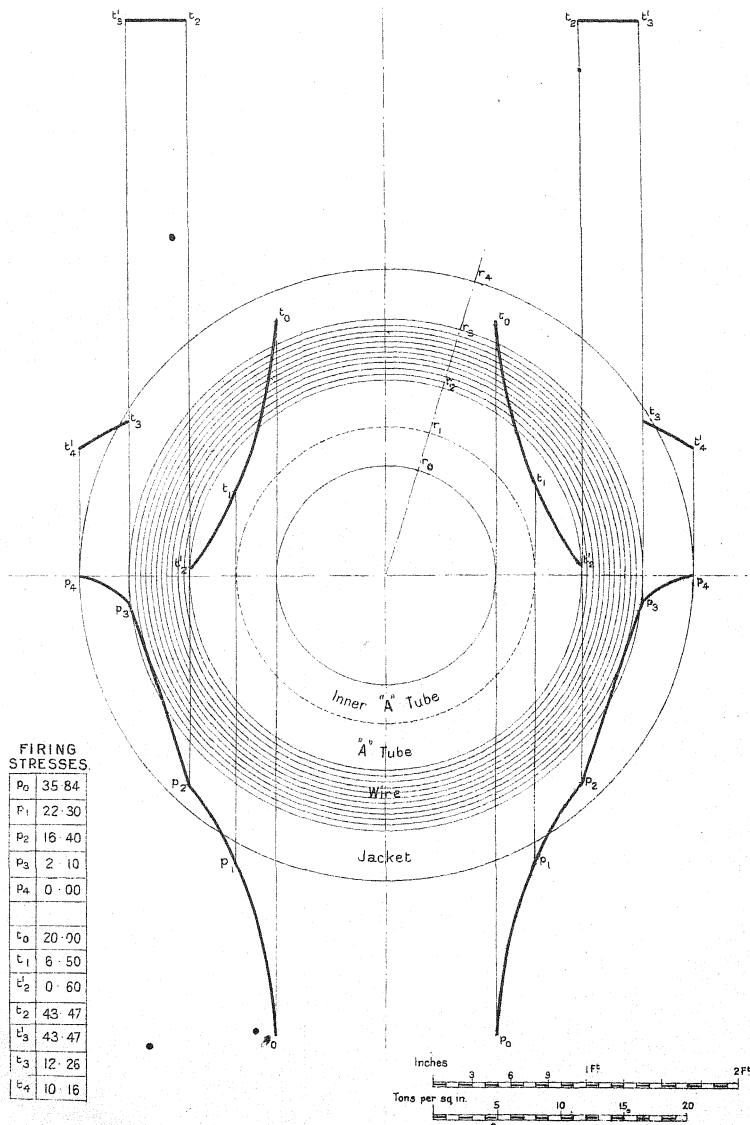




FIGURE 7.

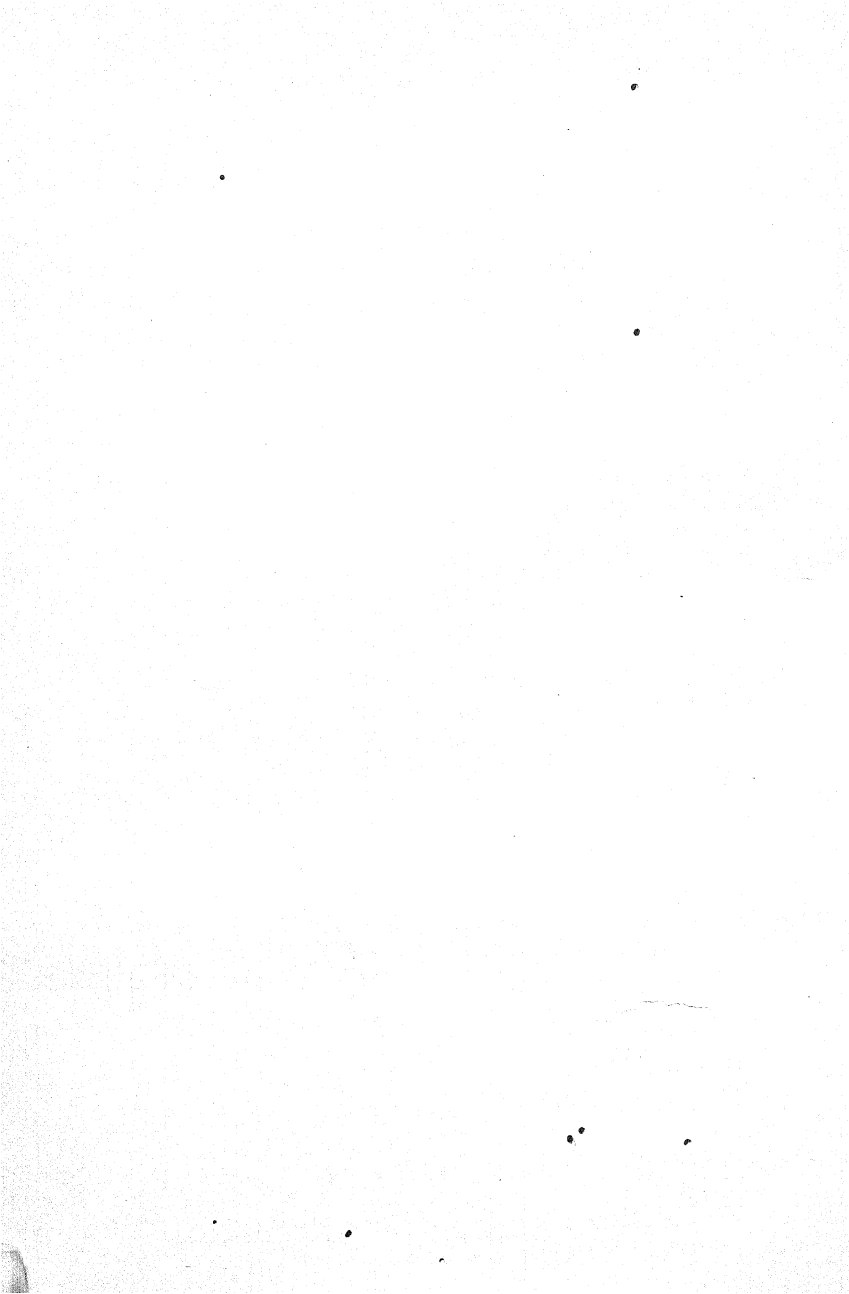
FIRING STRESSES.

ORDNANCE B.L. 12 IN. M⁵ X 1.



FIRING STRESSES.

p_0	35 84
p_1	22 30
p_2	16 40
p_3	2 10
p_4	0 00
t_0	20 90
t_1	6 50
t_2	0 60
t_3	43 47
t_4	43 47
t_5	12 26
t_6	10 16



Winding Tension.

As shown in Part I, p. 177, the requisite winding on tension at a point r is given by

$$\theta = (t) + \frac{r^2 + r_0^2}{r^2 - r_0^2} (p),$$

where (t) , (p) are the initial stresses at the point on the wire distant r from the centre of bore, and r_0 , r_1 , r_2 , r_3 are the internal radii of the inner A tube, A tube, wire coil, and jacket respectively.

The expression for θ becomes

$$\theta = \frac{A}{r} + \frac{B}{r - r_0} + \frac{C}{r + r_0},$$

where

$$A = -(t + p_3) r_3,$$

$$B = (t + p_2) r_3 - (t + p_0) r_0,$$

$$C = (t + p_3) r_3 + (t + p_0) r_0.$$

This last formula for θ is convenient, because A, B, and C are the same for each layer of wire.

The winding tension of the last layer of wire must be the same as the tension in repose, so that at r_3 ,

$$\theta_3 = (t_3),$$

and in the case of the 12-inch XI, across the middle of the powder chamber,

$$\theta_3 = (t_3) = 31 \cdot 21.$$

The winding on tension of the inner or first layer can be obtained directly from

$$\theta_2 = (t_2) + \frac{r_2^2 + r_0^2}{r_2^2 - r_0^2} (p_2) = 42 \cdot 7 = (t_2) + (t_3).$$

This value could be obtained equally well from

$$\theta_2 = \frac{A}{r_2} + \frac{B}{r_2 - r_0} + \frac{C}{r_2 + r_0}.$$

For practical purposes the $\theta_2 \theta_3$ curve is replaced by the straight line $\theta_2 \theta_3$, or else by a series of steps in which several layers are wound on at the same tension, followed by another series of layers at a tension reduced by the same amount from the previous series, and so on.

Shrinkage.

Shrinkage is the excess of the external diameter of the inner tube over the internal diameter of the outer tube, before they are put together and both are cold.

Fig. 8.

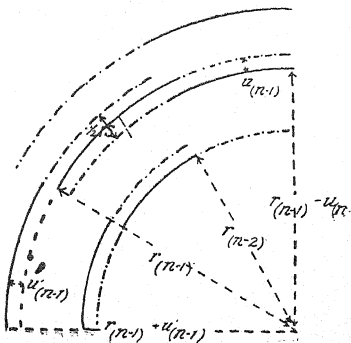
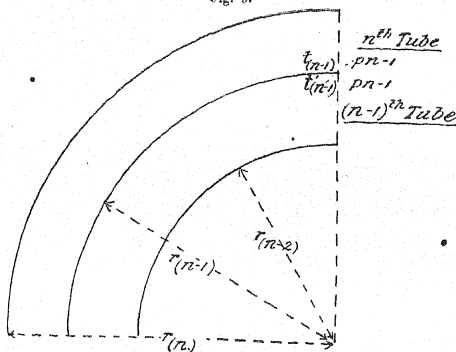


Fig. 9.



Hence, from the figure:—if S = shrinkage,

$$\frac{S}{2} = (r_{n-1} + u'_{n-1}) - (r_{n-1} - u_{n-1}) = u'_{n-1} + u_{n-1},$$

where u'_{n-1} and u_{n-1} are the radial displacements, from the unstrained position, of the outer and inner circumferential fibres of a tube and the superimposed hoop respectively, due to shrinkage.

Denoting the initial stresses at r_{n-1} in the completed gun by (t_{n-1}) , (t'_{n-1}) , and (p_{n-1}) , then from equation (a) of (5),

$$E \frac{u_{n-1}}{r_{n-1}} = E e_t = (t_{n-1}) + \sigma [(p_{n-1}) - R],$$

$$-E \frac{u'_{n-1}}{r_{n-1}} = -E e_t = (t'_{n-1}) + \sigma [(p_{n-1}) - R].$$

The negative sign is given against u'_{n-1} in the second equation because it denotes a compression, whereas u_{n-1} denotes an extension.

Hence by subtraction

$$\frac{E}{r_{n-1}} (u_{n-1} + u'_{n-1}) = (t_{n-1}) - (t'_{n-1}),$$

also

$$u_{n-1} + u'_{n-1} = \frac{S}{2},$$

therefore

$$_{n-1}S_n = \frac{2r_{n-1}}{E} [(t_{n-1}) - (t'_{n-1})].$$

This equation denotes the shrinkage between the $(n-1)^{\text{th}}$ and the n^{th} hoop; (t_{n-1}) is in tons/in.², $E = 12,500$; other dimensions are in inches.

The last equation may be written

$$E = \frac{(t_{n-1}) - (t'_{n-1})}{_{n-1}S_n / 2r_{n-1}},$$

so that the shrinkage is the elongation in a bar of the metal of unit section, and equal in length to the diameter $2r_{n-1}$ under a tension equal to the difference of the initial circumferential tensions at the common surface of the two hoops.

The values (σ_{n-1}) , (ℓ'_{n-1}) are the initial stresses, and as the powder pressure P_{n-1} at r_{n-1} increases them by equal amounts T_{n-1} to t_{n-1} , ℓ'_{n-1} the firing stresses, their difference is unaltered, so that

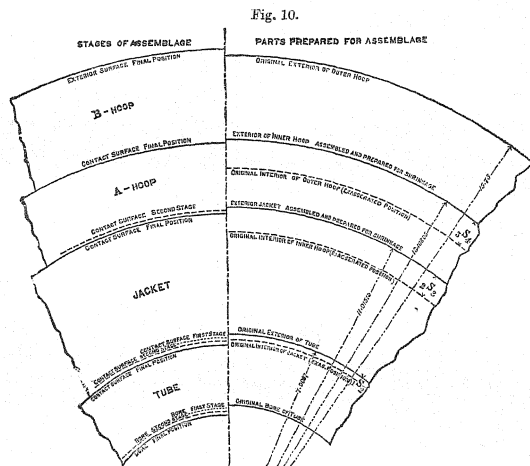
$$(\ell_{n-1}) - (\ell'_{n-1}) = t_{n-1} - \ell'_{n-1}.$$

Hence, so long as the shrinkage *between* hoops is considered, it can be calculated either from the firing or from the initial stresses; and it is independent of the shrinkage imparted at other surfaces of contact of the coils, provided it is calculated as the shrinkage of the parts before assemblage.

If, however, the shrinkage is estimated for the difference between the internal diameter of a coil and the external diameter of the finished portion of the gun, then the initial stresses already set up in the gun must be taken into account and deducted (*see* 1902 Edition, p. 258).

With several layers of metal the addition of each part that is shrunk on modifies the initial stresses previously existing.

This is illustrated in diagrams in the American "Notes on the Construction of Ordnance," Nos. 31, 33, 35, by Lieutenant Rogers Birnie, showing the shrinkage (enlarged 50 times) of the different finished parts, and the intermediate states during assemblage, and the final state, when a jacket and two hoops are shrunk over the A tube of an 8-inch gun, shown in longitudinal section in the annexed fig. 10.



In a state of repose the tension of the outer fibres of the outside hoop is 8.1 tons/in.², and the circumferential pressure in the interior of the bore is 19.9 tons/in.²; so that, with $r_0 = 5$, $r_s = 16$,

$$_0S_1 = 19.9 \times 10 \div 12,500 = 0.016,$$

or the contraction of the calibre is 16 thousandths of an inch, in consequence of the shrinkage while

$$_4S_5 = 8.1 \times 32 \div 12,500 = 0.021,$$

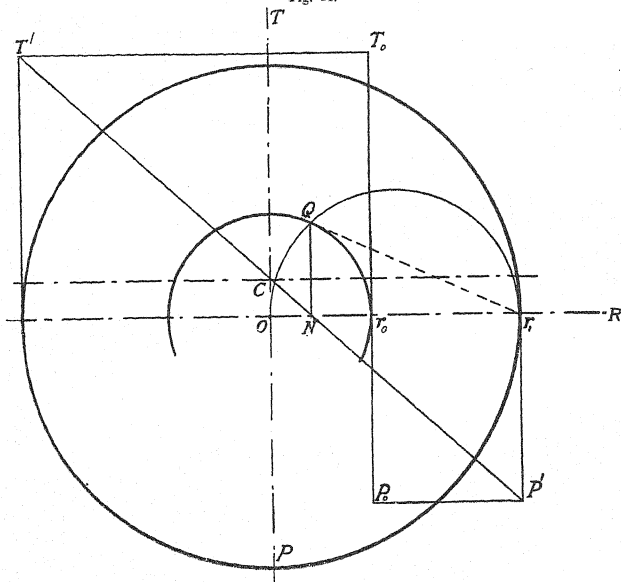
or the elongation of the external diameter due to the shrinkage is 21 thousandths of an inch.

The coefficient of expansion of steel per 1° F. is about $1 \div 150,000$: so that if ${}_nS_{n+1}$ denotes the shrinkage during manufacture, the temperature must be raised

$$150,000 \frac{{}_nS_{n+1}}{2r_n}$$

degrees Fahrenheit for the $(n+1)$ th coil to be expanded sufficiently so as to slip over the n th coil; and this rise of temperature can also be represented geometrically in a similar manner.

Fig. 11.



It is found (Treatise on Service Ordnance, 1908, p. 28) that a satisfactory practical plan to follow is, "shrinkage is given by making the internal diameter of a steel tube $\frac{1}{300}$ th smaller than the external diameter of the tube it goes over."

Graphical Construction of Gun Stresses.

The stresses in a gun can be graphically investigated by a rapid and simple method.

The following constructions are necessary:—

(a) Knowing that T and P are given by the equations $T = \frac{a}{r^2} + b$, $P = \frac{a}{r^2} - b$, and given at r_0 either $T = T_0$ or $P = P_0$, while at r_1 , $P_1 = 0$, to find the centre of the curves, see fig. 11.

Describe the semicircle on O_1R_1 cutting the r_0 circle in Q , then join r_1Q , this is a tangent from r_1 to the r_0 circle, since the angle OQ_1 is a right angle: drop the ordinate QN and join NT' or $P'N$ cutting OT in C ; this is the centre of the curves.

Proof.—With axes OR, OT the point T is $(-r_1, T_0)$, P' is (r_1, p_0) , also the co-ordinates of N are $(\frac{r_0^2}{r_1}, 0)$.

Since

$$T_0 = \frac{a}{r_0^2} + b,$$

and

$$P_1 = \frac{a}{r_1^2} - b = 0,$$

therefore

$$T_0 = b \frac{r_0^2 + r_1^2}{r_0^2} \quad \text{or} \quad b = T_0 \cdot \frac{r_0^2}{r_0^2 + r_1^2}.$$

The equation to line NT' is

$$\frac{T - T_0}{-T_0} = \frac{r + r_1}{\frac{r_0^2}{r_1^2} + r_1} = \frac{r_1(r + r_1)}{r_0^2 + r_1^2}.$$

This cuts OT where $r = 0$.

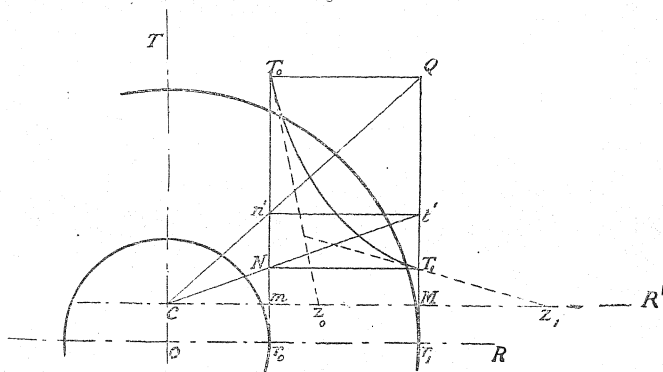
Therefore $OC = T_0 \frac{r_0^2}{r_0^2 + r_1^2}$, and this is equal to b .

Therefore C is the centre of the curves.

If P_0 is given instead of T_0 , a similar proof holds good (see fig. 11). If P_0 and T_0 only be known, then, by bisecting the length T_0P_0 , the centre is found.

(b) Given the axes of $T = \frac{a}{r^2} + b$ and one point (r_0, T_0) on the curve, to draw the curve (see fig. 12).

Fig. 12.



Make $OC = b$, referred to CT and a line CR' at right angles to CT as axes, the equation for T becomes $T = \frac{a}{r^2}$.

The construction to find the value T_1 at a point r_1 is as follows:—

Join CQ , Q being the point (r_1, T_0) , cutting r_0T_0 in n' : Draw $n't'$ parallel to OR , join Ct' cutting r_0T_0 in N , draw NT_1 . Then T_1 is a point on the curve, also if $Cz_0 = \frac{2}{3}r_0$ and

$C_1 = \frac{3}{2} r_1$, then $r_0 T_0$ and $r_1 T_1$ are tangents to the curve. Proof:—The equation to CQ is $r = \frac{T}{T_0}$, this cuts $r_0 T_0$ in q' , therefore

$$M' = m'q' = \frac{r_0}{r_1} T_0.$$

The equation to Cf is

$$\frac{r}{r_1} = \frac{T}{\frac{r_0}{r_1} T_0} = \frac{r_1}{r_0} \cdot \frac{T}{T_0}.$$

This cuts $r_0 T_0$ where

$$Nm = T_1 M = \frac{r_0^2}{r_1^2} T_0 = \frac{a}{r_1^2} \quad \text{since} \quad T_0 r_0^2 = a.$$

That is, T_1 lies on the curve $T = \frac{a}{r^2}$.

Again, the equation to the tangent at (r_1, T_1) is

$$\begin{aligned} T - T_1 &= \frac{dT}{dr} (r - r_1) \\ &= -\frac{2a}{r_1^3} (r - r_1) \quad \text{since} \quad T_1 = \frac{a}{r_1^2}. \end{aligned}$$

This cuts CR' where $T = 0$, therefore

$$-\frac{a}{r_1^2} = -\frac{2a}{r_1^3} (C_1 - r_1) \quad \text{or} \quad C_1 = \frac{3r_1}{2}.$$

(c) Given two points on a rectangular hyperbola and one asymptote to find the other asymptote, see fig. 13.

In the firing stresses of a wire coil, see (20),

$$(p + t)r = \text{a constant, a rectangular hyperbola;}$$

if the axes be changed to SX, SO, r_2 where $O_s = t$, the uniform firing tension, in the wire coil the equation becomes

$$pr = \text{a constant,}$$

so that SX, SO are the asymptotes, the wire coil lying within $r_2 p_3$.

Let $P_2 (r_2, p_2)$ and $P_3 (r_3, p_3)$ be the two given points and OP the given asymptote.

Complete the rectangle $p_2 p_3 p_4 p_5$.

Draw the diagonal $p_2 p_5$ to cut PO produced in S and draw SX parallel to OR. This is the required asymptote. The proof is obvious on completing the rectangles Sp_2 and Sp_3 , for then these rectangles are seen to be equal; and SX and SP must be the asymptotes of the rectangular hyperbola $pr = \text{constant}$.

If both asymptotes SX and SP are known, but only one point on the rectangular hyperbola, say (p_2, r_2) is known, then another point is at once found, see fig. 13.

Thus, to find p_2 at r_2 , complete the rectangle $r_2 p_3$; join Sp_3 and produce it to meet $r_3 p_3$ in p_2' ; then, drawing $p_2' p_2$ parallel to SX or OR to cut $r_2 p_3$ in p_2 gives p_2 the required value of the radial pressure at r_2 .

The proof is obvious, since the rectangle Sp_2 is seen to be equal to the rectangle Sp_3 .

By means of the three constructions first proved, all the powder and firing stresses in a wire gun, or in a gun built up with tubes shrunk on, can be graphically drawn.

FIG. 13.

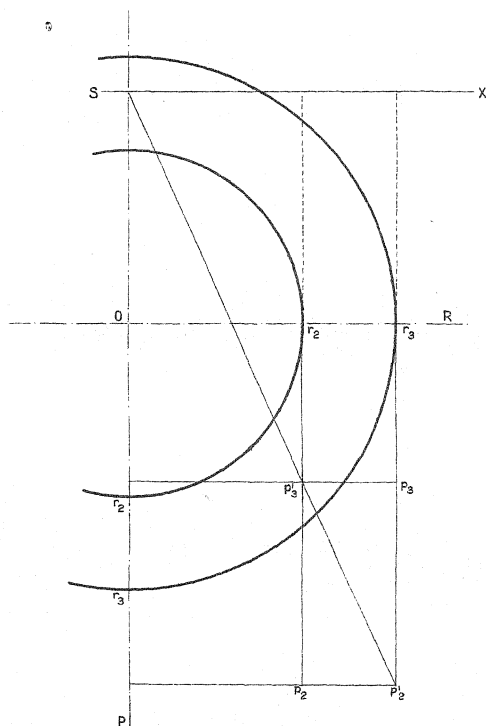
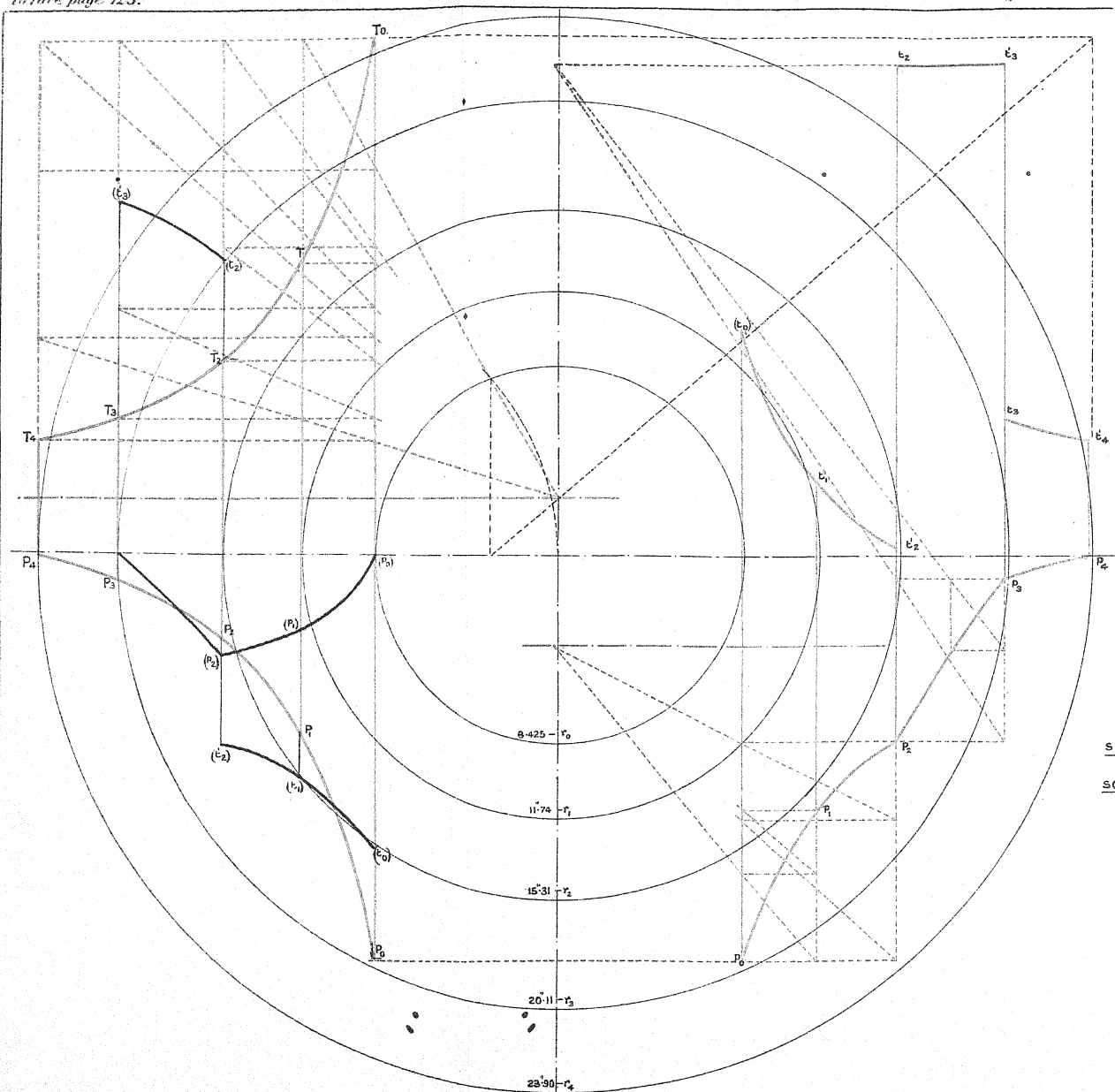


FIGURE 14.



STRESSES FOUND FROM FIGURE.							
P_0	35.9	P_0	35.9	(P_0)	0.0		
P_1	15.9	P_1	22.4	(P_1)	6.5		
P_2	7.4	P_2	16.4	(P_2)	9.0		
P_3	2.1	P_3	2.1	(P_3)	0.0		
P_4	0.0	P_4	0.0	(P_4)	0.0		
T_0	46.0	t_0	20.0	(t_0)	-26.0		
T_1	26.2	t_1	6.5	(t_1)	-19.7		
T_2	17.4	t_2	0.5	(t_2)	-16.9		
T_3	12.2	t_3	43.6	(t_3)	-26.2		
T_4	10.2	t_4	43.6	(t_4)	31.4		
		t_3	12.2	(t_3)	0.0		
		t_4	10.2	(t_4)	0.0		

SCALE OF SIZES, $\frac{1}{5}$

SCALE OF STRESSES, 1" = 10 TONS PER SQ IN.

The example taken is for the stresses across the centre of the chamber of the 12-inch B.L. XI gun, in which the data are (see fig. 14):—

$$r_0 = 8.425 \text{ inches,}$$

$$r_1 = 11.74 \quad ,,$$

$$r_2 = 15.31 \quad ,,$$

$$r_3 = 20.11 \quad ,,$$

$$r_4 = 23.9 \quad ,,$$

Also $l_0 = 20$, $(l_0) = -26$, so that

$$T_0 = 46,$$

then by following the procedure employed in the stress calculations already made, the powder stress curves for T and P are found graphically; knowing $T_0 = 46$ and $P_0 = 0$, the centre of the curves is first found, and then other points on the curves are obtained as before.

Next, in the firing stresses, $p_0 (= P_0)$ is now found, $l_0 = 20$ is known, thence the centre of the p , t curves is obtained at once, and the firing stress curves in the inner A and A tube are graphically found.

In the firing stresses for the wire coil p_2 is found, $p_2 = P_2$ is already found, and so two points (p_2 , r_2) (p_3 , r_3) are known, the asymptote giving the value of $t_2 = t = t_3'$ is graphically found, and any other points on the p_2 , p_3 curve can be found (as in fig. 13).

In the jacket the firing stresses are the same as the powder stresses.

The initial stresses are got by subtracting the powder stresses from the firing stresses.

Fig. 14 shows the complete stresses; the agreement of the various stresses is seen to be very close indeed to the calculated stresses (see p. 115).

Suppose, however, that in a wire gun the working maximum chamber pressure is fixed, say 18 tons/in.², and thence that P_0 is fixed at 36, also that the uniform maximum firing stress tension in the wire is also fixed, say $t_2' = t_3' = 45$; then by the constructions already given, the complete stresses can be found; the data in this case are

$$P_0 = 36,$$

$$t_2 = t_3' = 45.$$

The value of l_0 and of (l_0) can be deduced by this rapid graphical method; the procedure is as before when T_0 and (l_0) were given.

Captain Noble, I.O.M., A.O.D., who worked out the graphical method for obtaining fig. 14, also suggests the following for finding the radii of the A tube, wire coil, and jacket for given stresses in the metal (see figure).

Suppose the following data are given:—

$$P_0 = 36, \quad T_0 = 46, \quad r_0 = 6'', \quad l_0 = 20.$$

The centre of the powder P_0 , T curves is known at once, it is 5 tons/in.² above the centre line of bore. Knowing r_0 , P_0 , T_0 , the P and T curves can be drawn, where the P curve cuts the line OR gives r_4 .

On the right side of the figure the firing stress curves p and t can now be drawn, since l_0 and p_0 are known.

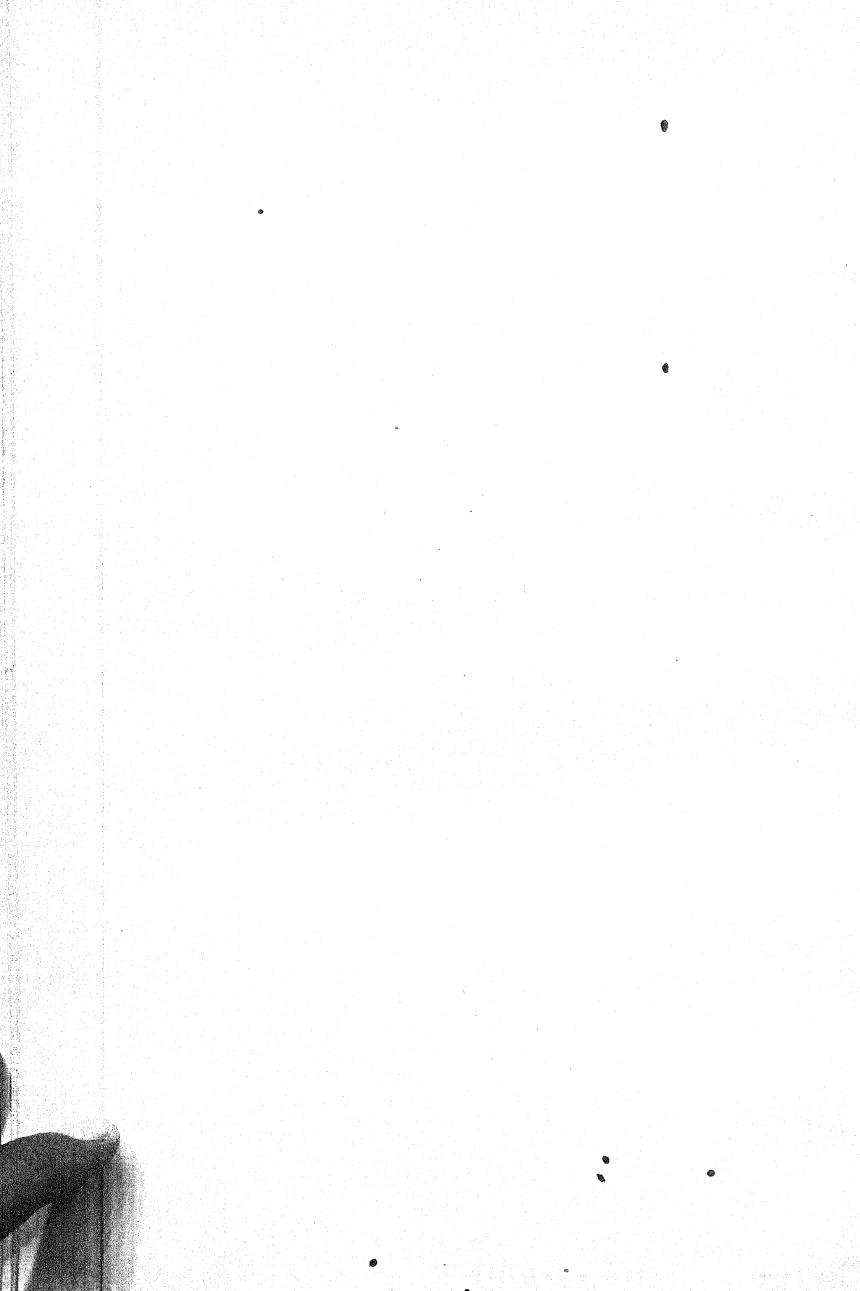
To find the wire layer, the firing tension of the outer ring of fibres of the A tube must be known; it is in most cases a small quantity. Assume it to be, say zero, so that $t_2' = 0$, and also that the fixed firing wire tension $t_2 = t_3' = 45$, then where the t curve just drawn cuts the centre line OR gives r_2 .

To find r_3 , the curve of firing pressure p in the jacket is the same as that for the powder pressure, therefore draw backwards from r_4 a copy of the P_4P_0 curve, where this is cut by the rectangular hyperbola $(p+t)r = \text{constant}$ of the wire coil radial pressure will give p_3 and r_3 .

And now, to draw this rectangular hyperbola. The two asymptotes are known, since $t_2 = t_3' = t = 45$, also it cuts OR where $p = 0$, and the point (r_2, r_2) on it is also known, hence as many points as are required can be drawn, where the hyperbola cuts the curve p_4p_3 gives p_3 , and drawing the ordinate gives r_3 .

It is a matter of choice as to the value of r_1 ; values can be selected for it. In this graphical method the values of r_2, r_3, r_4 are obtained and the complete stress diagram is drawn at the same time.

In the example worked out on fig. 15, r_1 is taken as equal to 8 inches.



Let BP_2 be the line or groove; at M,

$$\tan \alpha = \frac{\pi}{30}, n = 30.$$

Therefore

$$\frac{d^2y}{dx^2} = \text{constant} = c,$$

$$\tan \theta = \frac{dy}{dx} = cx + c_1, \text{ but when } x = 0, \frac{dy}{dx} = 0, \text{ therefore } c_1 = 0,$$

$$y = \frac{c}{2}x^2 + c_2, \text{ but when } x = 0, \frac{dy}{dx} = 0, \text{ therefore } c_2 = 0,$$

and when $x = l$ inches, where l represents the length of rifling, generally called the "length of travel" of the shot,

$$\tan \theta = \tan \alpha = \frac{\pi}{n},$$

therefore

$$cl = \tan \alpha = \frac{\pi}{n}, c = \frac{\pi}{nl},$$

so that

$$y = \frac{\pi}{2nl}x^2,$$

or

$$x^2 = \frac{2nl}{\pi}y \dots \dots \dots (2),$$

which is a parabola, whose vertex is at B and latus rectum is

$$\frac{2nl}{\pi}$$

Putting

$$x = l, \text{ gives } y = P_2M = l \frac{\pi}{2n};$$

$$P_1M = l \frac{\pi}{n}.$$

This shows that for a uniformly increasing twist from 0 to 1 in n , the angle turned through with uniformly increasing twist is half that turned with uniform twist; the velocity of rotation of the shot at the muzzle is the same.

The angle of rifling β at any point distant k inches from B is given by

$$\tan \beta = \frac{dy}{dx} = \frac{\pi}{nl}k = \frac{\pi}{q} \dots \dots \dots (3),$$

where q is the twist of rifling.

At the start, if the twist is 1 in m and increase uniformly to 1 in n at the muzzle M, then the required curve is obtained thus:—

$$\frac{d^2y}{dx^2} = \text{constant } c,$$

$$\frac{dy}{dx} = cx + c, \text{ but when } x = 0, \frac{dy}{dx} = \frac{\pi}{m},$$

$$\text{and when } x = l, \frac{dy}{dx} = \frac{\pi}{n}.$$

Therefore

$$\frac{dy}{dx} = \frac{1}{l} \left(\frac{\pi}{n} - \frac{\pi}{m} \right) x + \frac{\pi}{m},$$

$$y = \frac{1}{2l} \left(\frac{\pi}{n} - \frac{\pi}{m} \right) x^2 + \frac{\pi}{m} x, \text{ since } x = 0 \text{ when } y = 0.$$

At any point distant K inches from B the twist of rifling q is given by

$$\frac{\pi}{q} = \frac{1}{l} \left(\frac{\pi}{n} - \frac{\pi}{m} \right) K + \frac{\pi}{m} = \tan \beta,$$

where β is the angle of rifling, and

$$q = \frac{\pi}{l \left(\frac{1}{n} - \frac{1}{m} \right) K + \frac{\pi}{m}} = \frac{m}{1 - K \left(1 - \frac{m}{n} \right)} \quad (4).$$

Velocity of Rotation.

The velocity of rotation of the projectile at any point depends solely on the forward velocity and the twist of rifling at that point; so that at the muzzle it depends upon the muzzle velocity and the final twist of rifling at the muzzle.

If V denotes the forward axial velocity with which the shot leaves the muzzle, then the spin imparts to the points on the outside cylindrical surface a component velocity at right angles to the axis of magnitude—

$$V \tan \beta = \frac{\pi V}{n} \text{ f/s};$$

this is called the *linear velocity of rotation*.

The *angular velocity of rotation*, in radians/second, is obtained by dividing this by the radius of the shot in feet, a or $d \div 24$; it is, therefore,

$$\frac{\pi V}{na} \quad \text{or} \quad \frac{24\pi V}{nd};$$

and this again is converted into revolutions per second by dividing by 2π , since one revolution equals 2π radians; the shot, therefore, makes—

$$\frac{V}{2na} \quad \text{or} \quad \frac{12V}{nd} \text{ revs/sec.}$$

Thus, comparing the 6-inch gun and magazine rifle, in each of which $n = 30$; then for the same muzzle velocity, say 2000 f/s, the linear velocities of rotation will be the same, namely, 209 f/s, but the rifle bullet will make 2640 revs/second, against 133 revs/second of the 6-inch projectile.

Formerly it was considered requisite for a projectile to possess a given linear velocity of rotation to ensure its stability in flight, and for this reason the twist of rifling in howitzers, firing with low velocities, was made very quick, even up to one in 12 calibres.

But it is now found that the linear velocity of rotation should be a given fraction of the initial velocity, so that the same twist of rifling is suitable for high or low velocities, with a given projectile; but the determination of the appropriate twist from theoretical considerations is not a simple matter, and the twist must be settled by experiment to a great extent.

The investigation of the stability of an elongated projectile moving through the air in the direction of its axis with given angular velocity is very similar to that required for the

TABLE OF ROTATION FOR STABILITY OF PROJECTILES.

(Calculated from Sir George Greenhill's formula by Major Cundill, R.A., and extended by Mr. A. G. Hadcock, late R.A., *vide Proc. R.A.I.*, vol. xi, No. 2, and vol. xiv, No. 3.)

Length of projectile in calibres.	Minimum twist at muzzle of gun requisite to give stability = 1 turn in <i>n</i> calibres.			
	Cast-iron common shell; cavity = $\frac{1}{3}$ ths vol. of shell. (Density of cast iron 7.207.)	Palliser shell; cavity = $\frac{1}{4}$ th vol. of shell. (Density of chilled iron 8.000.)	Solid steel bullet. (Density of steel 8.000.)	Solid lead and tin bullets of similar composition to M.-H. bullets. (Density of alloy 10.9.)
	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>
2.0	63.87	71.08	72.21	84.29
2.1	59.84	66.59	67.06	78.98
2.2	56.31	62.67	63.67	74.32
2.3	53.19	59.19	60.14	70.20
2.4	50.31	56.10	57.09	66.53
2.5	47.91	53.32	54.17	63.24
2.6	45.65	50.81	51.62	60.26
2.7	43.61	48.53	49.30	57.55
2.8	41.74	46.45	47.19	55.09
2.9	40.02	44.54	45.25	52.72
3.0	38.45	42.79	43.47	50.74
3.1	36.99	41.16	41.82	48.82
3.2	35.64	39.66	40.30	47.04
3.3	34.39	38.27	38.84	45.38
3.4	33.22	36.97	37.56	43.84
3.5	32.13	35.75	36.33	42.40
3.6	31.11	34.62	35.17	41.05
3.7	30.15	33.55	34.09	39.79
3.8	29.25	32.55	33.07	38.61
3.9	28.40	31.61	32.11	37.48
4.0	27.60	30.72	31.21	36.43
4.1	26.85	29.88	30.36	35.43
4.2	26.13	29.08	29.55	34.49
4.3	25.45	28.33	28.78	33.59
4.4	24.81	27.61	28.05	32.74
4.5	24.20	26.93	27.36	31.94
4.6	23.65	26.22	26.74	31.21
4.7	23.06	25.66	26.08	30.44
4.8	22.53	25.08	25.48	29.74
4.9	22.03	24.51	24.91	29.07
5.0	21.56	23.98	24.36	28.44
5.1	21.08	23.46	23.84	27.83
5.2	20.64	22.97	23.34	27.24
5.3	20.22	22.50	22.86	26.68
5.4	19.81	22.05	22.40	26.14
5.5	19.42	21.61	21.96	25.63
5.6	19.04	21.19	21.53	25.13
5.7	18.68	20.79	21.12	24.66
5.8	18.33	20.40	20.73	24.20
5.9	18.00	20.03	20.35	23.75
6.0	17.67	19.67	19.98	23.33
7.0	14.99	16.68	16.95	19.78
8.0	13.02	14.48	14.72	17.18
9.0	11.50	12.80	13.00	15.18
10.0	10.31	11.47	11.45	13.60

stability of a top or gyrost, spinning with its axis vertical, and the behaviour of the bodies have a close analogy. The annexed table on p. 128 shows the result of such calculations.

When a top is spun, the motion of the axis is at first unsteady, but this unsteadiness soon disappears, and the top then spins upright, when it is said to go to sleep; after a time the friction of the point reduces the spin to such an extent that the vertical position becomes unstable, and the axis again begins to wobble; the axis inclines more and more from the upright position, until finally the top falls over on its side (*see* Sir G. Greenhill's Dynamics, p. 194 to end).

So, too, an elongated projectile fired from a rifled gun is at first rather unsteady from the first portion of its flight, but the friction of the air soon destroys the irregular gyrations, and the shot, if provided with sufficient spin, proceeds steadily in the direction of the axis.

If the spin of the projectile died away more rapidly than the forward velocity, the projectile, like the top, would again become unsteady.

But the forward retardation of the shot is much greater than the angular retardation, so that the shot moves as if on an increasing screw; and practically, if once steady, the shot will continue so throughout its trajectory, in consequence of this *overscrew*.

In high-angle fire, however, the motion tends to become unsteady in the descending branch, in consequence of the great curvature of the trajectory.

THE TOTAL ROTATING THRUST ON THE DRIVING FLANKS OF A DRIVING BAND.

Take a cross-section of a gun, let P be the top groove PZ looking from the breech end towards the muzzle, PZ being parallel to the axis of the bore. PX is vertical and at right angles to PZ, PY is perpendicular to the plane PZX (see fig. 17).

Draw the line PT so that ZPT = θ , the angle of rifling at P; similarly draw PN₀ in the plane PZYN₀, the quadrantal surface XZY is thus shifted round through the angle θ to the position XTN₀.

If the driving flank of the groove is radial to the bore along PX, then PN₀ in the plane XN₀ is the direction of the normal to the driving surface.

When the driving flank is not radial to the bore, but such that the normal to the driving flank, as measured in the cross-section of the bore, makes an angle δ with the radius of the bore, then the plane through the driving surface is now changed (from XPT) to QPT, and the angle QPX is $90^\circ - \delta$. The normal to this new driving surface is PN in the plane XPN₀, and PN₀ becomes displaced to PN in the plane XPN₀ through an angle N₀N = ψ .

Since T is the pole of the plane XN₀N, XQ' = N₀N = ψ , and the angle XTQ = ψ , so that in the right-angled triangle QYT

$$\left. \begin{aligned} YTQ &= 90^\circ - \psi \\ TY &= 90^\circ - \theta \\ QY &= \delta \\ \tan \psi \tan \delta &= \cos \theta \end{aligned} \right\} \dots \dots \dots (5).$$

and

The forces acting upon the shot in the bore are:—

- I. The thrust of the powder gas, G tons, in the direction of PZ.
- II. The normal reaction of the surfaces in contact, R tons, in the direction PN.
- III. The friction, μR tons, acting tangentially *backwards* in the direction TP.

The components of these impressed forces resolved parallel to the axis PZ are thus composed of

$$(G + R \cos ZN - \mu R \cos ZT) \text{ tons,}$$

hence (from the formula $W \frac{f}{g} = P$)

$$\frac{W}{2240 \times g} \cdot \frac{d^2 z}{dt^2} = G + R \cos ZN - \mu R \cos ZT \dots \dots \dots (6),$$

where W is the weight of the shot in pounds, and Z the distance it has advanced in feet, while turning through an angle ω radians. Now, taking moments about the axis of the shot, the impressed couple on the shot along the axis PY at right angles to PZ (which is parallel to the axis of the shot), is

$$(rR \cos YN - \mu rR \cos YT) \text{ tons/feet,}$$

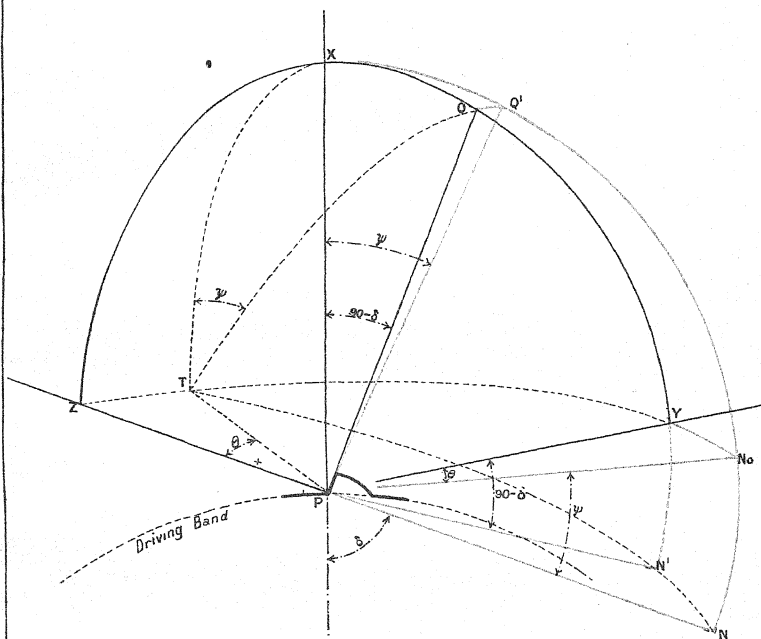
where r is the radius of the shot in feet, hence

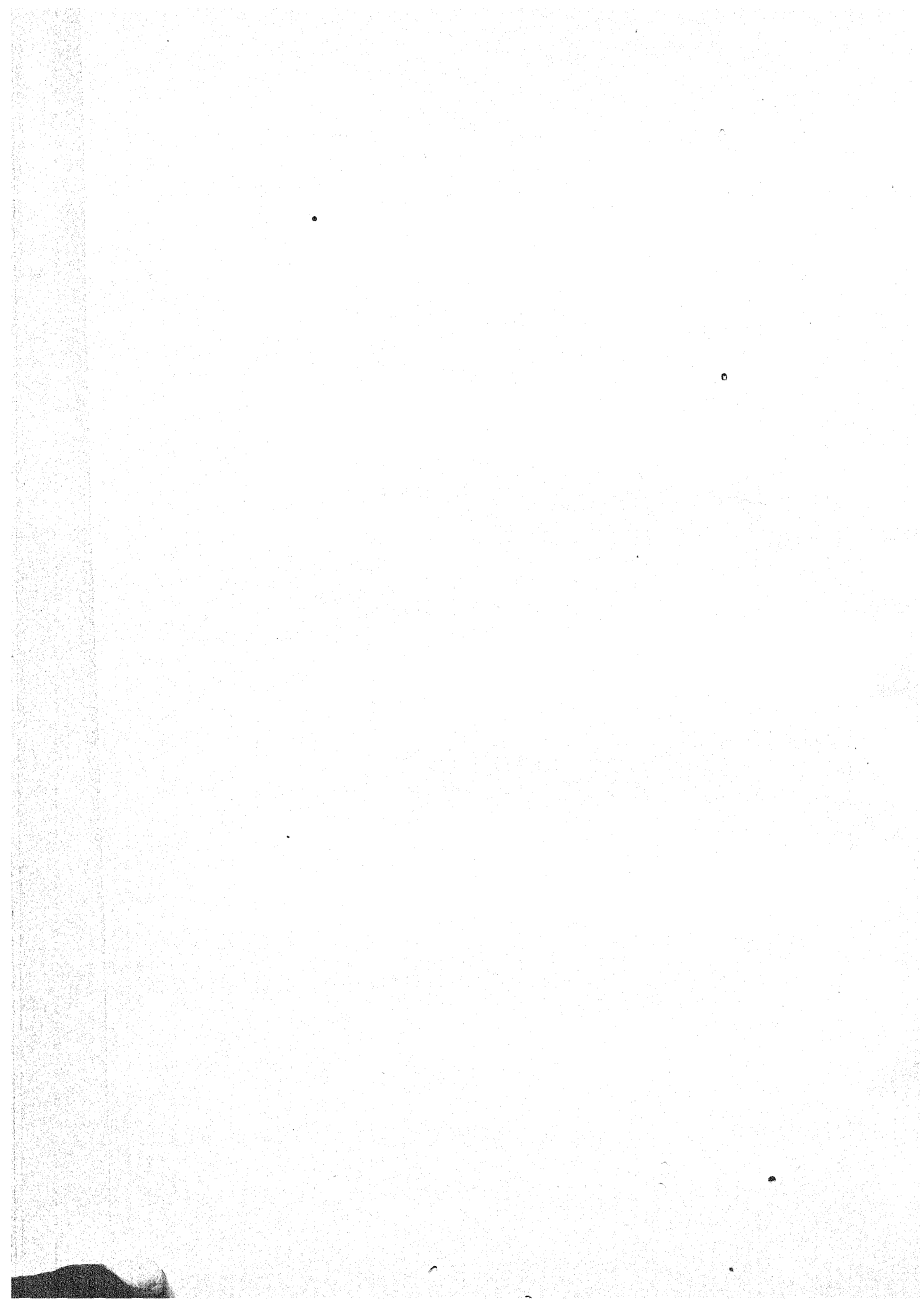
$$\frac{W\rho^2}{2240g} \cdot \frac{d^2 \omega}{dt^2} = rR \cos YN - \mu rR \cos YT \dots \dots \dots (7).$$

Let the curve of rifling be $y = f(z)$

$$r \frac{d\omega}{dt} = \frac{dy}{dt} = \frac{dy}{dz} \cdot \frac{dz}{dt}.$$

FIGURE 17.





Therefore ,

$$r \frac{d^2 \omega}{dt^2} = \frac{d^2 y}{dt^2} = \frac{d^2 y}{dz^2} \left(\frac{dz}{dt} \right)^2 + \frac{dy}{dz} \cdot \frac{d^2 z}{dt^2} \quad (8).$$

Also

$$\frac{dz}{dt} = v, \quad \frac{dy}{dz} = \tan \theta. \quad (9),$$

therefore

$$r \frac{d^2 \omega}{dt^2} = \frac{d^2 y}{dz^2} \cdot v^2 + \tan \theta \frac{d^2 z}{dt^2} \quad (10).$$

Also from (6)

$$\frac{d^2 z}{dt^2} = \frac{G + R (\cos ZN - \mu \cos ZT)}{W} \quad (11),$$

$$\frac{1}{2240 \times g}$$

and from (7)

$$r \frac{d^2 \omega}{dt^2} = \frac{r^2 R (\cos YN - \mu \cos YT)}{\rho^2 \frac{1}{2240 \times g}} \quad (12).$$

Substituting these values in equation (10) gives

$$\frac{r^2}{\rho^2} R \frac{\cos YN - \mu \cos YT}{\frac{1}{2240 \times g}} = \frac{d^2 y}{dz^2} v^2 + \frac{G \tan \theta + R (\cos ZN - \mu \cos ZT) \tan \theta}{\frac{1}{2240 \times g}},$$

or

$$R = \frac{G \tan \theta + \frac{W v^2}{2240 \times g} \cdot \frac{d^2 y}{dz^2}}{\frac{r^2}{\rho^2} (\cos YN - \mu \cos YT) - (\cos ZN - \mu \cos ZT) \tan \theta} \quad (13),$$

but

$$\cos YT = \sin \theta,$$

$$\cos ZT = \cos \theta,$$

$$\cos YN = \cos YN_0 \cos N_0 N = \cos \theta \cos \psi,$$

$$\cos ZN = \cos ZN_0 \cos N_0 N = \sin \theta \cos \psi,$$

and from (5)

$$\tan \psi = \cos \theta \cot \delta,$$

therefore

$$\cos \psi = \frac{\sin \delta}{\sqrt{\sin^2 \delta + \cos^2 \theta \cos^2 \delta}},$$

and

$$\cos YN = \frac{\sin \delta}{\sqrt{1 + \sin^2 \delta \tan^2 \theta}},$$

$$\cos ZN = -\frac{\tan \theta \sin \delta}{\sqrt{1 + \sin^2 \delta \tan^2 \theta}}$$

so that

$$R = \frac{G \tan \theta + \frac{W v^2}{2240 \times g} \cdot \frac{d^2 y}{dz^2}}{\frac{r^2}{\rho^2} + \tan^2 \theta} \sin \delta - \left(\frac{r^2}{\rho^2} - 1 \right) \mu \sin \theta \quad (14),$$

which gives the general expression for any curve of rifling $y = f(z)$ expressed in terms of θ and δ . Formula (14) and the above method of obtaining it is due to Sir G. Greenhill.

For a parabolic twist, starting at zero and increasing uniformly up to $\tan \theta = \frac{\pi}{n}$ at the muzzle

$$z^2 = \frac{2nl}{\pi} y = cy, \text{ say,}$$

where l is the length of rifling,

$$\tan \theta = \frac{dy}{dz} = \frac{2z}{c} = \frac{\pi z}{nl},$$

where z is the distance down the bore,

$$\frac{d^2y}{dz^2} = \frac{2}{c} = \frac{2\pi}{2nl} = \frac{\pi}{nl}.$$

Substituting these values of $\tan \theta$ and $\frac{d^2y}{dz^2}$ in (14) gives

$$R = \frac{2\rho^2 \left(Gz + \frac{Wv^2}{2240 \times g} \right)}{\frac{c^2 r^2 + 4\rho^2 z^2}{\sqrt{4z^2 \sin^2 \delta + c^2}} \sin \delta - \frac{2\mu c z (r^2 - \rho^2)}{\sqrt{4z^2 + c^2}}} \quad (15),$$

which is the formula given by Sir A. Noble in 'Phil. Magazine,' 1863 and 1864.

With a *uniform twist*, $\frac{d^2y}{dz^2} = 0$, $\tan \theta = \frac{\pi}{n}$, and if (as is now usual) the normal to the driving surface, that is, the line of action R is perpendicular to the radius, then the grooves have a radial flank and $\delta = 90^\circ$. The formula for R then becomes

$$\frac{R}{G} = \frac{\tan \theta}{\cos \theta \left(\frac{r^2}{\rho^2} + \tan^2 \theta \right) - \left(\frac{r^2}{\rho^2} - 1 \right) \mu \sin \theta},$$

since

$$\tan \theta = \frac{\pi}{n}, \quad \cos \theta = \frac{n}{\sqrt{n^2 + \pi^2}}, \quad \text{and} \quad \sin \theta = \frac{\pi}{\sqrt{n^2 + \pi^2}}.$$

(a) For a solid cylinder rotating about the longitudinal axis, $\rho = \frac{r}{\sqrt{2}}$.

(b) For a solid pointed shot, ρ is less than $\frac{r}{\sqrt{2}}$ because the mean radius of the head is less than r .

(c) For a hollow cylinder ρ will be greater than $\frac{r}{\sqrt{2}}$.

(d) For a pointed shell, ρ will be intermediate between (b) and (c) and approach (a) in value, that is $\rho = \frac{r}{\sqrt{2}}$.

The mathematical calculation of ρ for a pointed projectile is somewhat laborious especially when the shape of the cavity of a shell has to be considered; it is best to find ρ by graphical methods.

With $\frac{r^2}{\rho^2} = 2$.

$$\frac{R}{G} = \frac{\pi \sqrt{n^2 + \pi^2}}{2n^2 + \pi^2 - \mu \pi n} \quad (16).$$

Take a 12-inch gun, for which $n = 30$, $\mu = 0.2$, $d = 12$, and a maximum chamber pressure $p_1 = 18$ tons/in.².

$$G = \pi \frac{d^2}{4} p \text{ tons,}$$

$R = \pi \cdot d \cdot t$ where t represents the thrust in tons per inch of circumference of bore.

$$\frac{R}{G} = \frac{\pi d t}{\pi \frac{d^2}{4} p} = \frac{\pi \sqrt{n^2 + \pi^2}}{2n^2 + \pi^2 - \mu \pi n} = \frac{95}{1800 + 9.9 - 18.8} = .053,$$

which gives

$$R = .053 G = .053 \pi \frac{d^2}{4} p \quad \dots \quad (17).$$

At the point of maximum pressure $p = p_1 = 18$.

$$R = .053 \times \pi \times 36 \times 18 = 108 \text{ tons.}$$

$$t = \frac{.053}{4} p_1 d = 2.86 \text{ tons per inch of circumference at the time of maximum pressure.}$$

Lastly, take the case of uniform rifling, $\delta = 90^\circ$, and *neglecting friction*, then

$$\tan \theta = \frac{\pi}{n} = \frac{\pi}{30} \quad \text{for } n = 30,$$

$$\frac{d^2 y}{d^2 z} = 0, \quad \delta = 90^\circ, \quad \mu = 0;$$

hence from (14) and (16)

$$\frac{R}{G} = \frac{\pi (n^2 + \pi^2)^{1/2}}{2n^2 + \pi^2} = .051,$$

$$\left. \begin{aligned} R &= .051 G = .051 \pi \frac{d^2}{4} p = .04 d^2 p \\ t &= \frac{.051}{4} p d = .013 p d \end{aligned} \right\} \dots \dots \dots (18).$$

From (16), (17), and (18) for uniform rifling and $\delta = 90^\circ$ the circumferential thrust of R tons at any point is obtained at once when the pressure-space curve is known.

CHAPTER VII.

INTERIOR BALLISTICS.

WHEN a charge of cordite or other explosive is exploded in the chamber of the gun, the investigation of the relations connecting the pressure of the gases and the velocity of the projectile throughout the bore of a gun of known dimensions, with the various elements such as the capacity of the chamber, travel of the projectile, time, rate of burning of the explosive, &c., &c., is a part of the science called Interior Ballistics.

The subject of Interior Ballistics cannot be treated by rigid mathematical methods only, there exist such a number of factors which affect results that it is necessary to employ some empirical formulas. These formulas are deduced from the results of experiments made with closed-vessels and other instruments, but more especially from the results of firing guns of various calibre with various charges of the explosive which is used in the Service.

The following definitions are required in the practical treatment of Interior Ballistics:—

Let d represent the calibre of a gun in inches.

s the number of inches from the base of the projectile to any point along the axis of the bore: this is called the "travel" of the projectile to the selected point; the "total length of travel" is the distance from the base of the projectile to the muzzle = s_2 .

C the volume of the powder chamber in cubic inches (in.³).

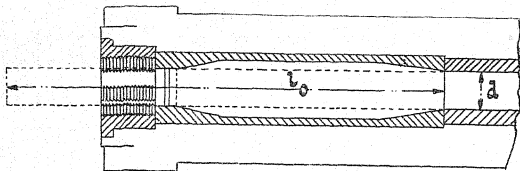
ω the weight of the charge in lbs.

Equivalent length of the powder chamber: l_0 .

If we take a cylinder whose cross-section is the same as that of the bore and whose volume is the same as the capacity of the chamber, the length (l_0) of this cylinder is called the equivalent length of the powder chamber; so that (fig. 1)

$$l_0 = \frac{C}{\pi \frac{d^2}{4}} \text{ inches} = 1.27 \frac{C}{d^2} \text{ inches} = 0.106 \frac{C}{d^2} \text{ feet; and } C = \pi \frac{d^2}{4} l_0.$$

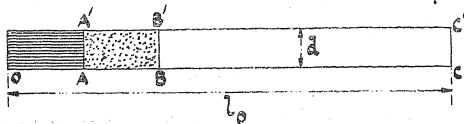
Fig. 1.



When a weight ω lbs. of explosive is in the powder chamber, the volume occupied by the charge includes a number of air interstices. In fig. 2, the chamber is represented as reduced to a cylinder whose cross-section is the same as that of the bore, its length $OC = l_0$ inches.

Suppose the air interstices of the charge removed, so that the volume occupied by the solid explosive itself is OA' and the volume occupied by the charge is OB' .

Fig. 2.



Specific gravity or density, δ , of the explosive.

$$\begin{aligned}\delta &= \frac{\text{weight of "solid" explosive}}{\text{weight of equal volume of water}} \\ &= \frac{\text{weight of explosive in } OA'}{\text{weight of water filling } OA'} = \frac{\varpi}{v_a/27.73}\end{aligned}$$

where v_a is the cubic inches in OA' (fig. 2), and 27.73 cubic inches of water weigh 1 lb. at the standard temperature of 62° F.; therefore

$$\delta = \frac{27.73\varpi}{v_a}$$

With cordite M.D., the density δ is 1.58, hence

$$v_a = OA' = \frac{27.73\varpi}{\delta} = \frac{27.73\varpi}{1.58}$$

The gravimetric density of the charge: γ .

Allowing for the air interstices of the charge, γ is the mean density of the contents of the volume actually occupied by the charge. In fig. 2, this is represented as $OB' = V$.

$$\begin{aligned}\gamma &= \frac{\text{weight of explosive in charge}}{\text{weight of water filling the volume occupied by the charge}} \\ &= \frac{\varpi}{v_a/27.73} = \frac{27.73\varpi}{v_a} = \frac{27.73\varpi}{\text{volume occupied by charge}}\end{aligned}$$

Density of loading: Δ .

This is the mean density of the contents of the whole powder chamber, represented in fig. 2 as $OC' = C$,

$$\Delta = \frac{\text{weight of charge in chamber}}{\text{weight of water filling chamber}}$$

This allows for air interstices of the charge as well as for "air-space," therefore

$$\Delta = \frac{\varpi}{C/27.73} = \frac{27.73\varpi}{C}$$

Thus, for a 12-inch B.L. IX, with a powder capacity 17930 cubic inches and a charge of 245 lbs. of cordite M.D.,

$$\Delta = \frac{27.73 \times 245}{17930} = 0.39.$$

Also the number of cubic inches allotted to each pound of cordite in the chamber is equal to $\frac{17930}{245} = 70.5$, hence 70.5 is the ratio of 27.73 to Δ , and $\frac{27.73}{0.39} = 70.5$.

When the number of cubic inches allotted to each pound of the charge in the chamber is 27.73, then the loading density is $\Delta = 1$.

$$\text{Expansion volume: } v_0 = \frac{1}{\Delta}.$$

When the contents of a whole charge are supposed evenly distributed throughout the space in the gun *behind the projectile*, then

$$v = \frac{\text{volume occupied per lb. of charge}}{27.73}$$

When the shot is rammed home, then before it moves,

$$v_0 = \frac{C}{27.73\omega} = \frac{1}{\Delta}.$$

When $\Delta = 1$, then $C = 27.73\omega$, $v_0 = 1$, and the unit expansion volume is 27.73 cubic inches per lb. of charge.

In the case of the 12-inch IX, $v_0 = \frac{1}{0.39} = 2.56$ expansion volumes in the chamber.

When the projectile has moved down the bore a distance s inches from the seat of the projectile, the space behind it is $\left(C + \frac{\pi}{4}d^2s\right)$ in.³, and then the number of expansion volumes is (*see fig. 3*)

$$= \frac{C + \frac{\pi}{4}d^2s}{27.73\omega} = \frac{\frac{\pi}{4}d^2l_0 + \frac{\pi}{4}d^2s}{27.73\omega} = \frac{\frac{\pi}{4}d^2(l_0 + s)}{27.73\omega} = \frac{\frac{\pi}{4}d^2l}{27.73\omega} = 0.0283 \frac{d^2l}{\omega};$$

so that, if the total volume of the chamber and bore to the muzzle be B in.³, then

$$B = C + \frac{\pi}{4}d^2s_2$$

where s_2 is the total travel, and

$$v_2 = \frac{B}{27.73\omega}.$$

In the case of the 12-inch IX, this gives

$$v_2 = \frac{63,100}{27.73 \times 245} = 9.3,$$

total expansion volumes in the bore.

v_1 is usually taken to represent the number of expansions to the point of maximum pressure: if this point be reached after a travel of s_1 inches, then

$$v_1 = \frac{C + \frac{\pi}{4}d^2s_1}{27.73\omega} = \frac{\frac{\pi}{4}d^2l_1}{27.73\omega} = 0.0283 \frac{d^2l_1}{\omega},$$

where

$$l_1 = l_0 + s_1.$$

Equivalent length of initial air space : z_0 .

In fig. 2 OA' represents the actual volume occupied by the charge as a solid ; AC' represents the total air space in the chamber ; the length AC = z_0 inches, so that

$$z_0 = OC - OA = \frac{C}{\frac{\pi}{4} d^2} - \frac{27 \cdot 73 \overline{w}}{\frac{\pi}{4} d^2 \delta}.$$

But

$$C = \frac{27 \cdot 73 \overline{w}}{\Delta},$$

therefore

$$\begin{aligned} z_0 &= \frac{27 \cdot 73 \overline{w}}{\pi d^2} \left(\frac{1}{\Delta} - \frac{1}{\delta} \right) \\ &= 35 \cdot 3 \left(\frac{1}{\Delta} - \frac{1}{\delta} \right) \frac{\overline{w}}{d^2} \text{ inches} \\ &= 2 \cdot 94 \frac{a^2 \overline{w}}{d^2} \text{ feet,} \end{aligned}$$

where

$$a^2 = \frac{1}{\Delta} - \frac{1}{\delta}.$$

This formula is made great use of in the treatment of Interior Ballistics by Colonel Ingalls, U.S. Artillery (see *Journal of the R.A.*, December, 1909, vol. XXXVI), "*Interior Ballistics after Lissak upon Ingalls*," by Captain J. H. Hardecastle, *p.a.c.*, late R.A. ; also in the confidential "*Internal Ballistics*," by Major N. B. Hefferman, *p.a.c.*, R.A., published in 1907.

From closed-vessel experiments the pressure corresponding to different densities of loading is obtained. Plotting these pressures as ordinates and the corresponding values of $\frac{1}{\Delta} = v$, that is expansion volumes as abscissæ, a curve of p and v can be drawn, and then, by a quadrature of the curve, the theoretical work E in foot/tons realised in a gun by the expansion of 1 lb. of an explosive from one value v_1 to another v_2 can be obtained, unit expansion volume per lb. of explosive being 27·73 cubic inches.

At any point of the curve let the pressure be p tons/in.², and suppose this constant while $v - \frac{dv}{2}$ changes to $v + \frac{dv}{2}$, that is, during a change of dv corresponding to $27 \cdot 73 dv$ cubic inches, the total pressure on the base of the shot in this interval is $p \times \frac{\pi}{4} d^2$, and the distance travelled by the shot is

$$\frac{27 \cdot 73 dv}{\frac{\pi}{4} d^2} \text{ inches,}$$

hence the work done is

$$27 \cdot 73 p dv \text{ inch tons} = \frac{27 \cdot 73}{12} p dv \text{ foot tons ;}$$

therefore

$$\Delta E = 2 \cdot 31 p dv \text{ foot tons,}$$

and the total work done from $v = v_1$ to $v = v_2$ is

$$E = 2 \cdot 31 \int_{v_1}^{v_2} p dv \text{ foot tons.}$$

By this method the theoretical work capable of being done by 1 lb. of gunpowder or of cordite, Mark I or M.D., in expanding from volume unity to another can be found.

For π lbs. of explosive

$$E = 2 \cdot 31 \pi \int_{v_1}^{v_2} p \, dv \text{ foot tons.}$$

If the curve connecting p and v be of an adiabatic nature, $pv^\gamma = \text{const.} = C$, then

$$\begin{aligned} \int_{v_1}^{v_2} p \, dv &= C \int \frac{dv}{v^\gamma} = \frac{C}{\gamma-1} \left[\frac{1}{v_1^{\gamma-1}} - \frac{1}{v_2^{\gamma-1}} \right] \\ &= \frac{p_1 v_1 - p_2 v_2}{\gamma-1} \quad \text{or} \quad \frac{p_1 v_1}{\gamma-1} \left[1 - \left(\frac{v_1}{v_2} \right)^{\gamma-1} \right], \end{aligned}$$

and then

$$E = 2 \cdot 31 \pi \frac{p_1 v_1 - p_2 v_2}{\gamma-1} \text{ foot tons}$$

or

$$2 \cdot 31 \pi \frac{p_1 v_1}{\gamma-1} \left[1 - \left(\frac{v_1}{v_2} \right)^{\gamma-1} \right],$$

so that E can be tabulated for all possible values of $\frac{v_1}{v_2}$.

When a gun is fired, the rate of ignition and the rate of combustion have a great effect on the pressures produced; of these two, the rate of combustion, however, has by far the greater effect. The greater the total surface of the explosive, the quicker the rate of evolution of the gases. The rate of combustion itself is a function of the pressure, so that the pressure is a function of the shape, size, chemical components, &c., of the explosive.

When the charge in a gun is ignited, a certain amount of pressure must be reached before the projectile's driving-band is "engraved." The rate of combustion is increased by the pressure, and before the projectile has moved far a high pressure has been reached. The projectile is meanwhile rapidly increasing its velocity and the volume behind it, so that very soon the pressure will begin to fall on account of this increasing volume and because of the loss of temperature of the gases from the conversion of heat into energy, also from the decreased surface of the explosive.

From the point of maximum pressure to the muzzle there should be a continuous fall of pressure, the decreasing pressure influencing the rate of combustion. The pressure-space curve along the whole length of travel is somewhat of the form in fig. 3.

The following method of treating interior ballistics involves no rigid theory; it is merely an attempt to simplify a very difficult problem and to arrive at a practical solution, namely, to find the weight and size of cordite M.D. which will produce in a given gun a required muzzle velocity and a given maximum pressure.

With cordite Mark I, or M.D., a charge is not, as a rule, wholly consumed when the point of maximum pressure is reached; it is however found that the pressure-space curve from this point to the muzzle is of the nature of an adiabatic.

(a) From the results of Sir A. Noble's experiments with closed vessel, &c., Mr. A. G. Haddock finds that from the point of maximum pressure to the muzzle, the curve connecting the pressure with the expansion volume, v , approximates very closely (for cordite M.D.) to the formula

$$(p-1)v^{1.28} = 107 \cdot 14.$$

(b) And that up to the point of maximum pressure the curve which connects p and v may be taken as a quadrant of an ellipse.

(c) Also that when employing (a) and (b) for finding the energy imparted to the projectile, a factor must be employed, which is given by

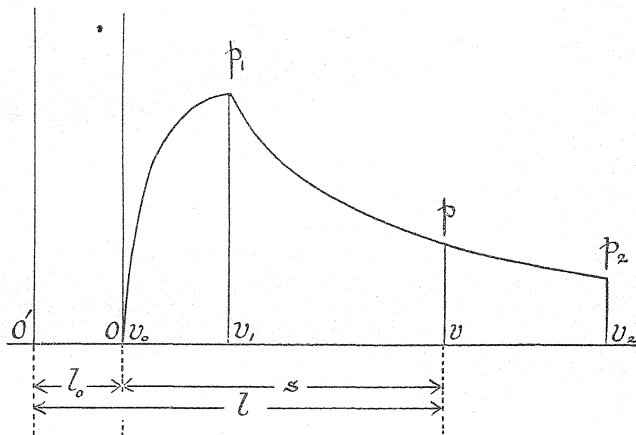
$$\text{factor} = 0.9 + 0.008d$$

where d is the calibre in inches.

For friction, &c., a deduction of 1 ton/in.² per foot of travel is made in foot tons, this is equal to

$$\text{area of bore} \times \text{shot travel in feet} = \frac{\pi d^2}{4} \cdot \frac{s}{12} \text{ foot tons.}$$

Fig. 3.



Accepting (a), (b), and (c), the following method is given for finding the weight of charge of cordite M.D., also the size which will just be burned at the muzzle to produce a given muzzle velocity and a given maximum chamber pressure.

The following data are required :—

- W lbs., weight of projectile.
- s_2 inches, total length of travel.
- d inches, calibre of gun.
- C cubic inches, capacity of chamber.
- V f/s, the required muzzle velocity.
- p_1 tons/in.², the maximum pressure allowed.

The following are then to be found :—

- ω lbs, weight of charge, cordite M.D.
- D inches, the actual diameter of the cord of cordite M.D.

In finding ω and D, the following curves are also obtained :—the pressure-space, velocity-space, time-space, film of Cordite-space; these give the pressure, velocity, time, and thickness of cordite burned, at any point along the bore of the gun.

The volume v in (a) and (b) is

$$v = \frac{\text{capacity}}{27 \cdot 73 \varpi} = \frac{\frac{\pi}{4} d^2 l}{27 \cdot 73 \varpi} = \frac{d^2 l}{35 \cdot 3 \varpi},$$

where l inches is the distance from any point in the bore, measured from the rear end of equivalent length of the chamber.

Substituting this value in the equation

$$(p-1) v^{1.28} = 107 \cdot 14,$$

then

$$(p-1) \left(\frac{\pi}{4} d^2 l \right)^{1.28} = 107 \cdot 14 (27 \cdot 73 \varpi)^{1.28},$$

$$\frac{1}{1.28} = 0.78, \text{ therefore } (p-1)^{0.78} \frac{d^2 l}{\varpi} = (107 \cdot 14)^{0.78} \frac{27 \cdot 73 \times 4}{\pi},$$

$$(p-1)^{0.78} d^2 l = 1361 \varpi \quad \dots \dots \dots (A).$$

In fig. 3, p_1 and v_1 are the pressure and volumes of expansion at the point of maximum pressure; p_2 , v_2 represent corresponding values at the muzzle; v_0 represents the volumes of expansion in the powder chamber itself. The area of the curve from v_0 to v_1 is $\frac{\pi}{4} (v_1 - v_0) p_1$, and from v_1 to v_2 the area is

$$\begin{aligned} \int_{v_1}^{v_2} p \, dv &= \int_{v_1}^{v_2} \left(\frac{107 \cdot 14}{v^{1.28}} + 1 \right) dv \\ &= \frac{107 \cdot 14}{0 \cdot 28} \left(\frac{1}{v_1^{0.28}} - \frac{1}{v_2^{0.28}} \right) + (v_2 - v_1) \\ &= 382 \cdot 5 \left(\frac{1}{v_1^{0.28}} - \frac{1}{v_2^{0.28}} \right) + (v_2 - v_1). \end{aligned}$$

The total area is therefore

$$(v_1 - v_0) p_1 \frac{\pi}{4} + 382 \cdot 5 \left(\frac{1}{v_1^{0.28}} - \frac{1}{v_2^{0.28}} \right) + (v_2 - v_1).$$

The work in foot tons per lb. of charge is

$$e = 2 \cdot 31 \left[(v_1 - v_0) p_1 \frac{\pi}{4} + 382 \cdot 5 \left(\frac{1}{v_1^{0.28}} - \frac{1}{v_2^{0.28}} \right) + (v_2 - v_1) \right] \quad \dots \dots \dots (B).$$

For ϖ lbs. of charge then, the work done on the projectile throughout the bore is equal to the muzzle energy in foot tons, and with the introduction of the factor and deduction for friction, &c.,

$$\begin{aligned} E &= \frac{WV^2}{2g \times 2240} \\ &= \text{factor} \left\{ 2 \cdot 31 \varpi \left[(v_1 - v_0) p_1 \frac{\pi}{4} + 382 \cdot 5 \left(\frac{1}{v_1^{0.28}} - \frac{1}{v_2^{0.28}} \right) + (v_2 - v_1) \right] \right. \\ &\quad \left. - \text{area of bore} \times \text{shot travel in feet} \right\}, \end{aligned}$$

or,

$$E = f \left(\varpi c - \frac{\pi}{4} d^2 \frac{s_2}{12} \right) \text{ foot tons};$$

this gives

$$\varpi c = \frac{E}{f} + \frac{\pi}{4} d^2 \frac{s_2}{12} \quad \dots \dots \dots (C).$$

From the equation

$$(p-1)r_1^{1.28} = 107 \cdot 14 \text{ (Hadcok's empirical formula for M.D. cordite),}$$

at point of maximum pressure,

$$r_1 = \left(\frac{107 \cdot 14}{p_1 - 1} \right)^{\frac{1}{1.28}} = \left(\frac{107 \cdot 14}{p_1 - 1} \right)^{0.78},$$

so that when p_1 is fixed, r_1 is also known, and in the equation (B), when p_1 is known, and hence also r_1 , then for any chosen value of r_2 this last equation becomes of the form

$$e = \lambda r_0 + \mu, \quad \tan \theta = \frac{e - \mu}{r_0} = \lambda,$$

which is a straight line, where λ and μ are constants, got from the known values of p_1 , r_1 , and the chosen value of r_2 , where

$$\lambda = -2 \cdot 31 p_1 \frac{\pi}{4} = \tan \theta,$$

$$\mu = 2 \cdot 31 \left[r_1 p_1 \frac{\pi}{4} + 382 \cdot 5 \left(\frac{1}{r_1^{0.28}} - \frac{1}{r_2^{0.28}} \right) + (r_2 - r_1) \right] \text{ in foot tons per lb. of cordite.}$$

Since λ does not contain r_2 , it follows that for a fixed value of p_1 , r_1 is known, and for various values of r_2 , the relation between e and r_0 can be represented by a series of parallel straight lines; r_0 being represented as abscissae and e as ordinates.

Example in drawing parallel lines for the case of $p_1 = 16$ tons/in.² (See Diagram A, by Captain R. K. Hezlet, R.A.)

Select any value of r_2 , say 10, then from (B) evaluate e for $r_0 = 2$, $r_0 = 3$, etc., as an example. The straight line got from joining the points (r_2 , $r_0 = 2$) and (r_2 , $r_0 = 3$) is drawn; e can now be read off this line for all values of r_0 and for $r_2 = 10$.

To draw the other lines it is necessary to find the co-ordinate values of e and r_0 for one point only and for another selected value of r_2 , say $r_2 = 11$; a line through this point parallel to the first gives e in terms of all values of r_0 and for $r_2 = 11$.

From the diagram of the series of parallel lines, e can be read off for all values of r_2 and all values of r_0 , so that the diagram gives a complete record of e for all values of r_0 and r_1 got from (B), when

$$p_1 = 16, \quad r_1 = 4 \cdot 64, \quad \lambda = \tan \theta = -29 \cdot 05$$

For any maximum pressure p_1 a similar diagram of parallel straight lines can be drawn.

On any one diagram for p_1 use is made of the equation $r_0 = \frac{C}{27 \cdot 73 \varpi}$, and that in any gun under discussion $\frac{r_2}{r_0} = \frac{B}{C} = \text{constant}$.

From the intersection of the plotting on the diagram of $r_0 = \frac{C}{27 \cdot 73 \varpi}$ and $\frac{r_2}{r_0} = \frac{B}{C}$, the weight of charge ϖ is read off.

An example will readily show the method of procedure, which is exactly the same for any gun.

Example for finding ϖ . (Diagram A.)

60-pr. B.L. :—

$$W = 60 \text{ lbs.}$$

$$d = 5 \text{ inches,}$$

$$s_2 = 11.5 \text{ feet.}$$

$$C = 618.75 \text{ ins.}^3.$$

$$B \text{ (total bore)} = 3361 \text{ ins.}^3.$$

It is required to find the charge of cordite M.D. which will give M.V. = 2030 f/s and

$$\text{Muzzle energy } E = \frac{p_1 = 16}{2g \times 2240} \frac{WV^2}{1} = 1818 \text{ foot tons}$$

$$\text{Factor, } f = 0.9 + (0.008 \times 5) = 0.94.$$

From

$$\begin{aligned} \varpi e &= \frac{E}{f} + \frac{\pi}{4} d^2 s_2 \\ &= \frac{1818}{0.94} + \frac{\pi}{4} \times 25 \times 11.5 = 2160, \\ \varpi &= \frac{2160}{e}. \end{aligned}$$

Now

$$\frac{1}{\Delta} = v_0 = \frac{C}{27.73\varpi} = \frac{618.75}{27.73\varpi},$$

therefore

$$\begin{aligned} \frac{v_0}{e} &= \frac{618.75}{27.73\varpi e} = \frac{618.75}{27.73 \times 2160} = \frac{618.75}{59,900} \\ &= \frac{2.07}{200} = \frac{3.1}{300} \text{ (a straight line).} \end{aligned}$$

From Diagram A :—

$$v_0 = 2.07 \text{ when } e = 200,$$

and

$$v_0 = 3.1 \text{ when } e = 300.$$

On the Diagram A, for $p_1 = 16$, plot any two selected points such as (2.07, 200), (3.1, 300), and draw the straight line through them.

Again,

$$\frac{v_2}{v_0} = \frac{B}{C} = \frac{3361}{618.75} = \frac{12}{2.21} = \frac{13}{2.395};$$

since

$$v_0 = 2.21 \text{ when } v_2 = 12 \text{ and } v_0 = 2.395 \text{ when } v_2 = 13.$$

Draw the straight line joining the points selected, (12, 2.21), (13, 2.395).

The intersection (see Diagram) of the two last drawn curves or straight lines gives a value of $e = 225$, from which

$$\varpi = \frac{2160}{225} = 9.6 \text{ lbs.}$$

DIAGRAM A.

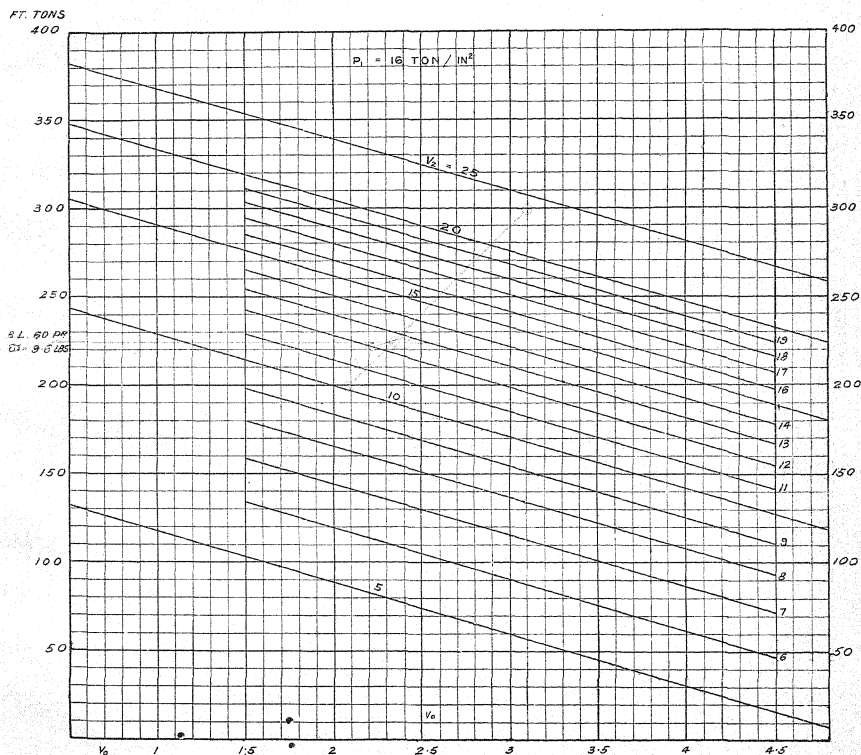
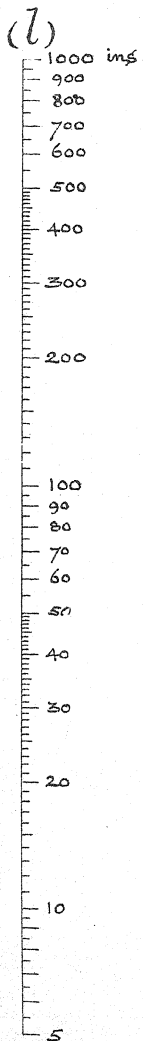
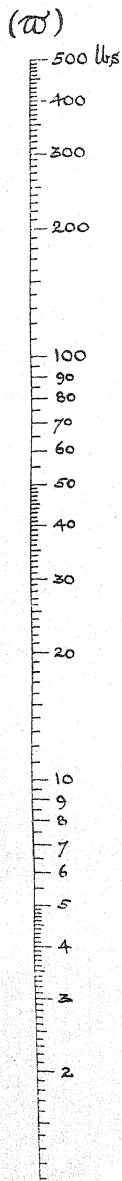


DIAGRAM B.

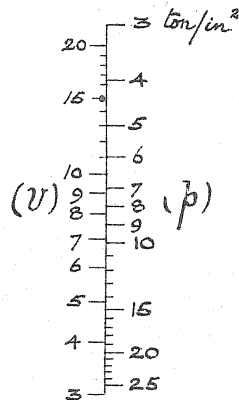
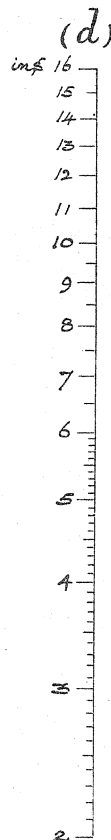
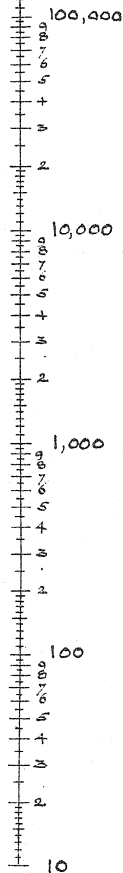
To face page 143. Chapter VII.



(C)

$$1,000,000 \text{ in}^3 \quad (p-1)v^{1.28} = 107.14$$

$$\frac{\pi d^3 l}{4} = C = 27.73 w v$$



SK
1909

The Pressure-space Curve between Point of Maximum Pressure and the Muzzle.

From page 140, formula (A),

$$(p-1)^{0.78} d^2 l = 1361 \varpi.$$

A graphic representation of this formula is most useful (see Diagram B).

Writing

$$d^2 l = 1361 \varpi (p-1)^{-0.78}$$

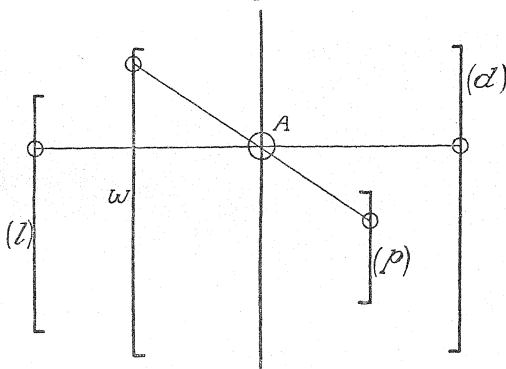
$$2 \log d + \log l = \log \varpi - 0.78 \log (p-1) + \log 1361,$$

which is of the form

$$\phi(d) + \psi(l) = f(\varpi) - F(p).$$

An equation of this form can, as shown by M. d'Ocagne in "Calcul-graphique et Nomographie," Paris, 1908, be represented by a "nomogram," in which the scales of d , l , ϖ , p lie on four parallel straight lines, while a fifth central undivided "line of reference" serves to connect the alignment of corresponding values of d (calibre) and l (travel plus E.L.C.), ϖ (weight of charge) and p (pressure) (fig. 4).

Fig. 4.



The nomogram is read in the following way:—

Supposing l , d , ϖ are known, and it is required to find the value of p to satisfy

$$(p-1)^{0.78} d^2 l = 1361 \varpi.$$

Join the given values of l and d on their respective scales by a straight line cutting the reference line in a point A (fig. 4).

Join the given value of ϖ on its scale to A and produce to cut the scale of p , the point of intersection will give the required value of p .

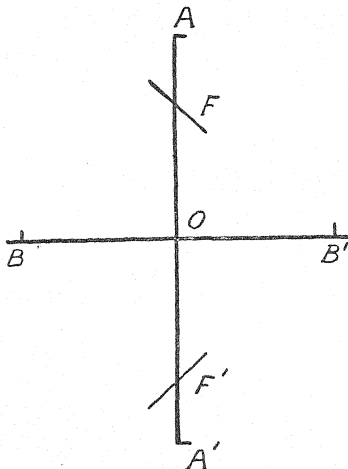
On Diagram B, worked out and drawn by Captain R. K. Hezlet, R.A., a complete representation is given which is applicable for variations of 1 to 500 lbs. for ϖ , 5 to 1000 inches for l , 10 to 1,000,000 cubic inches for capacity of chamber C, 2 to 16 inches for d , 3 to 25 tons/in.² for p ; l is measured from the rear end of the equivalent length of the chamber.

From Diagram B (the central scale has in this case been divided to show the actual capacities in cubic inches) a number of points can be at once plotted in the pressure-space curve between the point of maximum pressure and the muzzle; this part of the curve can be drawn (see Diagram C).

The Pressure-space Curve up to the Point of Maximum Pressure.

This is a quadrant of an ellipse (see p. 138), in which the semi-major axis is the length representing the maximum pressure p_1 tons/in.², the semi-minor axis is the length representing the travel s inches up to the point of maximum pressure, this " s " inches is found by means of Diagram B; p_1 , d , C , ϖ being known, l is found, hence $s = (l - l_0)$ inches is found, where l_0 is the equivalent length of the chamber.

Suppose $OA = a$, $OB = b$ represent the semi-major and semi-minor axes respectively. With centre B and radius equal to a describe a circle cutting AOA' in F and F', then F and F' are the foci.



Take a string of length $2a$ equal to the major axis, fasten the ends at F and F', then a pencil in the loop will describe a true ellipse; the required quadrant of the pressure-space curve up to the maximum pressure is thus quickly drawn with accuracy (see Diagram C).

Velocity along the Bore: Velocity-space Curve. (Diagram C.)

This is obtained quickly with the help of a planimeter, which gives the area, A , of the pressure-space curve up to any point along the bore; an example from the 60-pr. B.L. is given:—

60-pr. B.L.—

$$d = 5, \quad \varpi = 9.44 \text{ lbs.}, \quad p_1 = 16, \quad C = 618.75,$$

$$\text{E.L.C.} = l_0 = 31.5 \text{ inches}, \quad W = 60 \text{ lbs.}, \quad \text{total travel, 138 inches.}$$

Distance from end of equivalent length of chamber to point of maximum pressure is $s_1 = 62$ inches.

From the pressure-space diagram:—

$$1 \text{ square inch represents } 20 \text{ inch tons/in.}^2.$$

$$\text{Empirical factor} = 0.9 [0.9 + 0.008d] = 0.9 \times 0.94 = 0.846.$$

DIAGRAM C.

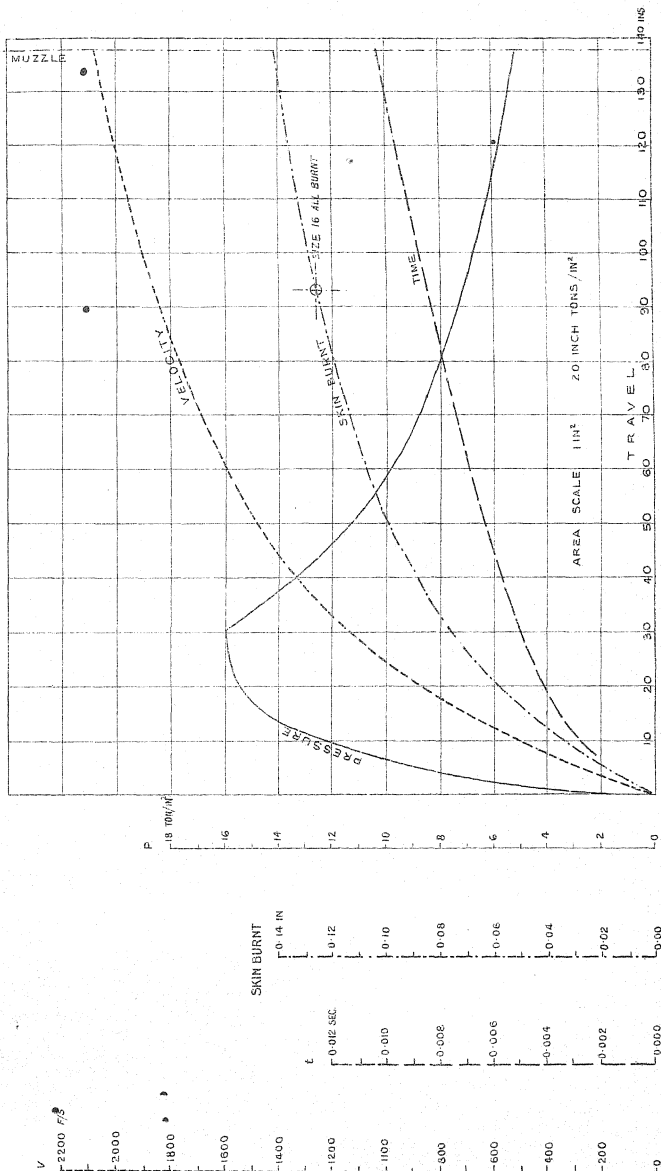
CHAMBER = 618.75 IN³ TRAVEL TO MAX^m PRESS 350.5 IN.
EQUIV LENGTH = 31.5 IN.

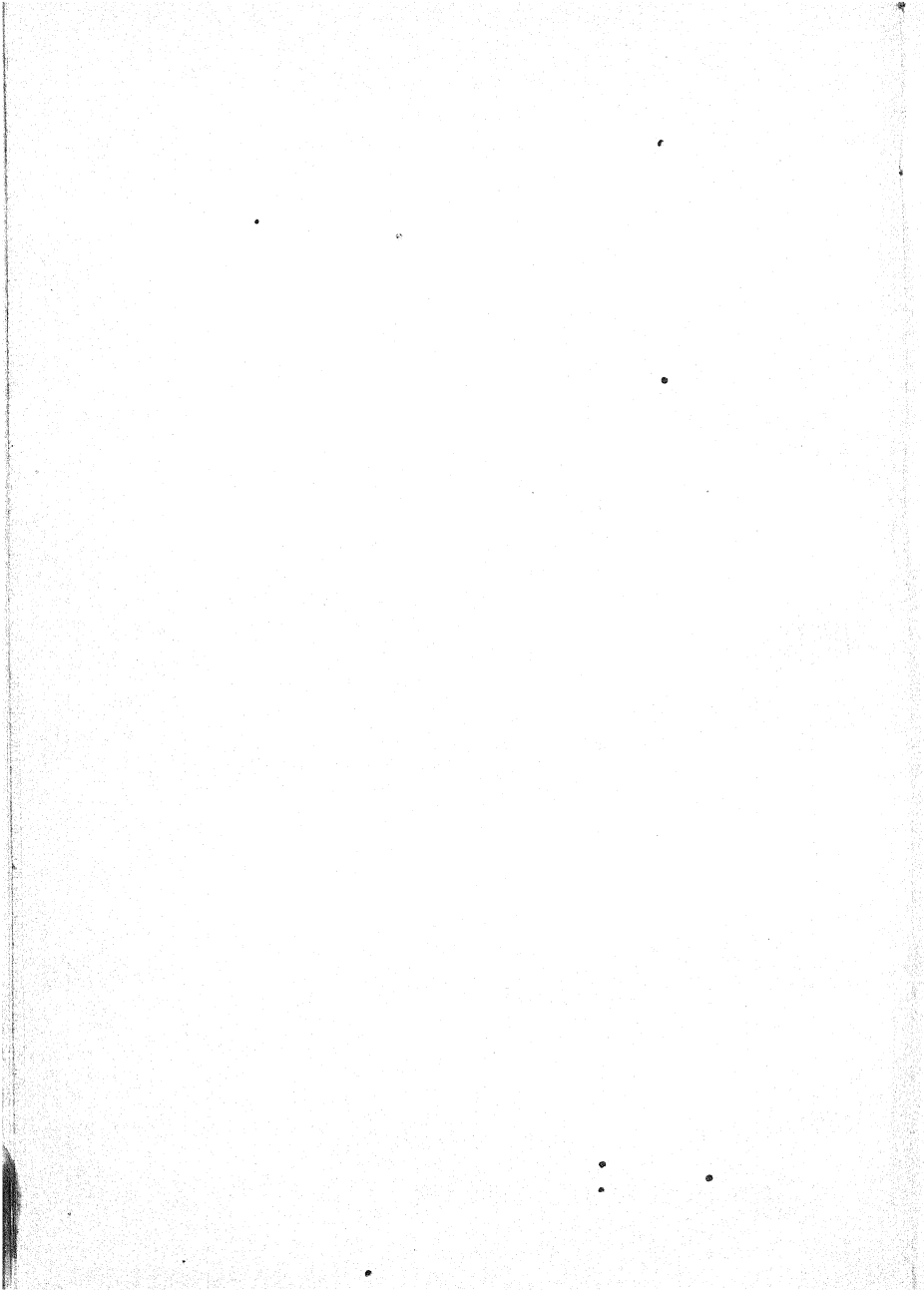
MAX^m PRESS. 16 TON./IN²
TRAVEL 136 IN.

$\omega = 9.44$ LB.
SIZE 16

$\delta = 5$ IN.
 $\omega = 60$ LB.

B. L. 60 PR





The velocity at any point is given by

$$\frac{Wv^2}{2g \times 2240} \text{ foot/tons} = A \times \frac{v^2}{12} \times 0.846 \times \pi \frac{r^2}{4},$$

$$v \text{ f/s} = \sqrt{A \times 20 \times 0.846 \times 62} = 255 \sqrt{A}.$$

Travel <i>s</i> inches.	<i>l</i> inches.	<i>p</i> tons/in. ² .	Area, sq. inches.	<i>v</i> = 255 \sqrt{A} , feet per second.
0	31.5	—	—	0
20	—	—	11.60	868
30.5	62	16	—	—
40	—	—	27.82	1345
60	—	9.8	39.37	1598
80	—	8.1	48.20	1770
100	—	6.1	55.61	1900
120	—	5.8	61.86	2005
138	—	5.1	66.70	2080

See Diagram C, which is worked out and drawn by Captain R. K. Hezlet, R.A.

Time-space, Film Burnt-space Curves, and Size of Cordite to be Used. (Diagram C.)

Use is made of Mansell's table for film of cordite burnt in 0.001 second under various pressures. Temperature of charge 80° F. For the 60-pr. B.L., the following table shows the tabulated figures which provide the data, and a similar procedure applies to any gun:—

Travel, <i>s</i> inches.	\bar{v} f/s, mean velocity over the interval.	Δt secs., time over the interval.	<i>t</i> secs., time to any point along bore.	\bar{p} tons/in. ² , mean pressure over the interval.	Mansell's factor $\times 1000$, Temperature of charge 80° F.	ΔF inches, film-burnt (reduction of diameter) during the interval.	F inches thickness burnt up to any point along bore.
0							
10	270	0.00309		8.6	11.78	0.0363	
20	700	0.00119	0.00309	14.7	19.92	0.0236	0.0363
30	1020	0.00082	0.00428	15.8	21.40	0.0175	0.0599
40	1250	0.00067	0.00510	14.7	19.92	0.0133	0.0774
40	1490	0.00112	0.00577	11.3	15.38	0.0172	0.0907
60	1700	0.00098	0.00689	8.8	12.04	0.0118	0.1079
80	1840	0.00091	0.00787	7.4	10.17	0.0092	0.1197
100	1960	0.00085	0.00878	6.3	8.70	0.0074	0.1289
120	2050	0.00073	0.00963	5.6	7.77	0.0057	0.1363
138			0.01036				0.1420*

* This is close to the actual size (0.1265) of size 16 M.D. Hence size 16 M.D. should be used. (See Diagram C.) This size 16 would be all burnt at a point about 40 inches from the muzzle.

GUNNERY TABLES.

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INTRODUCTION.

These ballistic tables are based upon experiments carried out at Shoeburyness in 1904-6 with projectiles of two-calibre head (Ordnance Committee Reports Nos. 1179, 1185 and 1192).

The laws of resistance of the air deduced from these experiments are expressed by the following equations in which C is the ballistic coefficient and r the retardation (in foot-seconds), or resistance per unit of projectile's mass :—

$$4000 \text{ f/s} > v > 2600 \text{ f/s,}$$

$$Cr = [7.1865702 - 10] e^{1.07}.$$

$$2600 \text{ f/s} > v > 2000 \text{ f/s,}$$

$$Cr = [7.7671157 - 10] e^{1.5}.$$

$$2000 \text{ f/s} > v > 1460 \text{ f/s,}$$

$$Cr = [6.7768067 - 10] e^{1.8}$$

$$1460 \text{ f/s} > v > 1190 \text{ f/s,}$$

$$Cr = [2.9795830 - 10] e^3.$$

$$1190 \text{ f/s} > v > 1040 \text{ f/s,}$$

$$Cr = [2.3689459 - 20] e^{4.5}.$$

$$1040 \text{ f/s} > v > 840 \text{ f/s,}$$

$$Cr = [2.7777107 - 10] e^8.$$

$$840 \text{ f/s} > v > 0 \text{ f/s,}$$

$$Cr = [6.8717017 - 10] e^{16}.$$

N.B.—The figures in the brackets are the logarithms of the coefficients of v .

TABLE I. gives values of K , Cr , and p for velocities from 100 f/s to 4000 f/s.

TABLES II., III., IV., V., VI., and VIII. are in appearance similar to those in Text Book of Gunnery, Part I., 1907, which they now supersede.

TABLE VII. is compiled from Glaisher's *Hygrometrical Tables*, and is useful for obtaining values of the coefficient of tenuity, τ , for all readings of the wet and dry bulb.

From this table, δ , the density in grains/ft.³ for the reading of the barometer in inches, and of the wet and dry bulb thermometer in degrees Fahrenheit of the meteorological record on the day of an experiment can be obtained, and thence the tenuity factor $\tau = \frac{\delta}{\Delta}$, where $\Delta = 534.22$ is the standard density in grains/ft.³ for dry air at 62° F. and a 30-inch barometric height.

To use the table, look out the number corresponding to the readings given by the wet and dry bulb; this gives the grains/ft.³ at a barometric height of 29 inches; and then the table of proportional parts (used in conjunction with the column "Difference for 1 inch in barometer") shows the addition to be made for the extra height of the barometer above 29 inches. The standard barometric* height in the table is taken very low, at 29 inches, in order to avoid negative proportional parts.

Examples :—

1. On March 7, 1879, the meteorological record was

$$\text{Thermometer } \left\{ \begin{array}{l} \text{Wet } 37^{\circ} \\ \text{Dry } 39^{\circ} \end{array} \right\} \text{Barometer } 29 \cdot 95 \left\{ \begin{array}{l} 538 \cdot 9 \\ 16 \cdot 7 \\ 0 \cdot 9 \end{array} \right.$$

$$\delta = 556 \cdot 5$$

$$\tau = \frac{\delta}{\Delta} = \frac{556 \cdot 5}{534 \cdot 22} = 1 \cdot 042.$$

2. On March 11, 1879, the readings were

$$\text{Thermometer } \left\{ \begin{array}{l} \text{Wet } 42^{\circ} \\ \text{Dry } 45^{\circ} \end{array} \right\} \text{Barometer } 30 \cdot 25 \left\{ \begin{array}{l} 532 \cdot 3 \\ 18 \cdot 3 \\ 3 \cdot 7 \\ 0 \cdot 9 \end{array} \right.$$

$$\delta = 555 \cdot 2$$

$$\tau = \frac{\delta}{\Delta} = \frac{555 \cdot 2}{534 \cdot 22} = 1 \cdot 039$$

TABLE I.

Values of K , C_r , and p .

v	K	$K \left(\frac{v}{1000} \right)^3$ $= C_r = pg$	p	v	K	$K \left(\frac{v}{1000} \right)^3$ $= C_r = pg$	p
<i>f/s.</i>			<i>lbs.</i>	<i>f/s.</i>			<i>lbs.</i>
100	1179.51	1.18	0.037	560	105.74	18.6	0.577
110	1032.18	1.37	0.043	580	100.67	19.6	0.610
120	913.80	1.58	0.049	600	96.01	20.7	0.644
130	816.93	1.79	0.056	620	91.70	21.9	0.679
140	736.41	2.02	0.063	640	87.71	23.0	0.714
150	668.61	2.26	0.070	660	84.01	24.2	0.750
160	610.85	2.50	0.078	680	80.57	25.3	0.787
170	561.14	2.76	0.086	700	77.37	26.5	0.824
180	517.99	3.02	0.094	720	74.38	27.8	0.862
190	480.23	3.29	0.102	740	71.58	29.0	0.901
200	446.95	3.58	0.111	760	68.96	30.3	0.940
210	417.44	3.87	0.120	780	66.49	31.6	0.980
220	391.12	4.16	0.129	800	64.18	32.9	1.02
230	367.52	4.47	0.139	820	62.00	34.2	1.06
240	346.26	4.79	0.149	840	59.94	35.5	1.10
250	327.04	5.11	0.159	860	59.94	38.1	1.18
260	309.56	5.44	0.169	880	59.94	40.8	1.27
270	293.63	5.78	0.180	900	59.94	43.7	1.36
280	279.04	6.13	0.190	920	59.94	46.7	1.45
290	265.67	6.48	0.201	940	59.94	49.8	1.55
300	253.36	6.84	0.212	960	59.94	53.0	1.65
310	241.99	7.21	0.224	980	59.94	56.4	1.75
320	231.47	7.58	0.236	1000	59.94	59.9	1.86
330	221.71	7.97	0.248	1020	59.94	63.6	1.98
340	212.63	8.36	0.260	1040	59.94	67.4	2.09
350	204.17	8.75	0.272	1060	64.01	76.2	2.37
360	196.28	9.16	0.284	1080	68.27	86.0	2.67
380	181.97	10.0	0.310	1100	72.73	96.8	3.01
400	169.36	10.8	0.337	1120	77.40	108.7	3.38
420	158.18	11.7	0.364	1140	82.27	121.9	3.79
440	148.21	12.6	0.392	1160	87.36	136.4	4.24
460	139.26	13.6	0.421	1180	92.67	152.3	4.73
480	131.21	14.5	0.451	1190	95.41	160.8	4.99
500	123.92	15.5	0.481	1200	95.41	164.9	5.12
520	117.30	16.5	0.512	1220	95.41	173.3	5.38
540	111.26	17.5	0.544	1240	95.41	181.9	5.65

Table I—continued.

v	K	$K \left(\frac{v}{1000} \right)^3$ $= Cr = pg$	p	v	K	$K \left(\frac{v}{1000} \right)^3$ $= Cr = pg$	p
f/s.			lbs.	f/s.			lbs.
1200	95.41	190.9	5.93	2280	53.73	630.8	19.78
1280	95.41	200.1	6.22	2300	53.03	645.2	20.04
1300	95.41	209.6	6.51	2320	52.35	659.7	20.30
1320	95.41	219.5	6.82	2340	51.68	662.1	20.57
1340	95.41	229.6	7.13	2360	51.02	670.6	20.83
1360	95.41	240.0	7.45	2380	50.38	679.2	21.10
1380	95.41	250.7	7.79	2400	49.75	687.8	21.36
1400	95.41	261.8	8.13	2420	49.13	696.4	21.63
1420	95.41	273.2	8.49	2440	48.53	705.0	21.90
1440	95.41	284.9	8.85	2460	47.94	713.7	22.17
1460	95.41	296.9	9.22	2480	47.36	722.4	22.44
1480	93.86	304.3	9.45	2500	46.80	731.2	22.71
1500	92.36	311.7	9.68	2520	46.24	740.0	22.99
1520	90.91	319.3	9.92	2540	45.70	748.8	23.26
1540	89.49	326.9	10.15	2560	45.16	757.7	23.54
1560	88.12	334.5	10.39	2580	44.64	766.6	23.81
1580	86.78	342.3	10.63	2600	44.12	775.5	24.09
1600	85.48	350.1	10.88	2620	43.67	783.5	24.40
1620	84.21	358.0	11.12	2640	43.24	793.5	24.71
1640	82.96	366.0	11.37	2660	42.80	803.6	25.03
1660	81.73	374.1	11.62	2680	42.38	815.8	25.34
1680	80.62	382.3	11.87	2700	41.96	826.0	25.66
1700	79.48	390.5	12.13	2720	41.55	836.2	25.98
1720	78.37	398.8	12.39	2740	41.15	846.5	26.30
1740	77.29	407.2	12.65	2760	40.75	856.8	26.62
1760	76.24	415.7	12.91	2780	40.36	867.2	26.94
1780	75.22	424.2	13.18	2800	39.98	877.7	27.26
1800	74.21	432.8	13.44	2820	39.60	888.1	27.59
1820	73.24	441.5	13.72	2840	39.23	898.7	27.92
1840	72.28	450.3	13.99	2860	38.87	909.3	28.25
1860	71.35	459.1	14.26	2880	38.51	919.9	28.58
1880	70.44	468.0	14.54	2900	38.17	930.6	28.91
1900	69.55	477.1	14.82	2920	37.81	941.4	29.24
1920	68.68	486.1	15.10	2940	37.47	952.2	29.58
1940	67.83	495.3	15.39	2960	37.13	963.0	29.92
1960	67.00	504.5	15.67	2980	36.80	973.9	30.25
1980	66.19	513.8	15.96	3000	36.48	984.8	30.59
2000	65.40	523.2	16.25	3020	36.15	995.8	30.93
2020	64.43	531.1	16.50	3040	35.84	1007	31.28
2040	63.49	539.0	16.74	3060	35.53	1018	31.62
2060	62.56	546.9	16.99	3080	35.22	1029	31.97
2080	61.66	554.9	17.24	3100	34.92	1040	32.32
2100	60.78	562.9	17.49	3120	34.62	1052	32.66
2120	59.93	571.0	17.74	3140	34.33	1063	33.01
2140	59.09	579.1	17.99	3160	34.04	1074	33.37
2160	58.27	587.2	18.24	3180	33.76	1086	33.72
2180	57.47	595.4	18.49	3200	33.48	1097	34.07
2200	56.69	603.6	18.75	3220	33.20	1108	34.43
2220	55.92	611.9	19.01	3240	32.93	1120	34.79
2240	55.18	620.1	19.26	3260	32.66	1132	35.15
2260	54.44	628.5	19.52	3280	32.39	1143	35.51

Table I—continued.

v	K	$K \left(\frac{v}{1000} \right)^3$ $= Cr = pg$	p	v	K	$K \left(\frac{v}{1000} \right)^3$ $= Cr = pg$	p
f/s			$lbs.$	f/s			$lbs.$
3300	32.13	1155	35.87	3660	28.00	1373	42.64
3320	31.88	1166	36.24	3680	27.80	1385	43.03
3340	31.62	1178	36.60	3700	27.60	1398	43.43
3360	31.37	1190	36.97	3720	27.40	1411	43.82
3380	31.13	1202	37.34	3740	27.21	1423	44.21
3400	30.88	1214	37.71	3760	27.01	1436	44.61
3420	30.64	1226	38.08	3780	26.82	1449	45.00
3440	30.40	1238	38.45	3800	26.64	1462	45.40
3460	30.17	1250	38.82	3820	26.45	1474	45.80
3480	29.94	1262	39.20	3840	26.27	1487	46.20
3500	29.71	1274	39.58	3860	26.09	1500	46.61
3520	29.49	1286	39.95	3880	25.91	1513	47.01
3540	29.27	1298	40.33	3900	25.73	1526	47.42
3560	29.05	1311	40.72	3920	25.56	1539	47.82
3580	28.83	1323	41.10	3940	25.38	1553	48.23
3600	28.62	1335	41.48	3960	25.21	1566	48.64
3620	28.41	1348	41.87	3980	25.05	1579	49.05
3640	28.20	1360	42.25	4000	24.88	1592	49.46

TABLE II.

Time t in seconds, between velocity V and v f/s. $t = C [T(V) - T(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	A.
f/s	0.186	1.027	1.855	2.670	3.472	4.262	5.040	5.807	6.562	7.306	+
10	8.040	8.762	9.474	10.177	10.869	11.552	12.225	12.889	13.544	14.190	.683
11	14.828	15.457	16.077	16.690	17.295	17.892	18.482	19.064	19.638	20.206	.967
12	20.767	21.321	21.868	22.408	22.942	23.470	23.991	24.507	25.016	25.519	.988
13	26.017	26.509	26.996	27.477	27.952	28.423	28.888	29.348	29.804	30.254	.971
14	30.069	31.140	31.576	32.008	32.435	32.857	33.276	33.690	34.100	34.506	.922
15	34.908	35.305	35.689	36.069	36.475	36.857	37.236	37.611	37.982	38.350	.882
16	38.715	39.076	39.433	39.788	40.139	40.487	40.831	41.173	41.512	41.847	.848
17	42.179	42.509	42.836	43.160	43.481	43.799	44.114	44.427	44.737	45.044	.818
18	45.319	45.622	45.922	46.219	46.514	46.806	47.126	47.414	47.699	47.982	.792
19	48.383	48.642	48.898	49.152	49.404	49.654	49.902	50.148	50.391	50.633	.770
20	51.310	51.510	51.706	51.920	52.132	52.322	52.470	52.716	52.961	53.204	.750
21	50.953	51.210	51.466	51.720	51.972	52.222	52.470	52.716	52.961	53.204	.750
22	53.445	53.684	53.921	54.157	54.391	54.624	54.855	55.084	55.311	55.537	.933
23	55.762	55.985	56.206	56.426	56.644	56.861	57.078	57.290	57.503	57.714	.917
24	57.923	58.131	58.338	58.544	58.748	58.951	59.152	59.352	59.551	59.749	.903
25	59.945	60.140	60.334	60.527	60.718	60.908	61.097	61.285	61.472	61.657	.910
26	61.842	62.025	62.207	62.388	62.568	62.747	62.924	63.101	63.277	63.451	.919
27	63.925	63.797	63.969	64.139	64.309	64.477	64.644	64.811	64.977	65.142	.968
28	65.305	65.468	65.630	65.791	65.951	66.110	66.268	66.425	66.582	66.738	.939
29	66.983	67.047	67.200	67.352	67.503	67.654	67.804	67.953	68.101	68.248	.931
30	68.394	68.540	68.685	68.829	68.973	69.116	69.258	69.399	69.540	69.680	.943
31	69.919	69.957	70.095	70.232	70.368	70.503	70.638	70.772	70.906	71.039	.935
32	71.302	71.433	71.563	71.693	71.822	71.950	72.078	72.205	72.331	72.456	.929
33	72.457	72.582	72.707	72.831	72.954	73.077	73.199	73.321	73.442	73.563	.923

34	7 3 683	3 802	3 921	4 039	4 157	4 274	4 391	4 507	4 622	4 737	0 117
35	4 852	4 966	5 079	5 192	5 304	5 416	5 528	5 639	5 749	5 859	112
36	5 969	6 078	6 186	6 294	6 402	6 509	6 615	6 721	6 827	6 932	107
37	7 7 087	7 141	7 244	7 348	7 451	7 554	7 656	7 758	7 859	7 960	108
38	8 080	8 160	8 260	8 359	8 457	8 555	8 653	8 751	8 848	8 945	098
39	9 041	9 137	9 232	9 327	9 422	9 516	9 610	9 704	9 797	9 890	094
40	7 9 982	*0 074	*0 166	*0 257	*0 348	*0 439	*0 529	*0 619	*0 709	*0 798	091
41	8 0 887	0 975	1 063	1 151	1 239	1 326	1 413	1 499	1 585	1 671	087
42	1 757	1 842	1 927	2 011	2 095	2 179	2 263	2 346	2 429	2 512	084
43	2 594	2 676	2 758	2 839	2 920	3 001	3 082	3 162	3 242	3 321	081
44	3 401	3 480	3 559	3 637	3 715	3 793	3 871	3 948	4 025	4 102	078
45	4 179	4 255	4 331	4 407	4 482	4 557	4 632	4 707	4 782	4 856	075
46	4 930	5 003	5 077	5 150	5 223	5 295	5 368	5 440	5 512	5 583	072
47	5 655	5 726	5 797	5 867	5 938	6 008	6 078	6 148	6 217	6 287	070
48	6 356	6 424	6 493	6 561	6 629	6 697	6 765	6 832	6 900	6 967	068
49	7 033	7 100	7 166	7 232	7 298	7 364	7 430	7 495	7 560	7 625	066
50	7 689	7 754	7 818	7 882	7 946	8 010	8 073	8 136	8 199	8 262	064
51	8 325	8 387	8 450	8 512	8 574	8 635	8 697	8 758	8 819	8 880	061
52	8 941	9 001	9 062	9 122	9 182	9 242	9 301	9 361	9 420	9 479	060
53	9 538	9 597	9 655	9 713	9 772	9 830	9 888	9 945	*0 003	*0 060	058
54	9 0 117	0 174	0 231	0 288	0 344	0 401	0 457	0 513	0 568	0 624	057
55	9 0 680	0 735	0 790	0 845	0 900	0 955	1 009	1 064	1 118	1 172	055
56	1 226	1 280	1 333	1 387	1 440	1 493	1 546	1 599	1 652	1 705	053
57	1 757	1 809	1 861	1 913	1 965	2 017	2 068	2 120	2 171	2 222	052
58	2 273	2 324	2 375	2 425	2 476	2 526	2 576	2 626	2 676	2 726	050
59	2 775	2 825	2 874	2 923	2 972	3 021	3 070	3 119	3 167	3 216	049
60	3 264	3 312	3 360	3 408	3 456	3 504	3 551	3 599	3 646	3 693	048
61	3 740	3 787	3 834	3 881	3 927	3 974	4 020	4 066	4 112	4 158	047
62	4 204	4 250	4 295	4 341	4 386	4 431	4 476	4 521	4 566	4 611	045
63	4 655	4 700	4 744	4 789	4 833	4 877	4 921	4 965	5 009	5 053	044
64	5 086	5 140	5 183	5 226	5 269	5 312	5 355	5 398	5 440	5 483	043
65	5 525	5 568	5 610	5 652	5 694	5 736	5 778	5 820	5 863	5 903	042
66	5 944	5 986	6 027	6 068	6 109	6 150	6 191	6 232	6 272	6 313	040
67	6 354	6 394	6 434	6 474	6 514	6 554	6 594	6 634	6 674	6 714	040
68	6 753	6 793	6 832	6 871	6 910	6 949	6 988	7 027	7 066	7 105	039
69	7 143	7 182	7 220	7 259	7 297	7 335	7 373	7 411	7 449	7 487	037

Table II—continued.
 $t = C[T(V) - T(e)]$

τ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											+
70	9 7 524	7 582	7 599	7 637	7 674	7 712	7 749	7 786	7 823	7 860	0 367
71	7 587	7 597	7 610	7 647	7 684	7 722	7 759	7 796	7 833	7 870	0 368
72	8 261	8 267	8 283	8 320	8 357	8 394	8 431	8 468	8 505	8 542	0 369
73	9 8 617	8 653	8 688	8 723	8 758	8 793	8 828	8 863	8 907	8 942	0 370
74	8 966	9 001	9 035	9 069	9 103	9 137	9 171	9 205	9 239	9 273	0 371
75	9 307	9 341	9 374	9 408	9 441	9 475	9 508	9 542	9 575	9 608	0 372
76	9 9 641	9 674	9 707	9 740	9 772	9 805	9 838	9 871	9 903	9 936	0 373
77	9 968	9 000	9 032	9 065	9 097	9 129	9 161	9 193	9 225	9 257	0 374
78	10 0 288	0 320	0 351	0 383	0 414	0 446	0 477	0 508	0 539	0 571	0 375
79	10 0 602	0 633	0 664	0 695	0 725	0 756	0 787	0 818	0 848	0 879	0 376
80	0 969	0 940	0 970	1 000	1 030	1 061	1 091	1 121	1 151	1 181	0 377
81	1 210	1 240	1 270	1 300	1 329	1 359	1 388	1 418	1 447	1 477	0 378
82	10 1 506	1 535	1 564	1 593	1 622	1 651	1 680	1 709	1 738	1 767	0 379
83	1 796	1 825	1 853	1 882	1 910	1 939	1 967	1 995	2 023	2 052	0 380
84	20 80	2 108	2 136	2 164	2 192	2 220	2 247	2 275	2 302	2 329	0 381
85	10 2 356	2 384	2 411	2 438	2 464	2 491	2 518	2 545	2 571	2 597	0 382
86	2 623	2 650	2 676	2 702	2 728	2 754	2 779	2 805	2 830	2 856	0 383
87	2 881	2 907	2 932	2 957	2 982	3 007	3 032	3 056	3 081	3 105	0 384
88	10 3 130	3 155	3 179	3 203	3 227	3 252	3 276	3 300	3 323	3 347	0 385
89	3 371	3 394	3 418	3 442	3 465	3 488	3 511	3 535	3 558	3 581	0 386
90	3 604	3 627	3 649	3 672	3 695	3 718	3 740	3 762	3 784	3 807	0 387
91	10 3 829	3 851	3 873	3 895	3 917	3 939	3 960	3 982	4 004	4 026	0 388
92	4 047	4 068	4 089	4 111	4 132	4 153	4 174	4 195	4 216	4 237	0 389
93	4 257	4 278	4 299	4 320	4 340	4 361	4 381	4 401	4 421	4 441	0 390
94	10 4 461	4 482	4 502	4 522	4 541	4 561	4 581	4 601	4 620	4 640	0 391
95	4 659	4 679	4 698	4 718	4 737	4 756	4 775	4 794	4 813	4 832	0 392
96	4 851	4 870	4 888	4 907	4 926	4 945	4 963	4 982	5 000	5 018	0 393
97	10 5 036	5 055	5 073	5 091	5 109	5 127	5 145	5 163	5 181	5 199	0 394
98	5 216	5 234	5 252	5 270	5 287	5 305	5 322	5 339	5 356	5 374	0 395
99	5 391	5 408	5 425	5 442	5 459	5 476	5 493	5 510	5 527	5 544	0 396

100	10 5 500	5 577	5 594	5 611	5 627	5 644	5 660	5 676	5 692	5 709	0 017
101	5 725	5 741	5 757	5 773	5 789	5 805	5 821	5 837	5 853	5 869	0 016
102	5 884	5 901	5 916	5 932	5 947	5 963	5 978	5 994	6 009	6 024	0 015
103	6 035	6 070	6 085	6 085	6 100	6 115	6 130	6 145	6 160	6 175	0 015
104	6 190	6 219	6 234	6 234	6 248	6 263	6 277	6 291	6 305	6 320	0 015
105	6 334	6 361	6 375	6 375	6 389	6 403	6 416	6 429	6 442	6 456	0 014
106	6 489	6 495	6 508	6 508	6 521	6 534	6 546	6 559	6 571	6 584	0 013
107	6 602	6 621	6 633	6 633	6 645	6 657	6 669	6 681	6 692	6 704	0 012
108	6 728	6 736	6 751	6 751	6 762	6 773	6 784	6 795	6 807	6 818	0 011
109	6 831	6 851	6 862	6 862	6 872	6 883	6 893	6 904	6 914	6 925	0 011
110	6 946	6 956	6 965	6 965	6 976	6 986	6 996	7 006	7 016	7 026	0 010
111	7 035	7 035	7 045	7 045	7 054	7 064	7 068	7 103	7 121	7 121	0 010
112	7 130	7 148	7 157	7 157	7 166	7 175	7 184	7 193	7 202	7 211	0 009
113	7 229	7 237	7 246	7 246	7 254	7 263	7 271	7 279	7 287	7 296	0 009
114	7 312	7 320	7 328	7 328	7 336	7 344	7 352	7 360	7 368	7 376	0 008
115	7 399	7 399	7 407	7 407	7 414	7 422	7 429	7 437	7 444	7 452	0 008
116	7 459	7 474	7 481	7 481	7 488	7 495	7 502	7 509	7 516	7 523	0 007
117	7 530	7 544	7 551	7 551	7 558	7 565	7 571	7 578	7 585	7 592	0 007
118	7 598	7 611	7 618	7 618	7 624	7 631	7 637	7 643	7 649	7 656	0 007
119	7 662	7 674	7 681	7 681	7 687	7 693	7 699	7 705	7 711	7 717	0 006
120	7 723	7 735	7 741	7 741	7 747	7 753	7 759	7 765	7 771	7 777	0 006
121	7 783	7 789	7 795	7 795	7 801	7 813	7 818	7 824	7 830	7 836	0 006
122	7 842	7 848	7 853	7 853	7 859	7 871	7 876	7 882	7 887	7 893	0 006
123	7 899	7 905	7 910	7 910	7 921	7 927	7 932	7 938	7 943	7 949	0 006
124	7 954	7 965	7 965	7 965	7 976	7 982	7 987	7 993	7 998	8 004	0 006
125	8 009	8 014	8 019	8 025	8 030	8 036	8 041	8 046	8 051	8 057	0 006
126	8 062	8 072	8 078	8 078	8 083	8 088	8 093	8 098	8 103	8 108	0 005
127	8 113	8 119	8 124	8 129	8 134	8 139	8 144	8 149	8 154	8 159	0 005
128	8 164	8 169	8 174	8 179	8 184	8 189	8 194	8 199	8 204	8 209	0 005
129	8 213	8 218	8 223	8 228	8 233	8 238	8 242	8 247	8 252	8 257	0 005
130	8 262	8 267	8 272	8 276	8 281	8 285	8 290	8 295	8 300	8 304	0 004
131	8 309	8 314	8 319	8 323	8 328	8 332	8 337	8 342	8 346	8 351	0 004
132	8 355	8 360	8 365	8 369	8 374	8 378	8 383	8 387	8 392	8 396	0 004
133	8 400	8 405	8 409	8 414	8 418	8 422	8 427	8 431	8 436	8 440	0 004
134	8 444	8 449	8 453	8 458	8 462	8 466	8 470	8 475	8 479	8 483	0 004
135	8 487	8 492	8 496	8 500	8 504	8 508	8 512	8 517	8 521	8 525	0 004

Table II—continued.
 $T = C [T(V) - T(v)]$

ν	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/λ											+
136	10 8 529	8 523	8 527	8 532	8 536	8 540	8 544	8 548	8 552	8 556	0 004
137	8 570	8 574	8 578	8 583	8 587	8 591	8 595	8 599	8 603	8 607	004
138	8 611	8 615	8 619	8 623	8 627	8 631	8 635	8 639	8 643	8 647	004
139	10 8 650	8 654	8 658	8 662	8 666	8 670	8 674	8 678	8 682	8 685	004
140	8 689	8 693	8 697	8 700	8 704	8 708	8 712	8 715	8 719	8 723	004
141	8 727	8 730	8 734	8 738	8 742	8 745	8 749	8 753	8 757	8 760	004
142	10 8 764	8 768	8 772	8 775	8 779	8 782	8 786	8 790	8 793	8 797	004
143	8 800	8 804	8 807	8 811	8 814	8 818	8 821	8 825	8 828	8 832	004
144	8 835	8 839	8 843	8 846	8 850	8 853	8 857	8 860	8 864	8 867	003
145	10 8 870	8 874	8 877	8 881	8 884	8 887	8 891	8 894	8 897	8 901	003
146	8 904	8 907	8 911	8 914	8 918	8 921	8 925	8 928	8 931	8 935	003
147	8 938	8 941	8 945	8 948	8 951	8 954	8 958	8 961	8 964	8 967	003
148	10 8 971	8 974	8 977	8 980	8 984	8 987	8 990	8 993	8 997	9 000	003
149	9 003	9 006	9 010	9 013	9 016	9 019	9 023	9 026	9 029	9 033	003
150	9 036	9 039	9 042	9 045	9 049	9 052	9 055	9 058	9 061	9 064	003
151	10 9 067	9 071	9 074	9 077	9 080	9 083	9 087	9 090	9 093	9 096	003
152	9 099	9 102	9 105	9 109	9 112	9 115	9 118	9 121	9 124	9 127	003
153	9 130	9 133	9 136	9 139	9 142	9 145	9 148	9 152	9 155	9 158	003
154	10 9 161	9 164	9 167	9 170	9 173	9 176	9 179	9 182	9 185	9 188	003
155	9 191	9 194	9 197	9 200	9 203	9 206	9 209	9 212	9 215	9 218	003
156	9 221	9 224	9 227	9 230	9 233	9 236	9 239	9 242	9 245	9 248	003
157	10 9 251	9 254	9 257	9 260	9 263	9 266	9 269	9 272	9 275	9 278	003
158	9 280	9 283	9 286	9 289	9 292	9 295	9 298	9 301	9 304	9 307	003
159	9 309	9 312	9 315	9 318	9 321	9 324	9 327	9 330	9 333	9 335	003
160	10 9 338	9 341	9 344	9 347	9 350	9 353	9 356	9 359	9 362	9 364	003
161	9 367	9 370	9 373	9 376	9 379	9 381	9 384	9 387	9 390	9 392	003
162	9 395	9 398	9 401	9 403	9 406	9 409	9 412	9 414	9 417	9 420	003
163	10 9 423	9 426	9 428	9 431	9 434	9 437	9 439	9 442	9 445	9 448	003
164	9 450	9 453	9 456	9 459	9 461	9 464	9 467	9 469	9 472	9 475	003
165	9 477	9 480	9 483	9 486	9 488	9 491	9 494	9 496	9 499	9 502	003

166	10	9-504	9-507	9-510	9-513	9-515	9-518	9-521	9-523	9-526	9-529	0-003
167		9-551	9-554	9-557	9-559	9-562	9-565	9-567	9-570	9-573	9-575	-003
168		9-557	9-560	9-562	9-565	9-567	9-570	9-573	9-575	9-578	9-581	-008
169	10	9-583	9-586	9-589	9-591	9-594	9-597	9-599	9-602	9-604	9-606	-003
170		9-600	9-611	9-614	9-616	9-619	9-621	9-624	9-626	9-629	9-631	-002
171		9-634	9-636	9-639	9-641	9-644	9-646	9-649	9-651	9-654	9-656	-002
172	10	9-659	9-661	9-664	9-666	9-669	9-671	9-674	9-676	9-679	9-681	-002
173		9-684	9-686	9-689	9-691	9-694	9-696	9-699	9-701	9-704	9-706	-002
174		9-709	9-711	9-714	9-716	9-719	9-721	9-724	9-726	9-729	9-731	-002
175	10	9-733	9-736	9-738	9-741	9-743	9-746	9-748	9-751	9-753	9-756	-002
176		9-738	9-761	9-763	9-766	9-768	9-770	9-773	9-775	9-777	9-780	-002
177		9-782	9-784	9-787	9-789	9-791	9-794	9-796	9-798	9-801	9-803	-002
178	10	9-805	9-808	9-810	9-812	9-815	9-817	9-819	9-822	9-824	9-826	-002
179		9-829	9-831	9-833	9-836	9-838	9-840	9-843	9-845	9-847	9-850	-002
180		9-852	9-854	9-857	9-859	9-861	9-864	9-866	9-868	9-871	9-873	-002
181	10	9-875	9-878	9-880	9-882	9-885	9-887	9-889	9-891	9-894	9-896	-002
182		9-898	9-900	9-903	9-905	9-907	9-909	9-912	9-914	9-916	9-918	-002
183		9-920	9-923	9-925	9-927	9-929	9-931	9-934	9-936	9-938	9-940	-002
184	10	9-942	9-945	9-947	9-949	9-951	9-954	9-956	9-958	9-960	9-963	-002
185		9-965	9-967	9-969	9-972	9-974	9-976	9-978	9-980	9-983	9-985	-002
186		9-987	9-989	9-991	9-993	9-996	9-998	9-000	9-002	9-004	9-006	-002
187	11	0-008	0-011	0-013	0-015	0-017	0-019	0-021	0-023	0-025	0-028	-002
188		0-020	0-032	0-034	0-036	0-038	0-040	0-042	0-045	0-047	0-049	-002
189		0-051	0-053	0-055	0-057	0-059	0-062	0-064	0-066	0-068	0-070	-002
190	11	0-072	0-074	0-076	0-078	0-080	0-083	0-085	0-087	0-089	0-091	-002
191		0-083	0-095	0-097	0-099	0-101	0-103	0-105	0-107	0-109	0-111	-002
192		0-113	0-115	0-117	0-119	0-121	0-124	0-126	0-128	0-130	0-132	-002
193	11	0-134	0-136	0-138	0-140	0-142	0-144	0-146	0-148	0-150	0-152	-002
194		0-154	0-156	0-158	0-160	0-162	0-164	0-166	0-168	0-170	0-172	-002
195		0-174	0-176	0-178	0-180	0-182	0-184	0-186	0-188	0-190	0-192	-002
196	11	0-194	0-196	0-198	0-200	0-202	0-204	0-206	0-208	0-210	0-212	-002
197		0-214	0-216	0-218	0-220	0-222	0-224	0-226	0-228	0-230	0-232	-002
198		0-234	0-236	0-238	0-240	0-242	0-244	0-246	0-248	0-250	0-252	-002
199	11	0-253	0-255	0-257	0-259	0-261	0-263	0-265	0-267	0-269	0-271	-002
200		0-272	0-274	0-276	0-278	0-280	0-283	0-285	0-286	0-288	0-290	-002
201		0-291	0-293	0-295	0-297	0-299	0-301	0-303	0-305	0-307	0-308	-002

Table II—continued.
 $t = G[T(V) - T(t)]$

θ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											+
202	11 0.310	0.312	0.314	0.316	0.318	0.320	0.322	0.323	0.325	0.327	0.002
203	0.329	0.331	0.333	0.335	0.336	0.338	0.340	0.342	0.344	0.346	0.002
204	0.347	0.349	0.351	0.353	0.355	0.357	0.359	0.361	0.362	0.364	0.002
205	11 0.366	0.368	0.370	0.372	0.374	0.375	0.377	0.379	0.381	0.383	0.002
206	0.384	0.386	0.388	0.390	0.392	0.393	0.395	0.397	0.399	0.401	0.002
207	0.402	0.404	0.406	0.408	0.410	0.412	0.414	0.415	0.417	0.419	0.002
208	11 0.421	0.423	0.425	0.427	0.429	0.430	0.432	0.434	0.436	0.438	0.002
209	0.429	0.441	0.443	0.444	0.446	0.448	0.449	0.451	0.453	0.455	0.002
210	0.456	0.458	0.460	0.462	0.464	0.465	0.467	0.469	0.471	0.473	0.002
211	11 0.474	0.476	0.478	0.480	0.482	0.483	0.485	0.487	0.489	0.491	0.002
212	0.492	0.494	0.496	0.498	0.499	0.501	0.503	0.504	0.506	0.508	0.002
213	0.509	0.511	0.513	0.515	0.516	0.518	0.520	0.521	0.523	0.525	0.002
214	11 0.526	0.528	0.530	0.532	0.534	0.535	0.537	0.539	0.541	0.542	0.002
215	0.544	0.546	0.548	0.549	0.551	0.553	0.554	0.556	0.558	0.559	0.002
216	0.561	0.563	0.564	0.566	0.568	0.569	0.571	0.573	0.574	0.576	0.002
217	11 0.578	0.580	0.581	0.583	0.585	0.587	0.588	0.590	0.592	0.594	0.002
218	0.595	0.597	0.599	0.601	0.602	0.604	0.605	0.607	0.608	0.610	0.002
219	0.611	0.613	0.614	0.616	0.618	0.620	0.621	0.623	0.625	0.627	0.002
220	11 0.628	0.630	0.632	0.634	0.635	0.637	0.638	0.640	0.641	0.643	0.002
221	0.644	0.646	0.648	0.650	0.651	0.653	0.654	0.656	0.658	0.660	0.002
222	0.661	0.663	0.665	0.667	0.668	0.670	0.671	0.673	0.674	0.676	0.002
223	11 0.677	0.679	0.681	0.683	0.684	0.686	0.687	0.689	0.690	0.692	0.002
224	0.693	0.695	0.697	0.699	0.700	0.702	0.703	0.705	0.706	0.708	0.002
225	0.709	0.711	0.713	0.715	0.716	0.718	0.719	0.722	0.724	0.725	0.002
226	11 0.725	0.727	0.728	0.730	0.731	0.733	0.735	0.736	0.738	0.740	0.002
227	0.741	0.743	0.745	0.746	0.748	0.749	0.751	0.752	0.754	0.755	0.002
228	0.757	0.758	0.760	0.762	0.763	0.765	0.767	0.768	0.770	0.772	0.002
229	11 0.773	0.775	0.776	0.778	0.779	0.781	0.782	0.784	0.785	0.787	0.002
230	0.788	0.790	0.792	0.794	0.795	0.797	0.798	0.800	0.801	0.803	0.002
231	0.804	0.806	0.807	0.809	0.810	0.812	0.813	0.815	0.816	0.818	0.002

232	11	0.819	0.821	0.822	0.824	0.825	0.827	0.828	0.830	0.831	0.833	0.002
233		0.819	0.836	0.837	0.839	0.840	0.842	0.843	0.845	0.846	0.848	0.002
234		0.849	0.851	0.852	0.854	0.855	0.857	0.858	0.860	0.861	0.863	0.002
235	11	0.864	0.865	0.867	0.869	0.870	0.872	0.873	0.875	0.876	0.878	0.002
236		0.879	0.881	0.882	0.884	0.885	0.887	0.888	0.890	0.891	0.893	0.002
237		0.894	0.896	0.897	0.899	0.900	0.902	0.903	0.905	0.906	0.908	0.002
238	11	0.909	0.911	0.912	0.914	0.915	0.917	0.918	0.920	0.921	0.923	0.002
239		0.924	0.926	0.927	0.929	0.930	0.932	0.933	0.934	0.936	0.937	0.002
240		0.938	0.940	0.941	0.943	0.944	0.946	0.947	0.949	0.950	0.952	0.002
241	11	0.953	0.955	0.956	0.958	0.959	0.961	0.962	0.963	0.965	0.966	0.001
242		0.967	0.969	0.970	0.972	0.973	0.974	0.976	0.977	0.978	0.980	0.001
243		0.981	0.982	0.984	0.985	0.987	0.988	0.990	0.991	0.993	0.994	0.001
244	11	0.996	0.997	0.999	1.000	1.002	1.003	1.005	1.006	1.007	1.009	0.001
245		1.010	1.011	1.013	1.014	1.016	1.017	1.019	1.020	1.022	1.023	0.001
246		1.024	1.026	1.027	1.029	1.030	1.032	1.033	1.034	1.036	1.037	0.001
247	11	1.038	1.040	1.041	1.042	1.044	1.045	1.046	1.048	1.049	1.050	0.001
248		1.052	1.053	1.054	1.056	1.057	1.058	1.060	1.061	1.062	1.064	0.001
249		1.065	1.066	1.068	1.069	1.071	1.072	1.074	1.075	1.077	1.078	0.001
250	11	1.079	1.081	1.082	1.084	1.085	1.086	1.088	1.089	1.090	1.092	0.001
251		1.093	1.094	1.096	1.097	1.098	1.100	1.101	1.102	1.104	1.105	0.001
252		1.106	1.108	1.109	1.111	1.112	1.113	1.115	1.116	1.117	1.119	0.001
253	11	1.120	1.121	1.123	1.124	1.125	1.127	1.128	1.129	1.131	1.132	0.001
254		1.133	1.135	1.136	1.137	1.139	1.140	1.141	1.143	1.144	1.145	0.001
255		1.147	1.148	1.149	1.151	1.152	1.153	1.155	1.156	1.157	1.159	0.001
256	11	1.160	1.161	1.162	1.164	1.165	1.166	1.168	1.169	1.170	1.172	0.001
257		1.173	1.174	1.175	1.177	1.178	1.179	1.181	1.182	1.183	1.185	0.001
258		1.186	1.187	1.188	1.190	1.191	1.192	1.194	1.195	1.196	1.198	0.001
259	11	1.199	1.200	1.201	1.203	1.204	1.205	1.207	1.208	1.209	1.211	0.001
260		1.212	1.215	1.216	1.217	1.219	1.220	1.221	1.222	1.223	1.224	0.001
261		1.225	1.227	1.228	1.229	1.230	1.232	1.233	1.234	1.235	1.237	0.001
262	11	1.238	1.239	1.240	1.242	1.243	1.244	1.245	1.247	1.248	1.249	0.001
263		1.250	1.252	1.253	1.254	1.256	1.257	1.258	1.259	1.261	1.262	0.001
264		1.263	1.264	1.266	1.267	1.268	1.269	1.271	1.272	1.273	1.274	0.001
265	11	1.276	1.277	1.278	1.279	1.281	1.282	1.283	1.284	1.286	1.287	0.001
266		1.289	1.290	1.291	1.292	1.293	1.294	1.295	1.297	1.298	1.299	0.001
267		1.300	1.301	1.303	1.304	1.305	1.306	1.308	1.309	1.310	1.311	0.001

Table II—continued.
 $t = C[T(V) - T(\rho)]$

f_s	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
268	1 313	1 314	1 315	1 316	1 317	1 319	1 320	1 321	1 322	1 323	+
269	1 325	1 326	1 327	1 328	1 329	1 331	1 332	1 333	1 334	1 335	0 001
270	1 337	1 338	1 339	1 340	1 341	1 343	1 344	1 345	1 346	1 347	0 001
271	1 349	1 350	1 351	1 352	1 353	1 355	1 356	1 357	1 358	1 359	0 001
272	1 361	1 362	1 363	1 364	1 365	1 367	1 368	1 369	1 370	1 371	0 001
273	1 373	1 374	1 375	1 376	1 377	1 379	1 380	1 381	1 382	1 383	0 001
274	1 385	1 386	1 387	1 388	1 389	1 391	1 392	1 393	1 394	1 395	0 001
275	1 397	1 398	1 399	1 400	1 401	1 402	1 404	1 405	1 406	1 407	0 001
276	1 408	1 410	1 411	1 412	1 413	1 414	1 415	1 417	1 418	1 419	0 001
277	1 420	1 421	1 422	1 424	1 425	1 426	1 427	1 428	1 429	1 430	0 001
278	1 432	1 433	1 434	1 435	1 436	1 437	1 438	1 439	1 441	1 442	0 001
279	1 443	1 444	1 445	1 446	1 447	1 448	1 449	1 451	1 452	1 453	0 001
280	1 454	1 455	1 456	1 458	1 459	1 460	1 461	1 462	1 463	1 464	0 001
281	1 466	1 467	1 468	1 469	1 470	1 471	1 472	1 474	1 475	1 476	0 001
282	1 477	1 478	1 479	1 480	1 482	1 483	1 484	1 485	1 486	1 487	0 001
283	1 488	1 490	1 491	1 492	1 493	1 494	1 495	1 496	1 498	1 499	0 001
284	1 500	1 501	1 502	1 503	1 504	1 505	1 507	1 508	1 509	1 510	0 001
285	1 511	1 512	1 513	1 514	1 515	1 516	1 518	1 519	1 520	1 521	0 001
286	1 522	1 523	1 524	1 525	1 526	1 527	1 529	1 530	1 531	1 532	0 001
287	1 533	1 534	1 535	1 536	1 537	1 538	1 540	1 541	1 542	1 543	0 001
288	1 544	1 545	1 546	1 547	1 548	1 549	1 550	1 551	1 552	1 553	0 001
289	1 554	1 556	1 557	1 558	1 559	1 560	1 561	1 562	1 563	1 564	0 001
290	1 565	1 566	1 567	1 568	1 569	1 570	1 571	1 572	1 574	1 575	0 001
291	1 576	1 577	1 578	1 579	1 580	1 581	1 582	1 583	1 584	1 585	0 001
292	1 586	1 588	1 589	1 590	1 591	1 592	1 593	1 594	1 595	1 596	0 001
293	1 597	1 598	1 599	1 600	1 601	1 602	1 604	1 605	1 606	1 607	0 001
294	1 608	1 609	1 610	1 611	1 612	1 613	1 614	1 615	1 616	1 617	0 001
295	1 618	1 620	1 621	1 622	1 623	1 624	1 625	1 626	1 627	1 628	0 001
296	1 629	1 630	1 631	1 632	1 633	1 634	1 635	1 636	1 637	1 638	0 001
297	1 639	1 640	1 641	1 642	1 643	1 644	1 645	1 646	1 647	1 648	0 001

298	11 1 649	1 650	1 651	1 652	1 653	1 654	1 655	1 656	1 657	1 658	001
299	1 659	1 661	1 662	1 663	1 664	1 665	1 666	1 667	1 668	1 669	001
300	1 670	1 671	1 672	1 673	1 674	1 675	1 676	1 677	1 678	1 679	001
301	1 680	1 681	1 682	1 683	1 684	1 685	1 686	1 687	1 688	1 689	001
302	1 690	1 691	1 692	1 693	1 694	1 695	1 696	1 697	1 698	1 699	001
303	1 700	1 701	1 702	1 703	1 704	1 705	1 706	1 707	1 708	1 709	001
304	1 710	1 711	1 712	1 713	1 714	1 715	1 716	1 717	1 718	1 719	001
305	1 720	1 721	1 722	1 723	1 724	1 725	1 726	1 727	1 728	1 729	001
306	1 730	1 731	1 732	1 733	1 734	1 735	1 736	1 737	1 738	1 739	001
307	1 739	1 740	1 741	1 742	1 743	1 744	1 745	1 746	1 747	1 748	001
308	1 749	1 750	1 751	1 752	1 753	1 754	1 755	1 756	1 757	1 758	001
309	1 759	1 760	1 761	1 762	1 763	1 764	1 765	1 766	1 767	1 768	001
310	1 769	1 770	1 771	1 772	1 773	1 774	1 775	1 776	1 777	1 778	001
311	1 779	1 780	1 781	1 782	1 783	1 784	1 785	1 786	1 787	1 788	001
312	1 789	1 790	1 791	1 792	1 793	1 794	1 795	1 796	1 797	1 798	001
313	1 799	1 800	1 801	1 802	1 803	1 804	1 805	1 806	1 807	1 808	001
314	1 809	1 810	1 811	1 812	1 813	1 814	1 815	1 816	1 817	1 818	001
315	1 819	1 820	1 821	1 822	1 823	1 824	1 825	1 826	1 827	1 828	001
316	1 829	1 830	1 831	1 832	1 833	1 834	1 835	1 836	1 837	1 838	001
317	1 839	1 840	1 841	1 842	1 843	1 844	1 845	1 846	1 847	1 848	001
318	1 849	1 850	1 851	1 852	1 853	1 854	1 855	1 856	1 857	1 858	001
319	1 859	1 860	1 861	1 862	1 863	1 864	1 865	1 866	1 867	1 868	001
320	1 869	1 870	1 871	1 872	1 873	1 874	1 875	1 876	1 877	1 878	001
321	1 879	1 880	1 881	1 882	1 883	1 884	1 885	1 886	1 887	1 888	001
322	1 889	1 890	1 891	1 892	1 893	1 894	1 895	1 896	1 897	1 898	001
323	1 899	1 900	1 901	1 902	1 903	1 904	1 905	1 906	1 907	1 908	001
324	1 909	1 910	1 911	1 912	1 913	1 914	1 915	1 916	1 917	1 918	001
325	1 919	1 920	1 921	1 922	1 923	1 924	1 925	1 926	1 927	1 928	001
326	1 929	1 930	1 931	1 932	1 933	1 934	1 935	1 936	1 937	1 938	001
327	1 939	1 940	1 941	1 942	1 943	1 944	1 945	1 946	1 947	1 948	001
328	1 949	1 950	1 951	1 952	1 953	1 954	1 955	1 956	1 957	1 958	001
329	1 959	1 960	1 961	1 962	1 963	1 964	1 965	1 966	1 967	1 968	001
330	1 969	1 970	1 971	1 972	1 973	1 974	1 975	1 976	1 977	1 978	001
331	1 979	1 980	1 981	1 982	1 983	1 984	1 985	1 986	1 987	1 988	001
332	1 989	1 990	1 991	1 992	1 993	1 994	1 995	1 996	1 997	1 998	001
333	1 999	2000	2001	2002	2003	2004	2005	2006	2007	2008	001

Table II—continued.
 $t = C[T(V) - T(v)]$

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											
334	11 1-085	1-086	1-087	1-088	1-089	1-090	1-091	1-092	1-092	1-093	+
335	1-085	1-086	1-087	1-087	1-088	1-089	1-089	2-000	2-001	2-002	0-001
336	2-002	2-003	2-004	2-005	2-006	2-006	2-007	2-008	2-009	2-010	-001
337	11 2-010	2-011	2-012	2-013	2-014	2-015	2-016	2-017	2-017	2-018	-001
338	2-019	2-020	2-021	2-022	2-023	2-023	2-024	2-025	2-026	2-027	-001
339	2-027	2-028	2-029	2-030	2-031	2-031	2-032	2-033	2-034	2-035	-001
340	11 2-035	2-036	2-037	2-038	2-039	2-040	2-040	2-041	2-042	2-043	-001
341	2-044	2-045	2-046	2-046	2-047	2-048	2-048	2-049	2-050	2-051	-001
342	2-052	2-052	2-053	2-054	2-055	2-056	2-056	2-057	2-058	2-059	-001
343	11 2-060	2-060	2-061	2-062	2-063	2-064	2-064	2-065	2-066	2-067	-001
344	2-068	2-068	2-069	2-070	2-071	2-072	2-072	2-073	2-074	2-075	-001
345	2-076	2-077	2-077	2-078	2-079	2-080	2-080	2-081	2-082	2-083	-001
346	11 2-084	2-084	2-085	2-086	2-087	2-088	2-088	2-089	2-090	2-091	-001
347	2-092	2-092	2-093	2-094	2-095	2-096	2-096	2-097	2-098	2-099	-001
348	2-100	2-100	2-101	2-102	2-103	2-104	2-104	2-105	2-106	2-107	-001
349	11 2-108	2-108	2-109	2-110	2-111	2-112	2-112	2-113	2-114	2-115	-001
350	2-116	2-116	2-117	2-118	2-119	2-120	2-120	2-121	2-122	2-123	-001
351	2-124	2-124	2-125	2-126	2-127	2-128	2-128	2-129	2-130	2-131	-001
352	11 2-131	2-132	2-133	2-134	2-135	2-135	2-136	2-137	2-138	2-139	-001
353	2-139	2-140	2-141	2-142	2-143	2-143	2-144	2-145	2-146	2-147	-001
354	2-147	2-148	2-149	2-150	2-150	2-151	2-152	2-153	2-154	2-155	-001
355	11 2-155	2-156	2-156	2-157	2-158	2-159	2-159	2-160	2-161	2-162	-001
356	2-162	2-163	2-164	2-165	2-165	2-166	2-167	2-168	2-169	2-170	-001
357	2-170	2-171	2-172	2-172	2-173	2-174	2-174	2-175	2-176	2-177	-001
358	11 2-177	2-178	2-179	2-180	2-180	2-181	2-182	2-183	2-184	2-185	-001
359	2-186	2-186	2-187	2-187	2-188	2-189	2-189	2-190	2-191	2-192	-001
360	2-192	2-193	2-194	2-195	2-196	2-196	2-197	2-198	2-199	2-200	-001

361	11	2 200	2 201	2 202	2 203	2 204	2 205	2 205	2 206	2 207	001
362		2 207	2 208	2 209	2 210	2 211	2 212	2 213	2 214	2 215	001
363		2 215	2 216	2 217	2 218	2 219	2 220	2 221	2 222	2 223	001
364	11	2 222	2 223	2 224	2 225	2 226	2 227	2 228	2 229	2 230	001
365		2 229	2 230	2 231	2 232	2 233	2 234	2 235	2 236	2 237	001
366		2 237	2 238	2 239	2 240	2 241	2 242	2 243	2 244	2 245	001
367	11	2 244	2 245	2 246	2 247	2 248	2 249	2 250	2 251	2 252	001
368		2 251	2 252	2 253	2 254	2 255	2 256	2 257	2 258	2 259	001
369		2 258	2 259	2 260	2 261	2 262	2 263	2 264	2 265	2 266	001
370	11	2 266	2 267	2 268	2 269	2 270	2 271	2 272	2 273	2 274	001
371		2 273	2 274	2 275	2 276	2 277	2 278	2 279	2 280	2 281	001
372		2 280	2 281	2 282	2 283	2 284	2 285	2 286	2 287	2 288	001
373	11	2 287	2 288	2 289	2 290	2 291	2 292	2 293	2 294	2 295	001
374		2 294	2 295	2 296	2 297	2 298	2 299	2 300	2 301	2 302	001
375		2 301	2 302	2 303	2 304	2 305	2 306	2 307	2 308	2 309	001
376	11	2 308	2 309	2 310	2 311	2 312	2 313	2 314	2 315	2 316	001
377		2 315	2 316	2 317	2 318	2 319	2 320	2 321	2 322	2 323	001
378		2 322	2 323	2 324	2 325	2 326	2 327	2 328	2 329	2 330	001
379	11	2 329	2 330	2 331	2 332	2 333	2 334	2 335	2 336	2 337	001
380		2 336	2 337	2 338	2 339	2 340	2 341	2 342	2 343	2 344	001
381		2 342	2 343	2 344	2 345	2 346	2 347	2 348	2 349	2 350	001
382	11	2 349	2 350	2 351	2 352	2 353	2 354	2 355	2 356	2 357	001
383		2 356	2 357	2 358	2 359	2 360	2 361	2 362	2 363	2 364	001
384		2 363	2 364	2 365	2 366	2 367	2 368	2 369	2 370	2 371	001
385	11	2 369	2 370	2 371	2 372	2 373	2 374	2 375	2 376	2 377	001
386		2 376	2 377	2 378	2 379	2 380	2 381	2 382	2 383	2 384	001
387		2 383	2 384	2 385	2 386	2 387	2 388	2 389	2 390	2 391	001
388	11	2 389	2 390	2 391	2 392	2 393	2 394	2 395	2 396	2 397	001
389		2 396	2 397	2 398	2 399	2 400	2 401	2 402	2 403	2 404	001
390		2 402	2 403	2 404	2 405	2 406	2 407	2 408	2 409	2 410	001
391	11	2 409	2 410	2 411	2 412	2 413	2 414	2 415	2 416	2 417	001
392		2 416	2 417	2 418	2 419	2 420	2 421	2 422	2 423	2 424	001
393		2 423	2 424	2 425	2 426	2 427	2 428	2 429	2 430	2 431	001
394	11	2 428	2 429	2 430	2 431	2 432	2 433	2 434	2 435	2 436	001
395		2 435	2 436	2 437	2 438	2 439	2 440	2 441	2 442	2 443	001
396		2 441	2 442	2 443	2 444	2 445	2 446	2 447	2 448	2 449	001

TABLE III.

Distance s in feet, between velocity V and v f/s. $s = C[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s.											+
10	3571.5	3656.1	3740.1	3823.6	3906.7	3989.2	4071.3	4153.0	4234.2	4314.9	82.6
11	4495.2	4475.0	4454.4	4433.4	4412.0	4390.2	4367.9	4345.3	4322.3	4308.8	78.2
12	5175.0	5250.8	5326.3	5401.3	5475.0	5550.4	5624.9	5698.5	5771.2	5844.2	74.4
13	5916.8	5989.0	6061.0	6132.6	6203.9	6274.8	6345.5	6415.8	6485.9	6555.6	71.0
14	6625.0	6694.2	6763.0	6831.6	6899.8	6967.8	7035.5	7102.9	7170.1	7237.0	68.0
15	7369.9	7369.9	7436.0	7501.8	7567.4	7632.7	7697.7	7762.5	7827.1	7891.4	65.3
16	7955.4	8019.3	8082.9	8146.2	8209.3	8272.2	8334.9	8397.3	8459.6	8521.6	62.9
17	8583.3	8644.9	8706.2	8767.3	8828.3	8889.0	8949.5	9009.7	9069.8	9129.7	60.7
18	9189.4	9248.9	9308.2	9367.3	9426.2	9484.9	9543.4	9601.7	9659.8	9717.8	58.7
19	9775.6	9833.2	9890.6	9947.8	*0004.9	*0061.8	*0118.5	*0175.0	*0231.4	*0287.6	56.9
20	1 0843.6	0899.4	0955.1	0910.6	0566.0	0621.2	0676.2	0731.1	0785.8	0840.3	55.2
21	0894.7	0949.0	1003.1	1057.0	1110.8	1164.4	1217.9	1271.2	1324.4	1377.5	53.6
22	1 1430.4	1483.1	1535.7	1588.2	1640.5	1692.7	1744.8	1796.7	1848.5	1900.1	52.2
23	1561.6	2003.0	2054.2	2105.3	2156.3	2207.1	2258.9	2309.4	2358.9	2409.2	50.8
24	2459.4	2509.5	2559.4	2609.3	2659.0	2708.5	2758.0	2807.3	2856.6	2905.7	49.5
25	1 2954.6	3003.5	3052.3	3100.9	3149.4	3197.8	3246.1	3294.3	3342.4	3390.3	48.4
26	3438.2	3485.9	3533.5	3581.0	3628.4	3675.7	3722.9	3770.0	3817.0	3863.9	47.3
27	3910.6	3957.3	4003.9	4050.3	4096.7	4142.9	4189.1	4235.1	4281.1	4327.0	46.2
28	1 4372.7	4418.4	4463.9	4509.4	4554.8	4600.0	4645.2	4690.3	4735.3	4780.2	45.2
29	4825.0	4869.7	4914.3	4958.9	5003.3	5047.6	5091.8	5136.0	5180.1	5224.1	44.3
30	5268.0	5311.8	5355.5	5399.1	5442.7	5486.2	5529.6	5572.9	5616.1	5659.2	43.5
31	1 5702.2	5745.2	5788.1	5830.9	5873.6	5916.2	5958.8	6001.3	6043.7	6086.0	42.6
32	6128.2	6170.3	6212.4	6254.4	6296.3	6338.1	6379.9	6421.6	6463.2	6504.7	41.8
33	6546.2	6587.6	6628.9	6670.1	6711.3	6752.4	6793.4	6834.3	6875.2	6916.0	41.1
34	1 6956.7	6997.3	7037.9	7078.4	7118.8	7159.2	7199.5	7239.7	7279.9	7320.0	40.4
35	7360.0	7399.9	7439.8	7479.6	7519.3	7559.0	7598.6	7638.2	7677.7	7717.1	39.7
36	7756.4	7795.7	7834.9	7874.0	7913.1	7952.1	7991.1	8030.0	8068.8	8107.6	39.0

Table III—continued.
 $s = C[S(V) - S(v)]$

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
$f_{1/2}$											+
37	8146.3	8184.9	8223.5	8262.0	8300.5	8338.9	8377.3	8415.5	8453.7	8491.8	38.4
38	8230.0	8267.9	8305.9	8343.8	8381.6	8419.4	8457.1	8494.8	8532.4	8570.0	37.8
39	8307.5	8344.9	8382.3	8419.6	8456.9	8494.1	8531.3	8568.4	8605.4	8642.4	37.2
40	8379.3	8416.2	8453.0	8489.8	8526.5	8563.2	8600.0	8636.7	8673.4	8710.0	36.7
41	8445.5	8482.0	8518.3	8554.5	8590.7	8626.8	8662.9	8698.9	8734.9	8770.9	36.1
42	8506.6	8542.4	8578.2	8613.9	8649.6	8685.2	8720.9	8756.5	8792.1	8827.7	35.6
43	8562.5	8597.7	8632.9	8668.0	8703.1	8738.2	8773.3	8808.4	8843.5	8878.6	35.1
44	8613.4	8648.1	8682.8	8717.5	8752.2	8786.9	8821.6	8856.3	8891.0	8925.7	34.7
45	8659.6	8693.7	8727.8	8761.9	8796.0	8830.1	8864.2	8898.3	8932.4	8966.5	34.2
46	8706.2	8739.7	8773.2	8806.7	8840.2	8873.7	8907.2	8940.7	8974.2	9007.7	33.8
47	8753.3	8786.4	8819.5	8852.6	8885.7	8918.8	8951.9	8985.0	9018.1	9051.2	33.3
48	8800.4	8833.1	8865.8	8898.5	8931.2	8963.9	8996.6	9029.3	9062.0	9094.7	32.9
49	8847.5	8879.8	8912.1	8944.4	8976.7	9009.0	9041.3	9073.6	9105.9	9138.2	32.5
50	8894.6	8926.6	8958.6	8990.6	9022.6	9054.6	9086.6	9118.6	9150.6	9182.6	32.1
51	8941.7	8973.4	9005.1	9036.8	9068.5	9100.2	9131.9	9163.6	9195.3	9227.0	31.8
52	8988.7	9020.1	9051.5	9082.9	9114.3	9145.7	9177.1	9208.5	9239.9	9271.3	31.4
53	9035.7	9066.8	9097.8	9128.8	9159.8	9190.8	9221.8	9252.8	9283.8	9314.8	31.0
54	9082.7	9113.4	9144.1	9174.8	9205.5	9236.2	9266.9	9297.6	9328.3	9359.0	30.7
55	9129.6	9159.9	9190.2	9220.5	9250.8	9281.1	9311.4	9341.7	9372.0	9402.3	30.3
56	9176.5	9206.4	9236.3	9266.2	9296.1	9326.0	9355.9	9385.8	9415.7	9445.6	30.0
57	9223.4	9252.9	9282.4	9311.9	9341.4	9370.9	9400.4	9429.9	9459.4	9488.9	29.7
58	9270.3	9299.4	9328.5	9357.6	9386.7	9415.8	9444.9	9474.0	9503.1	9532.2	29.4
59	9317.2	9345.9	9374.6	9403.3	9432.0	9460.7	9489.4	9518.1	9546.8	9575.5	29.1
60	9364.1	9392.4	9420.7	9449.0	9477.3	9505.6	9533.9	9562.2	9590.5	9618.8	28.8
61	9411.0	9438.9	9466.8	9494.7	9522.6	9550.5	9578.4	9606.3	9634.2	9662.1	28.5
62	9457.9	9485.4	9512.9	9540.4	9567.9	9595.4	9622.9	9650.4	9677.9	9705.4	28.3
63	9504.8	9531.9	9559.0	9586.1	9613.2	9640.3	9667.4	9694.5	9721.6	9748.7	28.0
64	9551.7	9578.4	9605.1	9631.8	9658.5	9685.2	9711.9	9738.6	9765.3	9792.0	27.7
65	9598.6	9624.9	9651.2	9677.5	9703.8	9730.1	9756.4	9782.7	9809.0	9835.3	27.5
66	9645.5	9671.4	9697.3	9723.2	9749.1	9775.0	9800.9	9826.8	9852.7	9878.6	27.2

67	2 7735 8	7762 9	7789 9	7816 9	7843 9	7870 9	7897 8	7924 7	7951 6	7978 5	27 0
68	8032 2	8032 2	8059 0	8085 8	8112 6	8139 3	8166 0	8192 4	8219 4	8246 6	26 7
69	8272 6	8299 2	8325 3	8352 3	8378 6	8405 3	8431 8	8457 4	8484 7	8511 1	26 5
70	2 8337 5	8363 9	8390 2	8416 6	8442 9	8469 2	8495 4	8521 6	8547 8	8574 7	26 3
71	9000 6	9026 4	9052 5	9078 6	9104 2	9130 1	9155 9	9181 7	9207 4	9233 2	26 1
72		9086 6	9112 5	9138 4	9164 2	9190 1	9215 9	9241 7	9267 4	9293 2	25 9
73	2 9318 9	9344 6	9370 3	9396 0	9421 6	9447 3	9473 0	9498 5	9524 0	9549 6	25 7
74	9575 1	9600 6	9626 1	9651 6	9677 0	9702 4	9727 8	9753 2	9778 5	9803 9	25 4
75	9529 2	9554 5	9579 8	9605 1	9630 3	9655 5	9680 7	*0003 0	*0031 0	*0056 2	25 2
76	3 0081 3	0106 4	0131 5	0156 5	0181 5	0206 6	0231 6	0256 5	0281 5	0306 4	25 0
77	0331 4	0356 3	0381 1	0406 0	0430 9	0455 7	0480 5	0505 3	0530 1	0554 8	24 8
78	0579 5	0604 2	0628 8	0653 6	0678 2	0702 9	0727 5	0752 1	0776 7	0801 2	24 7
79	3 0825 3	0850 3	0874 8	0899 3	0923 7	0948 2	0972 6	0997 0	1021 4	1045 8	24 5
80	1070 2	1094 5	1118 8	1143 1	1167 4	1191 7	1215 9	1240 1	1264 3	1288 5	24 3
81	1312 7	1336 9	1361 0	1385 1	1409 2	1433 3	1457 4	1481 4	1505 5	1529 5	24 1
82	3 1553 5	1577 5	1601 4	1625 4	1649 3	1673 2	1697 1	1721 0	1744 8	1768 7	23 9
83	1702 5	1816 3	1840 1	1863 9	1887 6	1911 4	1935 1	1958 8	1982 5	2006 2	23 7
84	2029 8	2053 4	2077 0	2100 5	2123 9	2147 3	2170 6	2193 9	2217 1	2240 3	23 4
85	3 2263 4	2286 5	2309 5	2332 5	2355 4	2378 2	2401 0	2425 8	2448 5	2469 1	22 9
86	2491 7	2514 2	2536 7	2559 1	2581 5	2603 8	2626 1	2648 3	2670 5	2692 6	22 3
87	2714 7	2736 7	2758 7	2780 6	2802 4	2824 2	2846 0	2867 7	2889 4	2911 0	21 8
88	3 2932 6	2954 1	2975 6	2997 0	3018 4	3039 7	3061 0	3082 2	3103 4	3124 5	21 3
89	3145 6	3166 6	3187 6	3208 6	3229 5	3250 3	3271 1	3291 9	3312 6	3333 3	20 8
90	3353 9	3374 5	3395 0	3415 5	3435 9	3456 3	3476 6	3496 9	3517 2	3537 4	20 4
91	3 3557 6	3577 6	3597 8	3617 8	3637 8	3657 8	3677 7	3697 5	3717 3	3737 1	20 0
92	3756 9	3776 6	3796 2	3815 8	3835 5	3854 3	3874 4	3893 8	3913 2	3932 6	19 5
93	3951 9	3971 1	3990 3	4009 5	4028 7	4047 8	4066 9	4085 9	4104 9	4123 8	19 1
94	3 4142 7	4161 6	4180 4	4199 2	4217 9	4236 6	4255 3	4273 9	4292 5	4311 0	18 7
95	4329 5	4348 0	4366 4	4384 8	4403 2	4421 5	4439 8	4457 8	4476 2	4494 3	18 3
96	4512 4	4530 5	4548 6	4566 6	4584 6	4602 5	4620 4	4638 3	4656 1	4673 9	17 9
97	3 4691 6	4709 3	4727 0	4744 7	4762 3	4779 8	4797 4	4814 9	4832 3	4849 7	17 5
98	4867 1	4884 5	4901 8	4919 1	4936 3	4953 5	4970 7	4987 5	5005 0	5022 1	17 2
99	5039 1	5056 1	5073 1	5090 0	5106 9	5123 8	5140 6	5157 4	5174 2	5190 9	16 9
100	3 5297 8	5314 9	5331 4	5347 7	5364 0	5380 3	5397 1	5413 6	5430 4	5447 4	16 5
101	5372 8	5389 1	5405 4	5421 7	5438 0	5454 2	5470 4	5486 5	5502 6	5518 7	16 2
102	5534 8	5550 8	5566 8	5582 7	5598 6	5614 5	5630 4	5646 2	5662 0	5677 8	15 9

Table III—continued.
 $s = C[S(V) - S(\nu)]$

ν	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f_s											+
103	3 5939 6	5700 3	5724 9	5740 6	5756 2	5771 8	5787 4	5802 9	5818 4	5833 9	15 6
104	5849 3	5804 7	5880 0	5895 2	5910 4	5925 4	5940 4	5955 3	5970 1	5984 9	15 0
105	5999 6	6014 2	6028 7	6043 1	6057 5	6071 8	6086 0	6100 2	6114 3	6128 3	14 3
106	3 6142 2	6156 1	6169 9	6183 6	6197 3	6210 9	6224 4	6237 9	6251 3	6264 6	13 6
107	6277 8	6291 0	6304 1	6317 1	6330 1	6343 0	6355 9	6368 7	6381 4	6394 0	12 4
108	6406 6	6419 1	6431 6	6444 0	6456 3	6468 6	6480 8	6493 0	6505 1	6517 1	12 3
109	3 6529 1	6541 0	6552 8	6564 6	6576 3	6588 0	6599 6	6611 2	6622 7	6634 2	11 7
110	6645 6	6656 9	6668 2	6679 4	6690 6	6701 7	6712 7	6723 7	6734 7	6745 6	11 1
111	6756 4	6767 2	6777 9	6788 6	6799 3	6809 9	6820 4	6830 9	6841 3	6851 7	10 6
112	3 6822 0	6822 3	6832 5	6842 7	6852 8	6862 9	6872 9	6882 9	6892 8	6902 7	10 1
113	6902 5	6912 7	6922 8	6932 9	6942 9	6952 9	6962 9	6972 9	6982 9	6992 9	9 6
114	7008 3	7007 6	7016 9	7026 2	7035 5	7044 5	7053 6	7062 7	7071 7	7080 7	9 1
115	3 7149 6	7158 5	7167 4	7176 2	7185 0	7193 7	7202 4	7211 1	7219 7	7228 2	8 7
116	7238 7	7245 2	7253 7	7262 1	7270 5	7278 8	7287 1	7295 3	7303 5	7311 7	8 3
117	7319 8	7327 9	7336 0	7344 0	7352 0	7360 0	7367 9	7375 8	7383 6	7391 4	8 0
118	3 7389 2	7406 9	7414 6	7422 3	7430 0	7437 5	7445 0	7452 5	7460 0	7467 5	7 6
119	7474 9	7482 3	7489 7	7497 1	7504 4	7511 8	7519 1	7526 4	7533 7	7541 0	7 4
120	7548 3	7555 6	7562 8	7570 1	7577 3	7584 6	7591 8	7599 0	7606 2	7613 4	7 3
121	3 7620 5	7627 7	7634 8	7641 9	7649 0	7656 1	7663 2	7670 3	7677 4	7684 5	7 1
122	7691 5	7698 6	7705 6	7712 6	7719 6	7726 6	7733 5	7740 5	7747 5	7754 4	7 0
123	7761 3	7768 3	7775 2	7782 1	7789 0	7795 9	7802 7	7809 6	7816 4	7823 3	6 9
124	3 7830 1	7836 9	7843 7	7850 5	7857 2	7864 0	7870 8	7877 6	7884 3	7891 0	6 8
125	7897 7	7904 4	7911 1	7917 8	7924 4	7931 1	7937 7	7944 4	7951 0	7957 7	6 7
126	7964 2	7970 8	7977 4	7984 0	7990 6	7997 2	8003 7	8010 2	8016 7	8023 2	6 6
127	3 8020 7	8026 2	8032 7	8039 2	8045 6	8052 1	8058 5	8064 9	8071 3	8077 8	6 5
128	8084 2	8100 6	8107 0	8113 4	8119 7	8126 1	8132 4	8138 8	8145 1	8151 4	6 4
129	8157 7	8164 0	8170 3	8176 6	8182 8	8189 1	8195 3	8201 6	8207 8	8214 0	6 3
130	3 8220 2	8226 6	8232 6	8238 8	8244 9	8251 1	8257 2	8263 3	8269 5	8275 6	6 2
131	8281 7	8287 8	8293 9	8300 0	8306 1	8312 2	8318 2	8324 3	8330 3	8336 4	6 1
132	8342 4	8348 4	8354 4	8360 4	8366 4	8372 3	8378 3	8384 2	8390 2	8396 1	6 0

133	3 8402.1	8408.0	8413.9	8419.8	8425.7	8431.6	8437.5	8443.3	8449.2	8455.0	5.9
134	8400.9	8406.7	8412.5	8418.3	8424.1	8429.9	8435.7	8441.5	8447.3	8453.1	5.8
135	8418.8	8424.6	8430.4	8436.2	8442.0	8447.8	8453.6	8459.4	8465.2	8471.0	5.7
136	3 8575.9	8581.7	8587.5	8593.3	8599.1	8604.9	8610.7	8616.5	8622.3	8628.1	5.6
137	8602.1	8607.9	8613.7	8619.5	8625.3	8631.1	8636.9	8642.7	8648.5	8654.3	5.5
138	8687.6	8693.4	8699.2	8705.0	8710.8	8716.6	8722.4	8728.2	8734.0	8739.8	5.5
139	3 8742.2	8748.0	8753.8	8759.6	8765.4	8771.2	8777.0	8782.8	8788.6	8794.4	5.4
140	8766.1	8771.9	8777.7	8783.5	8789.3	8795.1	8800.9	8806.7	8812.5	8818.3	5.3
141	8840.2	8846.0	8851.8	8857.6	8863.4	8869.2	8875.0	8880.8	8886.6	8892.4	5.2
142	3 8901.5	8907.3	8913.1	8918.9	8924.7	8930.5	8936.3	8942.1	8947.9	8953.7	5.1
143	8953.2	8959.0	8964.8	8970.6	8976.4	8982.2	8988.0	8993.8	8999.6	9005.4	5.0
144	9004.1	9009.9	9015.7	9021.5	9027.3	9033.1	9038.9	9044.7	9050.5	9056.3	5.0
145	3 9054.3	9060.1	9065.9	9071.7	9077.5	9083.3	9089.1	9094.9	9100.7	9106.5	4.9
146	9108.5	9114.3	9120.1	9125.9	9131.7	9137.5	9143.3	9149.1	9154.9	9160.7	4.8
147	9152.8	9158.6	9164.4	9170.2	9176.0	9181.8	9187.6	9193.4	9199.2	9205.0	4.8
148	3 9201.6	9207.4	9213.2	9219.0	9224.8	9230.6	9236.4	9242.2	9248.0	9253.8	4.8
149	9250.1	9255.9	9261.7	9267.5	9273.3	9279.1	9284.9	9290.7	9296.5	9302.3	4.8
150	9298.3	9304.1	9309.9	9315.7	9321.5	9327.3	9333.1	9338.9	9344.7	9350.5	4.8
151	3 9346.3	9352.1	9357.9	9363.7	9369.5	9375.3	9381.1	9386.9	9392.7	9398.5	4.7
152	9394.1	9400.0	9405.8	9411.6	9417.4	9423.2	9429.0	9434.8	9440.6	9446.4	4.7
153	9441.6	9447.4	9453.2	9459.0	9464.8	9470.6	9476.4	9482.2	9488.0	9493.8	4.7
154	3 9498.8	9504.6	9510.4	9516.2	9522.0	9527.8	9533.6	9539.4	9545.2	9551.0	4.7
155	9552.8	9558.6	9564.4	9570.2	9576.0	9581.8	9587.6	9593.4	9599.2	9605.0	4.6
156	9608.8	9614.6	9620.4	9626.2	9632.0	9637.8	9643.6	9649.4	9655.2	9661.0	4.6
157	3 9620.1	9625.9	9631.7	9637.5	9643.3	9649.1	9654.9	9660.7	9666.5	9672.3	4.6
158	9678.3	9684.1	9689.9	9695.7	9701.5	9707.3	9713.1	9718.9	9724.7	9730.5	4.6
159	9721.4	9727.2	9733.0	9738.8	9744.6	9750.4	9756.2	9762.0	9767.8	9773.6	4.5
160	3 9767.2	9773.0	9778.8	9784.6	9790.4	9796.2	9802.0	9807.8	9813.6	9819.4	4.5
161	9821.8	9827.6	9833.4	9839.2	9845.0	9850.8	9856.6	9862.4	9868.2	9874.0	4.5
162	9878.5	9884.3	9890.1	9895.9	9901.7	9907.5	9913.3	9919.1	9924.9	9930.7	4.5
163	3 9938.3	9944.1	9949.9	9955.7	9961.5	9967.3	9973.1	9978.9	9984.7	9990.5	4.5
164	9996.3	10002.1	10007.9	10013.7	10019.5	10025.3	10031.1	10036.9	10042.7	10048.5	4.5
165	3 9992.9	10001.8	10010.7	10019.6	10028.5	10037.4	10046.3	10055.2	10064.1	10073.0	4.5
166	10081.6	10090.5	10099.4	10108.3	10117.2	10126.1	10135.0	10143.9	10152.8	10161.7	4.4
167	4 0037.4	0043.2	0049.0	0054.8	0060.6	0066.4	0072.2	0078.0	0083.8	0089.6	4.4
168	0095.4	0101.2	0107.0	0112.8	0118.6	0124.4	0130.2	0136.0	0141.8	0147.6	4.4

Table III—continued.
 $s = C[S(V) - S(r)]$.

r	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											
160	4 0160-5	0178-9	0178-3	0182-6	0187-0	0191-4	0195-8	0200-1	0204-5	0208-9	4
161	0213-2	0217-0	0217-3	0220-3	0230-6	0234-9	0239-3	0243-6	0247-9	0252-3	4-3
170	0250-6	0260-0	0265-2	0269-5	0273-9	0278-2	0282-5	0286-9	0291-2	0295-5	4-3
171											
172	4 0360-8	0364-1	0368-4	0373-7	0377-0	0381-4	0385-7	0390-0	0394-3	0398-6	4-3
173	0392-9	0397-2	0401-5	0405-8	0410-1	0414-4	0418-7	0423-0	0427-3	0431-6	4-3
174	0385-7	0390-0	0394-2	0398-5	0402-8	0407-0	0411-3	0415-6	0419-8	0424-1	4-3
175	4 0428-3	0432-6	0436-8	0441-1	0445-3	0449-6	0453-8	0458-1	0462-3	0466-6	4-3
176	0470-8	0475-0	0479-2	0483-4	0487-6	0491-9	0496-1	0500-3	0504-6	0508-8	4-2
177	0513-0	0517-2	0521-4	0525-6	0529-8	0534-1	0538-3	0542-5	0546-7	0550-9	4-2
178	4 0555-1	0559-3	0563-5	0567-7	0571-0	0575-1	0579-3	0583-4	0587-6	0591-8	4-2
179	0596-9	0601-1	0605-3	0609-5	0613-7	0617-8	0622-0	0626-1	0630-3	0634-5	4-2
180	0638-6	0642-8	0646-9	0651-1	0655-2	0659-4	0663-5	0667-7	0671-8	0675-9	4-2
181											
182	4 0680-1	0684-2	0688-4	0692-5	0696-6	0700-8	0704-9	0709-0	0713-1	0717-3	4-1
183	0721-4	0725-5	0729-6	0733-7	0737-9	0742-0	0746-1	0750-2	0754-4	0758-5	4-1
184	0762-6	0766-7	0770-8	0774-9	0779-0	0783-1	0787-2	0791-3	0795-3	0799-4	4-1
185	4 0803-5	0807-6	0811-7	0815-8	0819-9	0824-0	0828-1	0832-2	0836-2	0840-3	4-1
186	0844-3	0848-4	0852-4	0856-5	0860-6	0864-6	0868-7	0872-8	0876-8	0880-9	4-1
187	0884-9	0889-0	0893-0	0897-1	0901-1	0905-1	0909-2	0913-2	0917-2	0921-3	4-0
188											
189	4 0925-3	0929-3	0933-4	0937-4	0941-4	0945-5	0949-5	0953-5	0957-6	0961-6	4-0
190	0965-6	0969-6	0973-6	0977-6	0981-6	0985-6	0989-6	0993-6	0997-6	1001-6	4-0
191	1005-6	1009-6	1013-6	1017-6	1021-6	1025-6	1029-5	1033-5	1037-5	1041-5	4-0
192	4 1045-5	1049-5	1053-5	1057-5	1061-5	1065-5	1069-4	1073-4	1077-4	1081-4	4-0
193	1085-3	1089-3	1093-2	1097-2	1101-2	1105-2	1109-1	1113-1	1117-0	1121-0	4-0
194	1124-9	1128-9	1132-8	1136-8	1140-7	1144-6	1148-6	1152-5	1156-4	1160-4	3-9
195	4 1164-3	1168-2	1172-1	1176-0	1180-0	1183-9	1187-8	1191-7	1195-7	1199-6	3-9
196	1203-5	1207-4	1211-3	1215-2	1219-1	1223-1	1227-0	1230-9	1234-8	1238-7	3-9
197	1242-6	1246-5	1250-4	1254-3	1258-2	1262-1	1266-0	1269-9	1273-8	1277-7	3-9
198	4 1261-6	1265-5	1269-4	1273-3	1277-2	1301-0	1304-9	1308-8	1312-6	1316-5	3-9
	1320-3	1324-2	1328-1	1332-0	1335-9	1339-8	1343-6	1347-5	1351-3	1355-2	3-9
	1369-0	1362-9	1366-7	1370-6	1374-4	1378-2	1382-1	1385-9	1389-7	1393-6	3-5

199	4	1387.4	1401.2	1405.0	1408.9	1412.7	1416.5	1420.4	1424.2	1428.0	1431.9	3.8
200		1436.7	1439.5	1443.3	1447.1	1450.9	1454.8	1458.6	1462.4	1466.2	1470.1	3.8
201		1473.9	1477.7	1481.5	1485.3	1489.1	1492.9	1496.7	1500.5	1504.3	1508.2	3.8
202	4	1512.0	1515.8	1519.6	1523.4	1527.2	1531.0	1534.8	1538.6	1542.4	1546.2	3.8
203		1559.0	1558.8	1557.6	1556.4	1555.2	1554.0	1552.8	1551.6	1550.4	1549.2	3.8
204		1587.9	1591.7	1595.5	1599.3	1603.1	1606.9	1610.6	1614.4	1618.2	1622.0	3.8
205	4	1625.7	1629.5	1633.3	1637.0	1640.8	1644.6	1648.3	1652.1	1655.9	1659.6	3.8
206		1665.4	1667.1	1668.8	1670.5	1672.2	1673.9	1675.6	1677.3	1679.0	1680.7	3.8
207		1701.0	1704.7	1708.5	1712.2	1716.0	1719.7	1723.5	1727.2	1731.0	1734.7	3.7
208	4	1738.5	1742.2	1746.0	1749.7	1753.5	1757.2	1761.0	1764.7	1768.5	1772.2	3.7
209		1776.0	1779.7	1783.4	1787.1	1790.9	1794.6	1798.3	1802.1	1805.8	1809.6	3.7
210		1813.3	1817.0	1820.8	1824.5	1828.2	1831.9	1835.7	1839.4	1843.1	1846.9	3.7
211	4	1850.6	1854.3	1858.0	1861.7	1865.4	1869.1	1872.8	1876.5	1880.3	1884.0	3.7
212		1887.7	1891.4	1895.1	1898.8	1902.5	1906.2	1909.9	1913.7	1917.4	1921.1	3.7
213		1924.8	1928.5	1932.2	1935.9	1939.6	1943.3	1947.0	1950.7	1954.4	1958.1	3.7
214	4	1961.8	1965.5	1969.2	1972.9	1976.6	1980.3	1984.0	1987.7	1991.4	1995.0	3.7
215		1998.7	2002.4	2006.1	2009.8	2013.5	2017.2	2020.9	2024.6	2028.2	2031.9	3.7
216		2035.0	2038.3	2041.6	2044.9	2048.2	2051.5	2054.8	2058.1	2061.4	2064.7	3.7
217	4	2072.3	2076.0	2079.7	2083.4	2087.0	2090.7	2094.4	2098.0	2101.7	2105.4	3.7
218		2100.0	2113.7	2116.3	2120.0	2123.6	2127.3	2130.9	2134.6	2138.2	2141.9	3.7
219		2145.5	2149.2	2152.8	2156.5	2160.1	2163.8	2167.4	2171.1	2174.7	2178.4	3.7
220	4	2182.0	2185.7	2189.3	2193.0	2196.6	2200.2	2203.9	2207.5	2211.1	2214.8	3.6
221		2218.4	2222.1	2225.7	2229.4	2233.0	2236.6	2240.3	2243.9	2247.5	2251.2	3.6
222		2254.8	2258.4	2262.0	2265.7	2269.3	2272.9	2276.5	2280.1	2283.8	2287.4	3.6
223	4	2291.0	2294.6	2298.2	2301.9	2305.5	2309.1	2312.7	2316.3	2320.0	2323.6	3.6
224		2327.2	2330.8	2334.4	2338.0	2341.6	2345.2	2348.8	2352.4	2356.0	2359.6	3.6
225		2363.2	2366.8	2370.4	2374.0	2377.6	2381.2	2384.8	2388.4	2392.0	2395.6	3.6
226	4	2399.2	2402.8	2406.4	2410.0	2413.6	2417.2	2420.8	2424.4	2428.0	2431.6	3.6
227		2435.2	2438.8	2442.4	2446.0	2449.6	2453.2	2456.8	2460.4	2464.0	2467.6	3.6
228		2471.0	2474.6	2478.2	2481.8	2485.3	2488.9	2492.5	2496.1	2499.6	2503.2	3.6
229	4	2506.8	2510.3	2513.9	2517.5	2521.1	2524.6	2528.2	2531.8	2535.4	2538.9	3.6
230		2542.5	2546.0	2549.6	2553.1	2556.7	2560.3	2563.9	2567.4	2571.0	2574.6	3.6
231		2578.1	2581.6	2585.2	2588.7	2592.3	2595.8	2599.4	2602.9	2606.5	2610.0	3.5
232	4	2613.6	2617.1	2620.7	2624.2	2627.8	2631.3	2634.9	2638.4	2642.0	2645.5	3.5
233		2652.6	2656.1	2659.7	2663.2	2666.7	2670.3	2673.8	2677.3	2680.9	2684.4	3.5
234		2687.9	2691.5	2695.0	2698.5	2702.1	2705.6	2709.1	2712.7	2716.2	2719.7	3.5

Table III—continued.
 $s = G[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											
235	4 2719 7	9733 2	9730 8	9730 3	2733 8	2737 3	2740 9	2744 4	2747 9	2751 5	+
236	2765 0	2763 5	2762 0	2762 5	2769 0	2772 6	2770 1	2779 6	2783 1	2786 5	3 5
237	2780 1	2783 6	2787 1	2800 6	2804 1	2807 7	2811 2	2814 7	2818 2	2821 7	3 5
238	4 2825 2	2828 7	2832 2	2835 7	2839 2	2842 7	2846 2	2849 7	2853 2	2856 7	3 5
239	2860 2	2863 7	2867 2	2870 7	2874 2	2877 7	2881 1	2884 6	2888 1	2891 6	3 5
240	2885 1	2898 6	2902 1	2905 6	2909 1	2912 6	2916 1	2919 6	2923 0	2926 5	3 5
241	4 2930 0	2933 5	2937 0	2940 5	2944 0	2947 5	2950 9	2954 4	2957 9	2961 4	3 5
242	2964 8	2968 3	2971 8	2975 2	2978 7	2982 2	2985 6	2989 1	2992 5	2996 0	3 5
243	2999 5	3002 9	3006 4	3009 8	3013 3	3016 8	3020 2	3023 7	3027 2	3030 6	3 5
244	4 3034 1	3037 5	3041 0	3044 5	3047 9	3051 4	3054 9	3058 3	3061 8	3065 3	3 5
245	3068 7	3072 2	3075 6	3079 1	3082 5	3086 0	3089 4	3092 9	3096 3	3099 8	3 5
246	3103 2	3106 7	3110 1	3113 6	3117 0	3120 5	3123 9	3127 4	3130 8	3134 3	3 5
247	4 3137 7	3141 1	3144 6	3148 0	3151 4	3154 9	3158 3	3161 7	3165 2	3168 6	3 4
248	3172 0	3175 4	3178 8	3182 3	3185 7	3189 2	3192 6	3196 0	3199 5	3202 9	3 4
249	3206 3	3209 7	3213 1	3216 5	3220 0	3223 4	3226 8	3230 3	3233 7	3237 1	3 4
250	4 3240 5	3243 9	3247 3	3250 8	3254 2	3257 6	3261 0	3264 4	3267 8	3271 3	3 4
251	3274 7	3278 1	3281 5	3284 9	3288 3	3291 8	3295 2	3298 6	3302 0	3305 4	3 4
252	3308 8	3312 2	3315 6	3319 0	3322 4	3325 8	3329 2	3332 6	3336 0	3339 4	3 4
253	4 3342 8	3346 2	3349 6	3353 0	3356 4	3359 8	3363 2	3366 6	3370 0	3373 4	3 4
254	3376 8	3380 2	3383 6	3387 0	3390 4	3393 8	3397 1	3400 5	3403 9	3407 2	3 4
255	3410 6	3414 0	3417 4	3420 8	3424 2	3427 6	3431 0	3434 4	3437 7	3441 1	3 4
256	4 3444 5	3447 9	3451 3	3454 7	3458 0	3461 4	3464 8	3468 1	3471 5	3474 9	3 4
257	3478 2	3481 6	3485 0	3488 3	3491 7	3495 1	3498 5	3501 9	3505 2	3508 6	3 4
258	3511 9	3515 3	3518 6	3522 0	3525 4	3528 7	3532 1	3535 5	3538 8	3542 2	3 4
259	4 3545 5	3548 9	3552 2	3555 6	3559 0	3562 3	3565 7	3569 1	3572 4	3575 8	3 4
260	3579 1	3582 5	3585 8	3589 2	3592 5	3595 9	3599 2	3602 6	3605 9	3609 3	3 4
261	3612 6	3616 0	3619 3	3622 7	3626 0	3629 3	3632 7	3636 0	3639 3	3642 7	3 4
262	4 3646 0	3649 3	3652 7	3656 0	3659 3	3662 7	3666 0	3669 3	3672 7	3676 0	3 4
263	3679 3	3682 6	3685 9	3689 2	3692 6	3695 9	3699 2	3702 6	3705 9	3709 2	3 4
264	3712 5	3715 8	3719 1	3722 5	3725 8	3729 1	3732 4	3735 8	3739 1	3742 4	3 4

265	4	3745	7	3749	0	3752	3	3755	6	3758	9	3762	2	3765	5	3768	8	3772	1	3775	4	3	3
266		3775	7	3780	3	3785	3	3788	6	3791	9	3795	2	3798	5	3801	8	3805	1	3808	4	3	3
267		3811	7	3815	0	3818	3	3821	6	3824	9	3827	2	3830	5	3833	8	3836	1	3839	4	3	3
268	4	3844	6	3847	9	3851	2	3854	4	3857	7	3861	0	3864	3	3867	5	3870	8	3874	1	3	3
269		3877	4	3880	7	3883	9	3887	2	3890	5	3893	7	3897	0	3900	3	3903	6	3906	9	3	3
270		3910	1	3913	4	3916	6	3919	9	3922	2	3925	5	3928	7	3931	0	3934	3	3937	5	3	3
271	4	3942	8	3946	1	3949	3	3952	6	3955	9	3959	1	3962	4	3965	6	3968	9	3972	2	3	3
272		3975	4	3978	7	3981	9	3984	2	3987	5	3990	7	3994	0	3997	3	4001	4	4004	7	3	3
273		4007	9	4011	1	4014	4	4017	6	4020	9	4024	1	4027	4	4030	6	4033	8	4037	1	3	2
274	4	4040	3	4043	6	4046	8	4050	0	4053	2	4056	5	4059	7	4062	9	4066	1	4069	4	3	2
275		4072	6	4075	8	4079	1	4082	3	4085	5	4088	7	4091	9	4095	2	4098	4	4101	6	3	2
276		4104	8	4108	0	4111	2	4114	5	4117	7	4120	9	4124	1	4127	4	4130	6	4133	8	3	2
277	4	4137	0	4140	2	4143	4	4146	6	4149	8	4153	1	4156	3	4159	5	4162	7	4165	9	3	2
278		4159	1	4172	3	4175	5	4178	7	4181	9	4185	3	4188	5	4191	7	4194	7	4197	9	3	2
279		4201	1	4204	3	4207	5	4210	7	4213	9	4217	1	4220	3	4223	5	4226	7	4229	9	3	2
280	4	4233	1	4236	3	4239	5	4242	7	4245	9	4249	0	4252	2	4255	4	4258	6	4261	7	3	2
281		4264	9	4268	1	4271	3	4274	5	4277	7	4280	8	4284	0	4287	2	4290	4	4293	5	3	2
282		4286	7	4289	9	4293	1	4296	3	4299	5	4302	7	4305	8	4309	0	4312	1	4315	3	3	2
283	4	4295	4	4301	6	4304	8	4308	0	4311	2	4314	3	4317	5	4320	7	4323	8	4326	9	3	2
284		4300	1	4303	3	4306	4	4309	6	4312	7	4315	9	4318	1	4321	3	4324	5	4327	7	3	2
285		4301	6	4304	8	4307	9	4310	1	4313	4	4316	6	4319	8	4322	1	4325	3	4328	5	3	2
286	4	4323	1	4326	3	4329	5	4332	6	4335	7	4338	9	4341	2	4344	3	4347	5	4350	7	3	2
287		4324	6	4327	8	4330	1	4333	4	4336	6	4339	8	4342	0	4345	2	4348	3	4351	5	3	1
288		4325	9	4328	0	4331	2	4334	5	4337	7	4340	9	4343	1	4346	4	4349	6	4352	8	3	1
289	4	4327	2	4330	3	4333	5	4336	7	4339	9	4342	1	4345	3	4348	5	4351	7	4354	9	3	1
290		4328	4	4331	5	4334	7	4337	9	4340	1	4343	3	4346	5	4349	7	4352	9	4355	1	3	1
291		4329	5	4332	6	4335	8	4338	0	4341	2	4344	4	4347	6	4350	8	4353	1	4356	3	3	1
292	4	4330	5	4333	6	4336	8	4339	0	4342	2	4345	4	4348	6	4351	8	4354	1	4357	3	3	1
293		4331	6	4334	7	4337	9	4340	1	4343	3	4346	5	4349	7	4352	9	4355	1	4358	3	3	1
294		4332	7	4335	8	4338	0	4341	2	4344	4	4347	6	4350	8	4353	1	4356	3	4359	5	3	1
295	4	4333	8	4336	9	4339	1	4342	3	4345	5	4348	7	4351	9	4354	1	4357	3	4360	5	3	1
296		4334	9	4337	0	4340	2	4343	4	4346	6	4349	8	4352	1	4355	3	4358	5	4361	7	3	1
297		4335	0	4338	1	4341	3	4344	5	4347	7	4350	9	4353	1	4356	3	4359	5	4362	7	3	1
298	4	4336	1	4339	2	4342	4	4345	6	4348	8	4351	0	4354	2	4357	4	4360	6	4363	8	3	1
299		4337	2	4340	3	4343	5	4346	7	4349	9	4352	1	4355	3	4358	5	4361	7	4364	9	3	1
300		4338	3	4341	4	4344	6	4347	8	4350	0	4353	2	4356	4	4359	6	4362	8	4365	0	3	1

Table III—continued.
 $s = C[S(V) - S(v)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f_{1a}											+
301	4886-9	4890-0	4893-0	4896-0	4899-1	4902-1	4905-1	4908-1	4911-2	4914-2	3-0
302	4917-2	4920-3	4923-3	4926-3	4929-4	4932-4	4935-4	4938-4	4941-5	4944-5	3-0
303	4950-6	4953-6	4956-6	4959-6	4962-7	4965-7	4968-7	4971-7	4974-7	4977-7	3-0
304	4977-7	4980-8	4983-8	4986-8	4989-8	4992-9	4995-9	4998-9	5001-9	5004-9	3-0
305	5007-9	5011-0	5014-0	5017-0	5020-0	5023-0	5026-0	5029-0	5032-0	5035-0	3-0
306	5038-0	5041-0	5044-0	5047-0	5050-0	5053-0	5056-0	5059-0	5062-0	5065-0	3-0
307	5068-0	5071-0	5074-0	5077-0	5080-0	5083-0	5086-0	5089-0	5092-0	5095-0	3-0
308	5097-9	5100-9	5103-9	5106-9	5109-9	5112-9	5115-9	5118-9	5121-9	5124-9	3-0
309	5127-9	5130-9	5133-9	5136-9	5139-9	5142-8	5145-8	5148-8	5151-8	5154-8	3-0
310	5157-7	5160-7	5163-7	5166-7	5169-7	5172-6	5175-6	5178-6	5181-6	5184-6	3-0
311	5187-5	5190-5	5193-5	5196-5	5199-4	5202-4	5205-4	5208-3	5211-3	5214-3	3-0
312	5217-2	5220-2	5223-1	5226-1	5229-1	5232-0	5235-0	5238-0	5241-9	5243-9	3-0
313	5246-8	5249-8	5252-7	5255-7	5258-7	5261-6	5264-6	5267-6	5270-5	5273-5	3-0
314	5276-4	5279-4	5282-3	5285-3	5288-2	5291-2	5294-1	5297-1	5300-0	5303-0	3-0
315	5305-0	5308-9	5311-8	5314-8	5317-7	5320-7	5323-6	5326-6	5329-5	5332-5	3-0
316	5335-4	5338-4	5341-3	5344-3	5347-2	5350-2	5353-1	5356-0	5359-0	5361-9	2-9
317	5364-8	5367-8	5370-7	5373-6	5376-6	5379-5	5382-4	5385-4	5388-3	5391-2	2-9
318	5394-1	5397-1	5400-0	5402-9	5405-8	5408-8	5411-7	5414-7	5417-6	5420-5	2-9
319	5426-4	5429-3	5432-2	5435-1	5438-1	5441-0	5443-9	5446-8	5449-7	5452-6	2-9
320	5452-6	5455-6	5458-5	5461-4	5464-3	5467-2	5470-1	5473-0	5475-9	5478-8	2-9
321	5481-7	5484-7	5487-6	5490-5	5493-4	5496-3	5499-2	5502-1	5505-0	5507-9	2-9
322	5510-8	5513-7	5516-6	5519-5	5522-4	5525-3	5528-2	5531-1	5534-0	5536-9	2-9
323	5539-8	5542-7	5545-6	5548-5	5551-4	5554-3	5557-2	5560-1	5563-0	5565-9	2-9
324	5568-8	5571-7	5574-6	5577-5	5580-4	5583-3	5586-2	5589-1	5592-0	5594-9	2-9
325	5597-7	5600-6	5603-5	5606-4	5609-3	5612-2	5615-1	5618-0	5620-8	5623-7	2-9
326	5626-5	5629-4	5632-3	5635-2	5638-1	5641-0	5643-9	5646-8	5649-6	5652-5	2-9
327	5655-3	5658-2	5661-1	5664-0	5666-8	5669-7	5672-6	5675-4	5678-3	5681-2	2-9
328	5686-9	5689-9	5692-7	5695-6	5698-5	5701-4	5704-3	5707-1	5710-0	5712-9	2-9
329	5715-6	5718-5	5721-3	5724-2	5727-1	5729-9	5732-8	5735-6	5738-5	5741-4	2-9
330	5744-2	5747-0	5749-9	5752-7	5755-6	5758-4	5761-3	5764-1	5767-0	5769-9	2-9

331	4	5769-8	5772-7	5775-5	5778-4	5781-2	5784-1	5786-9	5788-8	5789-6	5792-6	5795-5	2-9
332		5788-3	5801-7	5804-0	5806-9	5812-6	5812-6	5815-4	5818-7	5821-1	5824-0	5824-0	2-9
333		5826-8	5829-7	5832-5	5835-3	5838-2	5841-0	5843-8	5846-7	5849-5	5852-3	5852-3	2-8
334	4	5855-1	5858-0	5860-8	5863-7	5866-5	5869-3	5872-2	5875-0	5877-8	5880-7	5880-7	2-8
335		5883-5	5886-2	5889-2	5892-0	5894-8	5897-7	5900-5	5903-3	5906-1	5908-9	5908-9	2-8
336		5911-7	5914-6	5917-4	5920-2	5923-0	5925-9	5928-7	5931-5	5934-3	5937-1	5937-1	2-8
337	4	5939-9	5942-8	5945-6	5948-4	5951-2	5954-1	5956-9	5959-7	5962-5	5965-3	5965-3	2-8
338		5968-1	5971-8	5974-6	5977-4	5980-2	5983-0	5985-8	5988-6	5991-4	5994-2	5994-2	2-8
339		5996-2	5999-0	6001-8	6004-6	6007-4	6010-2	6013-0	6015-8	6018-6	6021-4	6021-4	2-8
340	4	6024-2	6027-0	6029-8	6032-6	6035-4	6038-2	6041-0	6043-8	6046-6	6049-4	6049-4	2-8
341		6052-2	6055-0	6057-8	6060-6	6063-4	6066-2	6069-0	6071-8	6074-6	6077-4	6077-4	2-8
342		6080-1	6082-9	6085-7	6088-5	6091-3	6094-1	6096-9	6099-7	6102-5	6105-3	6105-3	2-8
343	4	6108-0	6110-8	6113-6	6116-4	6119-2	6122-0	6124-7	6127-5	6130-3	6133-1	6133-1	2-8
344		6135-8	6138-6	6141-4	6144-2	6147-0	6149-8	6152-5	6155-3	6158-1	6160-9	6160-9	2-8
345		6153-6	6156-4	6159-2	6162-0	6164-7	6167-5	6170-3	6173-1	6175-9	6178-6	6178-6	2-8
346	4	6181-3	6184-1	6186-9	6189-7	6192-4	6195-2	6198-0	6200-8	6203-5	6206-3	6206-3	2-8
347		6219-0	6221-8	6224-6	6227-4	6230-1	6232-9	6235-6	6238-4	6241-1	6243-9	6243-9	2-8
348		6246-6	6249-4	6252-2	6254-9	6257-6	6260-4	6263-1	6265-9	6268-6	6271-4	6271-4	2-8
349	4	6274-1	6276-9	6279-6	6282-4	6285-1	6287-9	6290-6	6293-4	6296-1	6298-9	6298-9	2-8
350		6301-6	6304-4	6307-1	6309-9	6312-6	6315-4	6318-1	6320-9	6323-6	6326-3	6326-3	2-8
351		6329-1	6331-9	6334-6	6337-4	6340-1	6342-9	6345-6	6348-4	6351-1	6353-8	6353-8	2-7
352	4	6355-5	6358-3	6361-0	6364-8	6367-5	6370-3	6373-0	6375-7	6378-4	6381-1	6381-1	2-7
353		6385-8	6388-6	6391-3	6394-1	6396-9	6399-7	6402-4	6405-2	6407-9	6410-7	6410-7	2-7
354		6411-1	6413-9	6416-6	6419-4	6422-1	6424-8	6427-5	6430-3	6433-0	6435-7	6435-7	2-7
355	4	6438-3	6441-1	6443-8	6446-6	6449-3	6452-0	6454-7	6457-4	6460-1	6462-8	6462-8	2-7
356		6455-5	6458-3	6461-0	6463-8	6466-5	6469-3	6472-0	6474-8	6477-5	6480-3	6480-3	2-7
357		6492-6	6495-4	6498-1	6500-8	6503-5	6506-2	6508-9	6511-6	6514-3	6517-0	6517-0	2-7
358	4	6519-7	6522-5	6525-2	6527-9	6530-6	6533-3	6536-0	6538-7	6541-4	6544-1	6544-1	2-7
359		6546-8	6549-5	6552-2	6554-9	6557-6	6560-3	6563-0	6565-7	6568-4	6571-1	6571-1	2-7
360		6573-8	6576-5	6579-2	6581-9	6584-6	6587-3	6590-0	6592-7	6595-4	6598-1	6598-1	2-7
361	4	6600-7	6603-4	6606-1	6608-8	6611-5	6614-2	6616-9	6619-6	6622-3	6625-0	6625-0	2-7
362		6637-7	6640-4	6643-1	6645-8	6648-5	6651-2	6653-9	6656-6	6659-3	6662-0	6662-0	2-7
363		6654-4	6657-1	6659-8	6662-5	6665-2	6667-9	6670-6	6673-3	6676-0	6678-7	6678-7	2-7
364	4	6681-2	6683-9	6686-6	6689-3	6691-9	6694-6	6697-3	6699-9	6702-6	6705-3	6705-3	2-7
365		6716-9	6719-6	6722-3	6725-0	6727-7	6730-4	6733-1	6735-8	6738-5	6741-2	6741-2	2-7
366		6743-6	6746-3	6749-0	6751-7	6754-4	6757-1	6759-8	6762-5	6765-2	6767-9	6767-9	2-7

Table III—continued.
 $s = C[S(V) - S(\phi)]$

n.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	A
$f_{1/2}$											+
367	4 6701 -2	6768 -6	6766 -6	6769 -2	6771 -9	6774 -6	6777 -2	6779 -9	6782 -5	6785 -2	2 7
368	6767 -8	6760 -5	6763 -2	6765 -8	6768 -1	6801 -2	6800 -5	6806 -5	6809 -1	6811 -8	2 7
369	6814 -4	6817 -1	6819 -7	6822 -4	6825 -0	6827 -7	6830 -3	6833 -0	6835 -6	6838 -3	2 7
370	4 6840 -9	6843 -6	6846 -2	6848 -9	6851 -5	6854 -2	6856 -8	6859 -4	6862 -1	6864 -7	2 6
371	6867 -3	6870 -3	6872 -6	6875 -3	6877 -9	6880 -6	6883 -2	6885 -8	6888 -5	6891 -1	2 6
372	6893 -7	6896 -4	6899 -0	6901 -7	6904 -3	6907 -0	6909 -6	6912 -2	6914 -9	6917 -5	2 6
373	4 6920 -1	6922 -8	6925 -4	6928 -0	6930 -7	6933 -3	6935 -9	6938 -6	6941 -2	6943 -8	2 6
374	6946 -4	6949 -1	6951 -7	6954 -3	6956 -0	6959 -5	6962 -2	6964 -8	6967 -4	6970 -0	2 6
375	6972 -6	6975 -3	6977 -9	6980 -5	6983 -1	6985 -7	6988 -4	6991 -0	6993 -6	6996 -2	2 6
376	4 6998 -8	7001 -4	7004 -1	7006 -7	7009 -3	7011 -9	7014 -6	7017 -2	7019 -8	7022 -4	2 6
377	7025 -0	7027 -7	7030 -3	7032 -9	7035 -5	7038 -1	7040 -7	7043 -4	7046 -0	7048 -6	2 6
378	7051 -1	7053 -8	7056 -4	7059 -0	7061 -6	7064 -2	7066 -8	7069 -4	7072 -0	7074 -6	2 6
379	4 7077 -2	7079 -8	7082 -4	7085 -0	7087 -6	7090 -2	7092 -8	7095 -4	7098 -0	7100 -6	2 6
380	7103 -4	7105 -8	7108 -4	7111 -0	7113 -6	7116 -2	7118 -8	7121 -4	7124 -0	7126 -6	2 6
381	7129 -2	7131 -8	7134 -4	7137 -0	7139 -6	7142 -2	7144 -8	7147 -4	7150 -0	7152 -6	2 6
382	4 7155 -1	7157 -7	7160 -3	7162 -9	7165 -5	7168 -1	7170 -7	7173 -3	7175 -9	7178 -5	2 6
383	7181 -0	7183 -6	7186 -2	7188 -8	7191 -4	7194 -0	7196 -5	7199 -1	7201 -7	7204 -3	2 6
384	7206 -8	7209 -4	7212 -0	7214 -6	7217 -2	7219 -8	7222 -3	7224 -9	7227 -5	7230 -1	2 6
385	4 7232 -6	7235 -2	7237 -8	7240 -4	7243 -0	7245 -6	7248 -2	7250 -7	7253 -3	7255 -9	2 6
386	7258 -4	7261 -0	7263 -6	7266 -1	7268 -7	7271 -3	7273 -8	7276 -4	7279 -0	7281 -5	2 6
387	7284 -1	7286 -7	7289 -3	7291 -8	7294 -4	7297 -0	7299 -5	7302 -1	7304 -7	7307 -2	2 6
388	4 7309 -8	7312 -4	7314 -9	7317 -5	7320 -1	7322 -6	7325 -1	7327 -7	7330 -2	7332 -8	2 6
389	7335 -4	7338 -0	7340 -5	7343 -1	7345 -6	7348 -2	7350 -7	7353 -3	7355 -8	7358 -4	2 6
390	7380 -9	7383 -5	7386 -1	7388 -7	7391 -2	7393 -8	7396 -3	7398 -9	7381 -4	7384 -0	2 6
391	4 7386 -5	7389 -1	7391 -6	7394 -2	7396 -7	7399 -3	7401 -8	7404 -4	7406 -9	7409 -5	2 6
392	7412 -0	7414 -6	7417 -1	7419 -7	7422 -2	7424 -8	7427 -4	7429 -9	7432 -5	7434 -0	2 5
393	7437 -4	7440 -0	7442 -5	7445 -1	7447 -6	7450 -2	7452 -7	7455 -3	7457 -8	7460 -3	2 5
394	4 7462 -8	7465 -4	7467 -9	7470 -5	7473 -0	7475 -6	7478 -1	7480 -7	7483 -2	7485 -7	2 5
395	7488 -2	7490 -8	7493 -4	7495 -8	7498 -3	7500 -9	7503 -4	7505 -9	7508 -4	7511 -0	2 5
396	7513 -5	7516 -1	7518 -6	7521 -2	7523 -7	7526 -2	7528 -7	7531 -2	7533 -7	7536 -2	2 5

397	4 7538 7	7541 3	7543 8	7546 3	7548 9	7551 4	7553 9	7556 4	7558 9	7561 4	2 5
398	7564 0	7566 5	7569 0	7571 6	7574 1	7576 6	7579 1	7581 6	7584 1	7586 6	2 5
399	7589 2	7591 7	7594 2	7596 7	7599 2	7601 7	7604 3	7606 8	7609 3	7611 8	2 5
400	4 7614 3										

TABLE IV.

$$\tan \phi - \tan \theta = C [I(V) - I(v)], \text{ or } \phi - \theta = 57.3 C [I(V) - I(v)].$$

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/λ											$+$
50	0.00000	0.00415	0.00827	0.01238	0.01646	0.02052	0.02456	0.02858	0.03258	0.03655	466
51	0.00461	0.00876	0.01287	0.01695	0.02101	0.02506	0.02909	0.03310	0.03709	0.04104	396
52	0.00900	0.01315	0.01726	0.02134	0.02539	0.02942	0.03343	0.03741	0.04136	0.04528	367
53	0.01352	0.01767	0.02178	0.02585	0.02989	0.03391	0.03790	0.04187	0.04581	0.04971	320
54	0.01809	0.02224	0.02635	0.03041	0.03443	0.03842	0.04238	0.04631	0.05021	0.05407	286
55	0.02271	0.02686	0.03097	0.03503	0.03905	0.04303	0.04697	0.05087	0.05473	0.05855	253
56	0.02740	0.03155	0.03566	0.03972	0.04374	0.04772	0.05166	0.05556	0.05941	0.06321	224
57	0.03219	0.03634	0.04045	0.04451	0.04853	0.05250	0.05643	0.06031	0.06414	0.06792	191
58	0.03700	0.04115	0.04526	0.04931	0.05332	0.05729	0.06121	0.06508	0.06890	0.07267	164
59	0.04182	0.04597	0.05008	0.05414	0.05815	0.06212	0.06604	0.07000	0.07391	0.07777	137
60	0.04664	0.05079	0.05489	0.05894	0.06295	0.06691	0.07083	0.07479	0.07870	0.08256	111
61	0.05146	0.05561	0.05971	0.06376	0.06777	0.07173	0.07565	0.07952	0.08334	0.08711	84
62	0.05628	0.06043	0.06453	0.06858	0.07258	0.07654	0.08046	0.08433	0.08815	0.09192	58
63	0.06110	0.06525	0.06935	0.07340	0.07740	0.08136	0.08528	0.08915	0.09297	0.09674	33
64	0.06592	0.07007	0.07417	0.07822	0.08222	0.08618	0.09009	0.09395	0.09776	0.10152	7
65	0.07074	0.07489	0.07899	0.08304	0.08704	0.09100	0.09491	0.09877	0.10258	0.10634	-20
66	0.07556	0.07971	0.08381	0.08786	0.09186	0.09581	0.09972	0.10358	0.10739	0.11115	-46
67	0.08038	0.08453	0.08863	0.09268	0.09668	0.10063	0.10454	0.10840	0.11221	0.11597	-72
68	0.08520	0.08935	0.09345	0.09750	0.10150	0.10545	0.10936	0.11322	0.11703	0.12079	-98
69	0.09002	0.09417	0.09827	0.10232	0.10632	0.11027	0.11418	0.11804	0.12185	0.12561	-124
70	0.09484	0.09899	0.10309	0.10714	0.11114	0.11509	0.11900	0.12286	0.12667	0.13043	-150
71	0.09966	0.10381	0.10791	0.11196	0.11596	0.11991	0.12382	0.12768	0.13149	0.13525	-176
72	0.10448	0.10863	0.11273	0.11678	0.12078	0.12473	0.12864	0.13250	0.13631	0.14007	-202
73	0.10930	0.11345	0.11755	0.12160	0.12560	0.12955	0.13346	0.13732	0.14113	0.14489	-228
74	0.11412	0.11827	0.12237	0.12642	0.13042	0.13437	0.13828	0.14214	0.14595	0.14971	-254
75	0.11894	0.12309	0.12719	0.13124	0.13524	0.13919	0.14310	0.14696	0.15077	0.15453	-280
76	0.12376	0.12791	0.13199	0.13604	0.14004	0.14400	0.14791	0.15177	0.15558	0.15934	-306

77	0 44793	464928	465063	465197	465381	465464	465597	465729	465861	465992	133
78	466123	466254	466384	466514	466644	466772	466900	467028	467156	467283	129
79	467410	467536	467662	467788	467913	468037	468162	468286	468409	468532	124
80	0 468555	468777	468899	469020	469141	469262	469382	469502	469622	469741	121
81	469869	469978	470066	470214	470361	470518	470664	470811	470956	471091	117
82	471027	471141	471256	471370	471483	471596	471709	471822	471934	472046	113
83	0 472157	472268	472379	472490	472600	472709	472819	472928	473036	473145	109
84	473253	473363	473468	473574	473680	473786	473891	474000	474104	474203	106
85	474306	474409	474511	474613	474714	474815	474915	475015	475114	475213	101
86	0 475311	475409	475507	475604	475701	475797	475892	475987	476082	476177	96
87	476211	476314	476417	476520	476622	476724	476826	476927	477027	477127	92
88	477157	477276	477395	477514	477632	477750	477869	477984	478094	478198	88
89	0 478363	478448	478533	478618	478702	478786	478869	478952	479035	479118	84
90	479200	479281	479363	479444	479524	479604	479684	479764	479843	479922	80
91	479701	479779	479856	479934	480011	480088	480164	480240	480316	480392	77
92	0 480452	480542	480616	480690	480764	480838	480911	480984	481056	481128	74
93	481200	481272	481343	481414	481485	481556	481626	481696	481765	481834	71
94	481903	481972	482040	482108	482176	482244	482311	482378	482444	482511	68
95	0 482577	482643	482708	482773	482838	482903	482967	483032	483096	483159	65
96	483223	483286	483348	483411	483473	483535	483597	483659	483720	483781	62
97	483842	483902	483963	484023	484082	484142	484201	484260	484319	484378	60
98	0 484436	484494	484552	484610	484667	484724	484781	484838	484894	484951	57
99	485007	485063	485118	485174	485229	485284	485338	485393	485447	485501	55
100	485555	485608	485662	485715	485768	485821	485873	485925	485977	486029	53
101	0 486081	486133	486184	486235	486285	486337	486388	486438	486488	486538	51
102	486587	486637	486686	486735	486784	486833	486881	486928	486975	487026	49
103	487074	487121	487169	487216	487263	487310	487357	487403	487450	487500	47
104	0 487542	487588	487633	487678	487723	487767	487811	487855	487899	487942	44
105	487985	488028	488070	488112	488154	488195	488236	488277	488317	488358	41
106	488398	488437	488477	488516	488555	488593	488632	488670	488701	488745	38
107	0 488782	488819	488856	488893	488929	488965	489001	489036	489071	489106	36
108	489141	489176	489210	489244	489278	489311	489345	489378	489411	489444	33
109	489476	489508	489540	489572	489604	489635	489666	489697	489728	489759	31
110	0 489789	489819	489849	489878	489908	489938	489967	489995	490021	490053	29
111	490081	490109	490137	490165	490193	490220	490247	490274	490301	490328	27
112	490355	490381	490407	490433	490459	490484	490510	490535	490560	490585	25

Table IV—continued.
 $\tan \phi - \tan \theta = C[\Pi(V) - I(\psi)],$ or $\phi - \theta = 57.3 C[\Pi(V) - I(\psi)].$

τ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
$t/\text{s.}$											
113	0 40610	40635	40659	40683	40708	40732	40756	40780	40803	40826	+
114	40850	40873	40896	40919	40941	40964	40986	41008	41030	41052	24
115	41074	41096	41117	41138	41160	41181	41202	41222	41243	41264	25
116	0 41284	41304	41325	41345	41365	41384	41404	41423	41443	41462	19
117	41481	41500	41519	41538	41557	41575	41594	41612	41630	41648	18
118	41666	41684	41702	41720	41737	41752	41772	41789	41806	41825	17
119	0 41840	41857	41873	41890	41907	41923	41940	41956	41973	41989	16
120	42005	42021	42038	42054	42070	42086	42102	42118	42134	42149	16
121	42165	42181	42197	42212	42228	42243	42258	42271	42286	42299	15
122	0 42320	42335	42351	42366	42381	42396	42411	42426	42440	42455	15
123	42470	42485	42499	42514	42529	42543	42558	42572	42586	42601	14
124	42615	42629	42644	42658	42672	42686	42700	42714	42728	42742	14
125	0 42756	42769	42783	42797	42810	42824	42838	42851	42865	42878	14
126	42892	42905	42918	42932	42945	42958	42971	42984	42997	43010	13
127	43023	43036	43049	43062	43075	43088	43100	43113	43126	43139	13
128	0 43161	43176	43190	43201	43213	43226	43236	43250	43259	43263	12
129	43275	43287	43299	43311	43323	43335	43347	43359	43371	43383	12
130	43395	43407	43418	43430	43442	43453	43465	43477	43488	43500	12
131	0 43511	43522	43534	43545	43556	43568	43579	43590	43601	43613	12
132	43624	43635	43646	43657	43668	43679	43690	43701	43712	43722	11
133	43733	43744	43755	43765	43776	43787	43797	43808	43818	43829	11
134	0 43840	43850	43861	43871	43881	43892	43902	43912	43922	43933	11
135	43943	43953	43963	43973	43983	43993	44003	44013	44023	44033	10
136	44043	44053	44063	44072	44082	44092	44102	44111	44121	44130	10
137	0 44140	44150	44159	44169	44178	44188	44197	44206	44216	44225	10
138	44234	44243	44253	44262	44271	44281	44290	44300	44308	44317	9
139	44326	44335	44344	44353	44362	44371	44380	44389	44397	44406	9
140	0 44415	44424	44433	44441	44450	44459	44467	44476	44485	44493	9
141	44502	44510	44519	44527	44536	44544	44553	44561	44569	44578	8
142	44586	44594	44602	44611	44619	44627	44635	44643	44652	44660	8

143	0-94608	-94676	-94684	-94692	-94700	-94708	-94716	-94724	-94732	-94739	8
144	-94747	-94755	-94763	-94771	-94778	-94786	-94794	-94802	-94809	-94817	8
145	-94825	-94832	-94840	-94848	-94856	-94863	-94870	-94878	-94885	-94893	8
146	0-94900	-94907	-94915	-94922	-94930	-94937	-94944	-94952	-94959	-94966	7
147	-94974	-94981	-94988	-94995	-95003	-95010	-95017	-95024	-95031	-95039	7
148	-95046	-95053	-95060	-95067	-95074	-95081	-95088	-95095	-95103	-95110	7
149	0-95117	-95124	-95131	-95138	-95145	-95151	-95158	-95165	-95172	-95179	7
150	-95186	-95193	-95200	-95207	-95214	-95220	-95227	-95234	-95241	-95248	7
151	-95254	-95261	-95268	-95274	-95281	-95288	-95294	-95301	-95308	-95314	7
152	0-95321	-95328	-95334	-95341	-95348	-95354	-95361	-95367	-95374	-95380	7
153	-95398	-95405	-95411	-95418	-95426	-95432	-95439	-95446	-95453	-95459	6
154	-95468	-95474	-95481	-95488	-95495	-95502	-95509	-95516	-95523	-95530	6
155	0-95537	-95544	-95551	-95557	-95564	-95571	-95578	-95585	-95592	-95599	6
156	-95606	-95613	-95620	-95627	-95634	-95641	-95648	-95655	-95662	-95669	6
157	-95676	-95683	-95690	-95697	-95704	-95711	-95718	-95725	-95732	-95739	6
158	0-95698	-95705	-95712	-95719	-95726	-95733	-95740	-95747	-95754	-95761	6
159	-95768	-95775	-95782	-95789	-95796	-95803	-95810	-95817	-95824	-95831	6
160	-95838	-95845	-95852	-95859	-95866	-95873	-95880	-95887	-95894	-95901	6
161	0-95872	-95879	-95886	-95893	-95900	-95907	-95914	-95921	-95928	-95935	6
162	-95942	-95949	-95956	-95963	-95970	-95977	-95984	-95991	-95998	-96005	6
163	-96012	-96019	-96026	-96033	-96040	-96047	-96054	-96061	-96068	-96075	6
164	0-96087	-96094	-96101	-96108	-96115	-96122	-96129	-96136	-96143	-96150	5
165	-96157	-96164	-96171	-96178	-96185	-96192	-96199	-96206	-96213	-96220	5
166	-96227	-96234	-96241	-96248	-96255	-96262	-96269	-96276	-96283	-96290	5
167	0-96294	-96301	-96308	-96315	-96322	-96329	-96336	-96343	-96350	-96357	5
168	-96364	-96371	-96378	-96385	-96392	-96399	-96406	-96413	-96420	-96427	5
169	0-96434	-96441	-96448	-96455	-96462	-96469	-96476	-96483	-96490	-96497	5
170	-96504	-96511	-96518	-96525	-96532	-96539	-96546	-96553	-96560	-96567	5
171	0-96574	-96581	-96588	-96595	-96602	-96609	-96616	-96623	-96630	-96637	5
172	-96644	-96651	-96658	-96665	-96672	-96679	-96686	-96693	-96700	-96707	5
173	0-96714	-96721	-96728	-96735	-96742	-96749	-96756	-96763	-96770	-96777	5
174	-96784	-96791	-96798	-96805	-96812	-96819	-96826	-96833	-96840	-96847	5
175	0-96854	-96861	-96868	-96875	-96882	-96889	-96896	-96903	-96910	-96917	5
176	-96924	-96931	-96938	-96945	-96952	-96959	-96966	-96973	-96980	-96987	4
177	0-96994	-96999	-97004	-97009	-97014	-97019	-97024	-97029	-97034	-97039	4
178	-97044	-97049	-97054	-97059	-97064	-97069	-97074	-97079	-97084	-97089	4

Table IV—continued.
 $\tan \phi - \tan \theta = C[I(V) - I(\phi)]$, or $\phi - \theta = 57.3 C [I(V) - I(\phi)]$.

r	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
E_{10}											
179	0.96749	0.96754	0.96758	0.96762	0.96766	0.96770	0.96774	0.96779	0.96783	0.96787	+
180	0.96791	0.96795	0.96799	0.96804	0.96808	0.96812	0.96816	0.96820	0.96824	0.96828	4
181	0.96832	0.96836	0.96840	0.96844	0.96848	0.96852	0.96856	0.96860	0.96864	0.96868	4
182	0.96872	0.96876	0.96880	0.96884	0.96888	0.96892	0.96896	0.96900	0.96904	0.96908	4
183	0.96912	0.96916	0.96920	0.96924	0.96928	0.96932	0.96936	0.96940	0.96944	0.96947	4
184	0.96951	0.96955	0.96959	0.96963	0.96967	0.96971	0.96974	0.96978	0.96982	0.96986	4
185	0.96990	0.96994	0.96997	0.97001	0.97005	0.97009	0.97013	0.97017	0.97020	0.97024	4
186	0.97028	0.97032	0.97035	0.97039	0.97043	0.97047	0.97050	0.97054	0.97058	0.97062	4
187	0.97065	0.97069	0.97073	0.97076	0.97080	0.97084	0.97087	0.97091	0.97095	0.97098	4
188	0.97102	0.97106	0.97109	0.97113	0.97116	0.97120	0.97124	0.97127	0.97131	0.97134	4
189	0.97138	0.97142	0.97145	0.97149	0.97152	0.97156	0.97160	0.97163	0.97167	0.97170	4
190	0.97174	0.97178	0.97181	0.97185	0.97188	0.97192	0.97196	0.97199	0.97203	0.97206	4
191	0.97210	0.97213	0.97217	0.97220	0.97224	0.97227	0.97230	0.97234	0.97237	0.97241	3
192	0.97244	0.97248	0.97251	0.97254	0.97258	0.97261	0.97265	0.97268	0.97271	0.97275	3
193	0.97278	0.97282	0.97285	0.97288	0.97292	0.97295	0.97299	0.97302	0.97306	0.97309	3
194	0.97312	0.97315	0.97319	0.97322	0.97325	0.97329	0.97332	0.97335	0.97339	0.97342	3
195	0.97348	0.97351	0.97354	0.97358	0.97361	0.97364	0.97368	0.97372	0.97375	0.97378	3
196	0.97378	0.97381	0.97385	0.97388	0.97391	0.97394	0.97398	0.97401	0.97404	0.97407	3
197	0.97411	0.97414	0.97417	0.97420	0.97423	0.97427	0.97430	0.97433	0.97436	0.97439	3
198	0.97442	0.97446	0.97449	0.97452	0.97455	0.97458	0.97461	0.97464	0.97467	0.97471	3
199	0.97474	0.97477	0.97480	0.97483	0.97486	0.97489	0.97492	0.97495	0.97498	0.97502	3
200	0.97505	0.97508	0.97511	0.97514	0.97517	0.97520	0.97523	0.97526	0.97529	0.97532	3
201	0.97535	0.97538	0.97541	0.97544	0.97547	0.97551	0.97554	0.97557	0.97560	0.97563	3
202	0.97566	0.97569	0.97572	0.97575	0.97578	0.97581	0.97584	0.97587	0.97590	0.97593	3
203	0.97595	0.97598	0.97601	0.97604	0.97607	0.97610	0.97613	0.97616	0.97619	0.97622	3
204	0.97625	0.97628	0.97631	0.97634	0.97637	0.97640	0.97643	0.97645	0.97648	0.97651	3
205	0.97654	0.97657	0.97660	0.97663	0.97665	0.97668	0.97671	0.97674	0.97677	0.97680	3
206	0.97683	0.97686	0.97689	0.97691	0.97694	0.97697	0.97700	0.97703	0.97706	0.97708	3
207	0.97711	0.97714	0.97717	0.97720	0.97723	0.97725	0.97728	0.97731	0.97734	0.97737	3
208	0.97739	0.97742	0.97745	0.97748	0.97750	0.97753	0.97756	0.97759	0.97761	0.97764	3

209	0-97767	97770	97772	97775	97778	97781	97783	97786	97789	97792	3
210	97794	97797	97800	97803	97806	97809	97811	97813	97816	97819	3
211	97821	97824	97827	97830	97832	97835	97838	97840	97843	97846	3
212	0-97848	97851	97854	97856	97859	97862	97864	97867	97870	97872	3
213	97875	97877	97880	97883	97885	97888	97891	97893	97896	97898	3
214	97901	97903	97906	97909	97911	97914	97916	97919	97921	97924	3
215	0-97927	97929	97932	97934	97937	97940	97942	97945	97947	97950	3
216	97952	97955	97957	97960	97962	97965	97967	97970	97972	97975	3
217	97977	97980	97982	97985	97987	97990	97992	97995	97997	98000	3
218	0-98002	98005	98007	98010	98012	98015	98017	98020	98022	98024	3
219	98027	98030	98032	98034	98037	98039	98041	98044	98046	98049	3
220	98051	98054	98056	98058	98061	98063	98065	98068	98070	98073	3
221	0-98075	98078	98080	98082	98085	98087	98090	98092	98094	98097	3
222	98099	98102	98104	98106	98109	98111	98113	98116	98118	98120	3
223	98125	98127	98130	98132	98135	98137	98139	98141	98144	98147	3
224	0-98149	98151	98153	98155	98158	98160	98162	98165	98167	98169	3
225	98169	98171	98174	98176	98178	98180	98183	98185	98187	98189	3
226	98194	98196	98198	98201	98203	98205	98208	98210	98212	98214	3
227	0-98214	98216	98219	98221	98223	98225	98228	98230	98232	98234	3
228	98227	98230	98231	98233	98236	98238	98240	98243	98245	98247	3
229	98250	98251	98253	98255	98258	98260	98262	98265	98267	98270	3
230	0-98261	98263	98265	98268	98270	98272	98275	98276	98278	98280	3
231	98282	98284	98286	98288	98291	98293	98295	98298	98300	98302	3
232	98304	98306	98309	98311	98313	98315	98317	98319	98321	98323	3
233	98324	98326	98328	98330	98332	98334	98336	98338	98340	98343	3
234	0-98345	98347	98349	98351	98353	98355	98357	98359	98361	98364	3
235	98365	98368	98370	98372	98374	98376	98378	98380	98382	98384	3
236	98386	98388	98390	98392	98394	98396	98398	98401	98403	98405	3
237	0-98407	98409	98411	98413	98415	98417	98419	98421	98423	98425	3
238	98427	98429	98431	98433	98435	98437	98439	98441	98443	98445	3
239	98447	98449	98451	98453	98455	98457	98459	98461	98463	98465	3
240	0-98467	98469	98471	98473	98475	98477	98479	98481	98482	98484	3
241	98486	98488	98490	98492	98494	98496	98498	98500	98502	98504	3
242	98506	98508	98509	98511	98513	98515	98517	98519	98521	98523	3
243	0-98525	98527	98528	98530	98532	98534	98536	98538	98540	98542	3
244	98546	98547	98549	98550	98551	98552	98553	98554	98555	98556	3
245	98558	98559	98560	98561	98562	98563	98564	98565	98566	98567	3

Table IV—continued.
 $\tan \phi - \tan \theta = C [I(V) - I(\psi)]$, or $\phi^\circ - \theta^\circ = 57.3 C [I(V) - I(\psi)]$.

ψ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/λ .											
245	0.98581	0.98583	0.98585	0.98587	0.98589	0.98591	0.98592	0.98594	0.98596	0.98598	+
246	0.98600	0.98602	0.98604	0.98606	0.98607	0.98609	0.98611	0.98613	0.98615	0.98616	2
247	0.98620	0.98620	0.98622	0.98624	0.98625	0.98627	0.98629	0.98631	0.98633	0.98634	2
248	0.98638	0.98638	0.98640	0.98642	0.98643	0.98645	0.98647	0.98649	0.98650	0.98652	2
249	0.98654	0.98655	0.98658	0.98660	0.98661	0.98663	0.98665	0.98666	0.98668	0.98670	2
250	0.98672	0.98673	0.98675	0.98677	0.98679	0.98680	0.98682	0.98684	0.98686	0.98687	2
251	0.98689	0.98691	0.98693	0.98694	0.98696	0.98698	0.98700	0.98701	0.98703	0.98705	2
252	0.98707	0.98708	0.98710	0.98712	0.98714	0.98715	0.98717	0.98719	0.98720	0.98722	2
253	0.98724	0.98725	0.98727	0.98729	0.98731	0.98732	0.98734	0.98736	0.98738	0.98739	2
254	0.98741	0.98743	0.98744	0.98746	0.98748	0.98749	0.98751	0.98753	0.98754	0.98756	2
255	0.98758	0.98760	0.98761	0.98763	0.98764	0.98766	0.98768	0.98769	0.98771	0.98773	2
256	0.98774	0.98776	0.98778	0.98779	0.98781	0.98783	0.98784	0.98786	0.98788	0.98789	2
257	0.98791	0.98792	0.98794	0.98796	0.98797	0.98799	0.98801	0.98802	0.98804	0.98805	2
258	0.98807	0.98809	0.98810	0.98812	0.98813	0.98815	0.98817	0.98818	0.98820	0.98821	2
259	0.98823	0.98825	0.98826	0.98828	0.98829	0.98831	0.98833	0.98834	0.98836	0.98837	2
260	0.98839	0.98841	0.98842	0.98844	0.98845	0.98847	0.98849	0.98850	0.98852	0.98853	2
261	0.98855	0.98857	0.98858	0.98860	0.98861	0.98863	0.98865	0.98866	0.98868	0.98869	2
262	0.98871	0.98873	0.98874	0.98876	0.98877	0.98879	0.98881	0.98882	0.98884	0.98885	2
263	0.98887	0.98888	0.98890	0.98891	0.98893	0.98895	0.98896	0.98898	0.98899	0.98901	2
264	0.98902	0.98904	0.98905	0.98907	0.98908	0.98910	0.98911	0.98913	0.98914	0.98916	2
265	0.98917	0.98919	0.98920	0.98922	0.98923	0.98925	0.98926	0.98928	0.98929	0.98931	2
266	0.98924	0.98926	0.98928	0.98930	0.98931	0.98933	0.98934	0.98936	0.98937	0.98939	2
267	0.98947	0.98949	0.98950	0.98952	0.98953	0.98955	0.98956	0.98958	0.98959	0.98961	2
268	0.98963	0.98964	0.98965	0.98967	0.98968	0.98970	0.98971	0.98973	0.98974	0.98975	2
269	0.98977	0.98978	0.98980	0.98981	0.98983	0.98984	0.98986	0.98987	0.98988	0.98990	1
270	0.98991	0.98993	0.98994	0.98996	0.98997	0.98999	0.99000	0.99001	0.99003	0.99004	1
271	0.99006	0.99007	0.99009	0.99010	0.99012	0.99013	0.99014	0.99016	0.99017	0.99019	1
272	0.99020	0.99021	0.99023	0.99024	0.99026	0.99027	0.99028	0.99030	0.99031	0.99033	1
273	0.99034	0.99035	0.99037	0.99038	0.99040	0.99041	0.99042	0.99044	0.99045	0.99047	1
274	0.99048	0.99049	0.99051	0.99052	0.99053	0.99055	0.99056	0.99058	0.99059	0.99060	1

Table IV—continued.
 $\tan \phi - \tan \theta = C[I(V) - I(\phi)],$ or $\phi^\circ - \theta^\circ = 57.3 C[I(V) - I(\phi)]$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s_e											
311	0 99482	99483	99484	99485	99486	99487	99488	99489	99490	99491	+
312	99492	99493	99494	99495	99496	99497	99498	99499	99500	99501	1
313	99502	99503	99504	99505	99506	99507	99508	99509	99510	99511	1
314	0 99511	99512	99513	99514	99515	99516	99517	99518	99519	99520	1
315	99521	99522	99523	99524	99525	99526	99527	99528	99529	99530	1
316	99531	99532	99533	99534	99535	99536	99537	99538	99539	99540	1
317	0 99541	99542	99543	99544	99545	99546	99547	99548	99549	99550	1
318	99551	99552	99553	99554	99555	99556	99557	99558	99559	99560	1
319	99561	99562	99563	99564	99565	99566	99567	99568	99569	99570	1
320	0 99568	99569	99570	99571	99572	99573	99574	99575	99576	99577	1
321	99578	99579	99580	99581	99582	99583	99584	99585	99586	99587	1
322	99588	99589	99590	99591	99592	99593	99594	99595	99596	99597	1
323	0 99595	99596	99597	99598	99599	99600	99601	99602	99603	99604	1
324	99605	99606	99607	99608	99609	99610	99611	99612	99613	99614	1
325	99615	99616	99617	99618	99619	99620	99621	99622	99623	99624	1
326	0 99621	99622	99623	99624	99625	99626	99627	99628	99629	99630	1
327	99631	99632	99633	99634	99635	99636	99637	99638	99639	99640	1
328	99641	99642	99643	99644	99645	99646	99647	99648	99649	99650	1
329	0 99647	99648	99649	99650	99651	99652	99653	99654	99655	99656	1
330	99657	99658	99659	99660	99661	99662	99663	99664	99665	99666	1
331	99667	99668	99669	99670	99671	99672	99673	99674	99675	99676	1
332	0 99673	99674	99675	99676	99677	99678	99679	99680	99681	99682	1
333	99683	99684	99685	99686	99687	99688	99689	99690	99691	99692	1
334	99693	99694	99695	99696	99697	99698	99699	99700	99701	99702	1
335	0 99697	99698	99699	99700	99701	99702	99703	99704	99705	99706	1
336	99707	99708	99709	99710	99711	99712	99713	99714	99715	99716	1
337	99717	99718	99719	99720	99721	99722	99723	99724	99725	99726	1
338	0 99721	99722	99723	99724	99725	99726	99727	99728	99729	99730	1
339	99731	99732	99733	99734	99735	99736	99737	99738	99739	99740	1
340	99741	99742	99743	99744	99745	99746	99747	99748	99749	99750	1

341	0-90745	99745	99747	99748	99749	99750	99751	99751	1
342	99752	99753	99754	99755	99756	99757	99758	99759	1
343	99760	99762	99763	99764	99765	99766	99767	99767	1
344	0-90768	99769	99770	99771	99772	99773	99774	99774	1
345	99775	99776	99777	99778	99779	99780	99781	99781	1
346	99783	99784	99785	99786	99787	99788	99789	99789	1
347	0-90790	99791	99792	99793	99794	99795	99796	99797	1
348	99797	99799	99800	99801	99802	99803	99804	99804	1
349	99805	99806	99807	99808	99809	99810	99811	99811	1
350	0-90812	99813	99814	99815	99816	99817	99818	99818	1
351	99819	99820	99821	99822	99823	99824	99825	99826	1
352	99826	99827	99828	99829	99830	99831	99832	99833	1
353	0-90833	99834	99835	99836	99837	99838	99839	99840	1
354	99841	99842	99843	99844	99845	99846	99847	99847	1
355	99847	99848	99849	99850	99851	99852	99853	99854	1
356	0-90854	99855	99856	99857	99858	99859	99860	99861	1
357	99861	99862	99863	99864	99865	99866	99867	99868	1
358	99868	99869	99870	99871	99872	99873	99874	99874	1
359	0-90875	99876	99877	99878	99879	99880	99881	99881	1
360	99882	99883	99884	99885	99886	99887	99888	99888	1
361	99888	99889	99890	99891	99892	99893	99894	99894	1
362	0-90895	99896	99897	99897	99898	99899	99900	99901	1
363	99901	99902	99903	99904	99905	99906	99907	99907	1
364	99908	99909	99910	99911	99912	99913	99914	99914	1
365	0-90914	99915	99916	99917	99918	99919	99920	99920	1
366	99921	99922	99923	99924	99925	99926	99927	99927	1
367	99927	99928	99929	99930	99931	99932	99933	99933	1
368	0-90934	99935	99936	99936	99937	99938	99939	99939	1
369	99940	99941	99942	99943	99944	99945	99946	99946	1
370	99946	99947	99948	99949	99950	99951	99952	99952	1
371	0-90952	99953	99954	99955	99956	99957	99958	99958	1
372	99958	99959	99960	99961	99962	99963	99964	99964	1
373	99965	99966	99967	99967	99968	99969	99970	99970	1
374	0-90971	99972	99973	99973	99974	99975	99976	99976	1
375	99977	99978	99979	99980	99981	99982	99983	99983	1
376	99983	99984	99985	99986	99987	99988	99989	99989	1

Table IV—continued,
 $\tan \phi - \tan \theta = C[I(V) - I(v)],$ or $\phi^\circ - \theta^\circ = 57.3 C[I(V) - I(v)].$

[illegible]

TABLE V.

Altitude Function, $A(r)$.

r	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f_k	0.00	0.05	0.26	0.50	1.06	1.95	2.37	3.22	4.20	5.31	+
51	6.54	7.89	9.37	10.98	12.70	14.53	16.40	18.58	20.78	23.10	1.83
52	25.54	28.10	30.74	33.52	36.41	39.41	42.52	45.74	49.07	52.52	3.00
53	56.07	59.73	63.50	67.37	71.34	75.41	79.59	83.88	88.37	92.76	4.07
54	97.35	102.03	106.82	111.70	116.68	121.76	126.95	132.23	137.58	143.04	5.08
55	148.40	154.24	160.17	165.80	171.72	177.74	183.84	190.03	196.31	202.67	6.02
56	208.12	215.06	222.28	228.98	235.78	242.67	249.63	256.67	263.80	271.01	6.80
57	278.30	285.66	293.11	300.65	308.27	315.96	323.71	331.55	339.47	347.47	7.50
58	355.54	363.08	371.89	380.19	388.56	397.00	405.51	414.09	422.75	431.47	8.44
59	440.27	449.14	458.07	467.08	476.16	485.30	494.51	503.78	513.13	522.54	9.14
60	532.01	541.54	551.14	560.81	570.54	580.33	590.19	600.11	610.10	620.16	9.79
61	630.29	640.38	650.72	661.01	671.39	681.77	692.24	702.70	713.34	723.98	10.41
62	734.08	745.33	756.23	767.09	778.01	788.99	800.03	811.13	822.28	833.49	10.98
63	844.76	856.08	867.45	878.87	890.34	901.86	913.43	925.05	936.72	948.44	11.52
64	960.20	972.01	983.87	995.78	1007.74	1019.76	1031.83	1043.95	1056.12	1068.34	12.02
65	1080.61	1092.62	1104.28	1116.88	1130.12	1143.61	1156.14	1167.71	1180.32	1192.97	12.40
66	1205.07	1218.31	1231.20	1244.04	1256.92	1269.84	1282.81	1295.82	1308.87	1321.97	12.82
67	1335.11	1348.29	1361.51	1374.77	1388.06	1401.30	1414.76	1428.17	1441.61	1455.09	13.33
68	1468.91	1482.17	1495.77	1509.41	1523.08	1536.79	1550.54	1564.33	1578.16	1592.03	13.71
69	1603.94	1619.88	1635.86	1651.91	1667.93	1683.99	1700.10	1716.24	1732.41	1748.62	14.08
70	1746.86	1761.14	1775.45	1789.79	1804.17	1818.58	1833.02	1847.50	1862.01	1876.55	14.41
71	1891.13	1905.74	1920.38	1935.05	1949.75	1964.48	1979.23	1994.01	2008.82	2023.66	14.73
72	2038.53	2053.43	2068.36	2083.32	2098.31	2113.33	2128.38	2143.46	2158.57	2173.71	15.02
73	188.88	204.07	210.29	216.54	222.81	229.11	235.43	241.78	248.15	254.54	15.30
74	241.95	247.39	252.85	258.34	263.86	269.41	274.99	280.60	286.24	291.91	15.55
75	297.41	303.33	309.27	315.23	321.21	327.21	333.23	339.27	345.33	351.40	15.80
76	355.69	361.80	367.93	374.08	380.24	386.41	392.60	398.81	405.03	411.26	16.02

Table V—continued.
Altitude Function, A (c).

ϕ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	A
f/s											+
77	2 816 02	832 20	848 40	864 61	880 83	897 07	913 32	929 58	945 86	962 15	16 24
78	978 46	1044 73	*011 15	*027 33	*043 94	*060 37	*076 83	*093 31	*109 81	*126 34	16 43
79	3 142 89	136 46	176 04	192 63	209 24	225 86	242 49	259 14	275 80	292 47	16 62
80	3 309 16	325 87	342 60	359 35	376 12	392 91	409 72	426 55	443 40	460 27	16 79
81	3 477 10	404 06	420 98	437 91	454 85	471 80	488 77	505 75	522 74	539 74	16 95
82	646 77	683 81	680 86	687 43	715 02	732 12	749 24	766 38	783 53	800 70	17 10
83	3 817 89	835 09	852 30	869 53	886 77	904 02	921 28	938 55	955 83	973 12	17 25
84	990 42	*007 72	*025 01	*042 28	*059 54	*076 78	*094 01	*111 23	*128 43	*145 62	17 42
85	4 162 80	179 96	197 10	214 22	231 32	248 41	265 48	282 53	299 56	316 57	17 59
86	4 333 56	350 54	367 50	384 44	401 37	418 28	435 17	452 04	468 90	485 74	16 91
87	5 002 56	519 36	536 15	552 92	569 67	586 40	603 11	619 80	636 47	653 13	16 73
88	669 77	686 39	702 99	719 57	736 13	752 67	769 20	785 71	802 20	818 67	16 54
89	4 885 12	857 55	867 96	884 35	900 72	917 08	933 42	949 74	966 04	982 32	16 36
90	998 58	*014 32	*031 04	*047 24	*063 42	*079 58	*095 73	*111 86	*127 97	*144 06	16 16
91	5 160 13	176 13	192 21	208 22	224 21	240 18	256 13	272 06	287 97	303 86	15 97
92	5 319 72	335 56	351 39	367 20	382 99	398 76	414 51	430 24	445 96	461 69	15 77
93	477 34	493 00	508 64	524 26	539 86	555 44	570 99	586 52	602 03	617 52	15 58
94	652 99	648 45	663 89	679 31	694 71	710 08	725 43	740 76	756 07	771 36	15 37
95	5 786 63	801 88	817 11	832 32	847 51	862 68	877 84	892 98	908 10	923 20	15 17
96	998 28	993 34	968 38	983 40	998 40	*013 38	*028 34	*043 27	*058 18	*073 07	14 98
97	6 067 94	102 79	117 62	132 43	147 23	162 01	176 77	191 51	206 23	220 93	14 78
98	6 235 61	250 27	264 91	279 53	294 13	308 71	323 27	337 81	352 33	366 83	14 58
99	281 30	293 76	310 20	324 92	339 82	354 46	369 42	384 40	399 42	414 38	14 38
100	325 02	539 29	553 54	567 77	581 98	596 17	610 34	624 48	638 60	652 70	14 19
101	6 696 78	680 95	694 00	708 93	722 94	736 93	750 90	764 85	778 78	792 70	13 99
102	846 40	820 40	834 34	848 18	862 00	875 80	889 58	903 34	917 08	930 80	13 80
103	944 40	958 17	971 83	985 47	999 10	*012 71	*026 30	*039 87	*053 42	*066 95	13 61
104	7 080 46	108 92	107 32	120 66	133 94	147 16	160 32	173 42	186 46	199 44	13 22
105	212 36	225 22	238 02	250 76	263 44	276 05	288 60	301 09	313 52	325 89	13 01
106	338 20	350 46	362 66	374 81	386 90	398 93	410 91	422 83	434 70	446 51	12 03

107	7 458 27	489 97	481 62	408 21	504 75	516 24	527 67	539 05	550 37	561 64	11 49
108	372 86	584 03	505 15	408 21	617 22	628 18	639 00	619 46	660 76	671 52	10 46
109	682 23	682 88	708 50	714 06	724 58	735 05	745 47	755 84	766 16	776 43	10 47
110	7 786 05	796 83	806 96	817 05	827 09	837 08	847 03	856 93	866 78	876 50	9 99
111	7 886 35	806 07	805 74	915 37	924 96	934 50	944 00	953 87	962 87	972 24	9 54
112	981 57	990 85	*000 00	*009 29	*018 45	*027 57	*036 65	*045 69	*054 68	*063 63	9 12
113	8 072 54	881 41	080 24	069 03	107 78	116 49	125 16	133 79	142 39	150 95	8 71
114	159 47	167 45	176 39	184 70	193 15	201 48	209 77	218 00	226 23	234 40	8 33
115	212 54	250 64	288 78	266 73	274 72	282 68	290 60	298 46	306 35	314 17	7 96
116	321 96	329 71	337 43	345 11	352 76	360 37	367 95	375 49	383 00	390 47	7 61
117	387 90	427 01	427 67	429 01	432 32	441 85	444 07	456 26	463 40	470 53	7 28
118	470 53	477 62	484 68	491 71	498 71	505 68	512 61	519 51	526 38	533 22	6 97
119	546 03	546 83	553 62	560 40	567 16	573 91	580 65	587 38	594 10	600 81	6 75
120	607 51	614 20	620 88	627 55	634 21	640 86	647 50	654 13	660 75	667 37	6 65
121	673 98	680 58	687 17	693 75	700 32	706 87	713 41	719 94	726 46	732 97	6 55
122	738 47	745 96	752 45	758 93	765 40	771 86	778 31	784 75	791 18	797 60	6 46
123	804 01	810 41	816 80	823 18	829 55	835 91	842 26	848 61	854 95	861 28	6 36
124	867 60	873 91	880 21	886 50	892 78	899 05	905 31	911 56	917 80	924 04	6 27
125	938 49	944 70	950 99	957 09	962 27	967 44	972 60	977 76	982 91	988 05	6 18
126	992 05	*004 30	*010 41	*016 51	*022 60	*028 68	*034 75	*040 82	*046 88	*052 91	6 09
127	9 052 93	058 97	065 00	071 03	077 05	083 06	089 06	095 05	101 03	107 00	6 01
128	9 112 96	118 91	124 86	130 80	136 73	142 65	148 56	154 46	160 35	166 23	5 92
129	172 10	177 97	183 83	189 68	195 53	201 37	207 20	213 03	218 85	224 66	5 84
130	230 46	236 25	242 03	247 80	253 57	259 35	265 08	270 82	276 55	282 27	5 76
131	9 287 98	293 69	299 30	305 08	310 76	316 44	322 11	327 77	333 42	339 06	5 68
132	314 60	320 32	325 04	330 55	336 15	341 75	347 34	352 92	358 49	364 06	5 60
133	400 62	406 17	411 71	417 24	422 77	428 29	433 80	439 31	444 81	450 30	5 52
134	9 455 78	461 26	466 73	472 19	477 64	483 08	488 51	493 94	499 36	504 77	5 44
135	510 18	515 58	520 95	526 35	531 73	537 10	542 46	547 82	553 17	558 51	5 37
136	568 84	569 17	574 49	579 80	585 10	590 40	595 69	600 97	606 24	611 51	5 30
137	9 616 77	622 02	627 27	632 51	637 74	642 96	648 18	653 39	658 59	663 79	5 22
138	668 98	674 16	679 34	684 51	689 67	694 82	699 97	705 11	710 25	715 38	5 15
139	720 50	725 61	730 72	735 82	740 91	746 00	751 08	756 15	761 21	766 27	5 09
140	9 771 32	776 36	781 40	786 43	791 45	796 47	801 48	806 49	811 49	816 49	5 02
141	821 48	826 46	831 43	836 40	841 36	846 31	851 26	856 20	861 13	866 06	4 96
142	870 98	875 89	880 80	885 70	890 59	895 48	900 36	905 23	910 10	914 96	4 89

Table V—continued.
Abundance Function, $\Delta(\nu)$.

ν	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
$f_{1/2}^*$											+
133	9 010 '82	994 '67	929 '51	934 '35	939 '18	944 '00	948 '82	953 '63	958 '44	963 '24	4 '82
143	068 '63	972 '82	977 '60	982 '37	987 '14	991 '00	996 '45	*001 '40	*006 '14	*010 '88	4 '76
144	020 '33	025 '05	029 '76	034 '47	039 '17	043 '87	048 '57	053 '24	057 '92	062 '60	4 '70
145	10 015 '61										
146	10 062 '50	067 '26	071 '03	076 '59	081 '25	085 '91	090 '56	095 '21	099 '86	104 '51	4 '66
147	109 '75	113 '79	118 '43	123 '07	127 '71	132 '34	136 '97	141 '60	146 '23	150 '86	4 '63
148	155 '38	164 '72	169 '34	173 '96	178 '57	183 '18	187 '79	192 '40	196 '00	200 '61	4 '61
149	201 '60	206 '20	210 '80	215 '40	219 '99	224 '58	229 '17	233 '76	238 '35	242 '93	4 '59
150	247 '51	252 '09	256 '67	261 '25	265 '82	270 '39	274 '96	279 '53	284 '09	288 '65	4 '57
151	293 '21	297 '77	302 '33	306 '88	311 '43	315 '98	320 '53	325 '08	329 '62	334 '16	4 '55
152	10 338 '70	343 '24	347 '78	352 '31	356 '84	361 '37	365 '90	370 '42	374 '94	379 '46	4 '53
153	383 '98	388 '50	393 '01	397 '52	402 '03	406 '54	411 '05	415 '55	420 '05	424 '55	4 '51
154	429 '05	433 '55	438 '04	442 '53	447 '02	451 '51	456 '00	460 '48	464 '96	469 '44	4 '49
155	10 473 '92	478 '40	482 '87	487 '34	491 '81	496 '28	500 '75	505 '21	509 '67	514 '13	4 '47
156	518 '50	523 '05	527 '50	531 '95	536 '40	540 '85	545 '30	549 '74	554 '18	558 '62	4 '45
157	563 '06	567 '50	571 '93	576 '36	580 '79	585 '22	589 '65	594 '08	598 '50	602 '92	4 '43
158	10 607 '34	611 '76	616 '18	620 '60	625 '00	629 '41	633 '82	638 '22	642 '62	647 '02	4 '41
159	651 '42	655 '82	660 '21	664 '60	668 '99	673 '38	677 '77	682 '16	686 '54	690 '92	4 '39
160	695 '30	699 '68	704 '06	708 '44	712 '81	717 '18	721 '55	725 '92	730 '28	734 '64	4 '37
161	10 739 '00	743 '36	747 '72	752 '07	756 '42	760 '77	765 '12	769 '47	773 '81	778 '15	4 '35
162	782 '40	786 '83	791 '17	795 '51	799 '84	804 '17	808 '50	812 '83	817 '16	821 '48	4 '33
163	825 '80	830 '12	834 '44	838 '76	843 '08	847 '39	851 '70	856 '01	860 '32	864 '62	4 '31
164	10 865 '92	873 '22	877 '52	881 '82	886 '12	890 '41	894 '70	898 '99	903 '28	907 '57	4 '29
165	911 '85	916 '13	920 '40	924 '66	928 '97	933 '25	937 '53	941 '80	946 '07	950 '34	4 '28
166	954 '61	958 '88	963 '14	967 '40	971 '66	975 '92	980 '18	984 '43	988 '68	992 '93	4 '26
167	10 997 '18	*001 '43	*005 '68	*009 '92	*014 '16	*018 '40	*022 '64	*026 '88	*031 '11	*035 '34	4 '24
168	11 039 '57	043 '80	048 '03	052 '26	056 '48	060 '70	064 '92	069 '14	073 '36	077 '57	4 '22
169	081 '78	085 '99	090 '20	094 '41	098 '61	102 '81	107 '01	111 '21	115 '41	119 '61	4 '20
170	12 123 '80	127 '99	132 '18	136 '37	140 '56	144 '75	148 '94	153 '12	157 '30	161 '48	4 '19
171	165 '06	169 '84	174 '01	178 '18	182 '35	186 '52	190 '69	194 '86	199 '02	203 '18	4 '17
172	207 '34	211 '50	215 '66	219 '82	223 '97	228 '12	232 '27	236 '42	240 '57	244 '71	4 '15

173	11	248.85	359.60	257.13	351.27	205.41	309.54	273.67	277.80	281.93	286.06	4.13
174		260.15	361.31	268.43	362.55	306.67	310.70	314.91	319.02	323.13	327.24	4.12
175		331.35	335.46	339.57	343.67	347.77		351.87	360.07	364.16	368.25	4.10
176	11	372.34	376.43	380.52	384.61	388.69	392.77	396.85	400.93	405.01	409.09	4.08
177		413.17	417.24	421.31	425.38	429.45	433.53	437.60	441.66	445.72	449.78	4.07
178		453.84	457.90	461.96	466.01	470.06	474.11	478.16	482.21	486.25	490.29	4.05
179	11	494.33	498.37	502.41	506.45	510.49	514.52	518.55	522.58	526.61	530.64	4.03
180		534.64	538.69	542.71	546.73	550.75	554.77	558.79	562.81	566.82	570.83	4.02
181		574.84	578.85	582.86	586.87	590.87	594.87	598.87	602.87	606.87	610.86	4.00
182	11	614.85	618.84	622.83	626.82	630.81	634.80	638.79	642.77	646.75	650.73	3.99
183		654.71	658.67	662.62	666.54	670.41	674.28	678.15	682.02	685.89	689.45	3.97
184		694.41	698.37	702.33	706.29	710.25	714.20	718.15	722.10	726.05	730.00	3.95
185	11	733.95	737.90	741.84	745.78	749.72	753.66	757.60	761.54	765.47	769.40	3.94
186		773.33	777.26	781.19	785.12	789.04	792.96	796.88	800.80	804.72	808.64	3.92
187		812.56	816.48	820.39	824.30	828.21	832.12	836.03	839.94	843.84	847.74	3.91
188	11	831.64	835.54	839.44	843.34	847.23	851.12	855.01	858.90	862.79	866.68	3.89
189		860.57	864.45	868.33	872.21	876.09	879.97	883.85	887.73	891.60	895.47	3.88
190		929.34	933.21	937.08	940.95	944.82	948.68	952.54	956.40	960.26	964.12	3.86
191	11	967.97	971.82	975.67	979.52	983.37	987.22	991.07	994.92	998.77	1002.62	3.85
192	12	1006.46	1010.30	1014.14	1017.98	1021.82	1025.65	1029.48	1033.31	1037.14	1040.97	3.83
193		1044.79	1048.62	1052.44	1056.26	1060.08	1063.90	1067.72	1071.54	1075.36	1079.17	3.82
194	12	1082.98	1086.79	1090.60	1094.41	1098.22	1102.02	1105.82	1109.62	1113.42	1117.22	3.80
195		1124.02	1127.83	1131.63	1135.43	1139.23	1143.03	1146.83	1150.63	1154.43	1158.23	3.79
196		1185.92	1189.70	1193.48	1197.26	1201.04	1204.82	1208.60	1212.38	1216.15	1219.92	3.78
197	12	1205.60	1209.46	1213.28	1217.10	1220.92	1224.73	1228.54	1232.34	1236.15	1239.95	3.76
198		1243.32	1247.12	1250.92	1254.71	1258.50	1262.29	1266.07	1269.86	1273.65	1277.43	3.75
199		1271.79	1275.53	1279.27	1283.01	1286.75	1290.48	1294.21	1297.94	1301.67	1305.40	3.73
200	12	1300.13	1303.86	1307.59	1311.32	1315.05	1318.77	1322.50	1326.21	1329.93	1333.65	3.72
201		1340.37	1344.09	1347.81	1351.53	1355.25	1358.96	1362.67	1366.38	1370.09	1373.80	3.71
202		1383.51	1387.22	1390.93	1394.64	1398.35	1402.06	1405.77	1409.48	1413.18	1416.88	3.71
203	12	1420.58	1424.28	1427.98	1431.68	1435.38	1439.08	1442.78	1446.48	1450.18	1453.88	3.70
204		1457.57	1461.26	1464.95	1468.64	1472.33	1476.02	1479.71	1483.40	1487.09	1490.78	3.69
205		1494.47	1498.16	1501.85	1505.54	1509.23	1512.92	1516.60	1520.28	1523.96	1527.64	3.69
206	12	1531.32	1535.00	1538.68	1542.36	1546.04	1549.72	1553.40	1557.07	1560.74	1564.41	3.68
207		1565.08	1568.75	1572.42	1576.09	1579.76	1583.43	1587.10	1590.76	1594.42	1598.08	3.67
208		1604.74	1608.40	1612.06	1615.72	1619.38	1623.04	1626.70	1630.36	1634.02	1637.68	3.66

Table V—continued.
Altitude Function, $\Delta(p)$.

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											
209	12 641.34	645.00	648.06	652.31	655.96	659.61	663.26	666.91	670.56	674.21	3.65
210	677.86	681.51	685.16	688.81	692.46	696.11	699.75	703.39	707.03	710.67	3.65
211	714.31	717.95	721.59	725.23	728.87	732.51	736.15	739.79	743.42	747.05	3.64
212	750.98	754.31	757.94	761.57	765.20	768.83	772.46	776.09	779.72	783.35	3.63
213	786.98	790.91	794.83	798.75	802.67	806.59	810.51	814.43	818.35	822.27	3.62
214	823.19	826.81	830.43	834.05	837.66	841.27	844.88	848.49	852.10	855.71	3.61
215	859.32	862.93	866.54	870.15	873.76	877.37	880.98	884.59	888.20	891.80	3.61
216	895.40	899.00	902.60	906.20	909.80	913.40	917.00	920.60	924.20	927.80	3.60
217	931.40	935.00	938.59	942.18	945.77	949.36	952.95	956.54	960.13	963.72	3.59
218	967.31	970.90	974.49	978.08	981.67	985.25	988.83	992.41	995.99	999.57	3.58
219	13 003.15	006.73	010.31	013.89	017.47	021.05	024.63	028.21	031.79	035.36	3.58
220	038.93	042.50	046.07	049.64	053.21	056.78	060.35	063.92	067.49	071.06	3.57
221	074.63	078.20	081.77	085.34	088.91	092.47	096.03	099.59	103.15	106.71	3.56
222	110.27	113.83	117.40	120.95	124.51	128.07	131.63	135.18	138.73	142.28	3.56
223	145.83	149.38	152.93	156.48	160.03	163.58	167.13	170.68	174.23	177.77	3.55
224	181.31	184.85	188.39	191.93	195.47	199.01	202.55	206.09	209.63	213.17	3.54
225	216.71	220.25	223.79	227.33	230.87	234.41	237.95	241.48	245.01	248.54	3.54
226	252.07	255.60	259.13	262.66	266.19	269.72	273.25	276.78	280.31	283.83	3.53
227	289.35	292.87	296.39	299.91	303.43	306.95	310.47	313.99	317.51	321.03	3.52
228	327.55	331.07	334.59	338.11	341.63	345.15	348.67	352.19	355.71	359.23	3.51
229	357.68	361.19	364.70	368.21	371.72	375.23	378.74	382.25	385.75	389.25	3.51
230	392.75	396.25	399.75	403.25	406.75	410.25	413.75	417.25	420.75	424.25	3.50
231	427.75	431.25	434.75	438.25	441.75	445.25	448.75	452.25	455.75	459.25	3.49
232	462.69	466.18	469.67	473.16	476.65	480.14	483.63	487.11	490.59	494.07	3.49
233	497.55	501.03	504.51	507.99	511.47	514.95	518.43	521.91	525.39	528.87	3.48
234	529.35	532.83	536.31	539.78	543.25	546.72	550.19	553.66	557.13	560.60	3.47
235	567.07	570.54	574.01	577.48	580.95	584.42	587.89	591.36	594.82	598.28	3.47
236	601.74	605.20	608.66	612.12	615.58	619.04	622.50	625.96	629.42	632.88	3.46
237	636.34	639.80	643.26	646.72	650.17	653.62	657.07	660.52	663.97	667.42	3.45
238	670.87	674.32	677.77	681.22	684.67	688.12	691.57	695.02	698.46	701.90	3.45

289	13	705.34	708.78	712.22	715.66	719.10	722.54	725.98	729.42	732.86	736.30	3.44
290	240	739.74	743.18	746.62	750.06	753.49	756.92	760.35	763.78	767.21	770.64	3.43
241		774.07	777.50	780.93	784.36	787.79	791.22	794.65	798.08	801.51	804.93	3.43
242	13	808.35	811.77	815.19	818.61	822.03	825.45	828.87	832.29	835.71	839.13	3.42
243	243	845.97	849.39	852.81	856.23	859.65	863.07	866.49	869.91	873.33	876.75	3.41
244		876.69	880.10	883.51	886.92	890.33	893.74	897.15	900.56	903.97	907.38	3.41
245	13	910.78	914.18	917.58	920.98	924.38	927.78	931.18	934.58	937.98	941.38	3.40
246	246	944.18	948.18	951.58	954.98	958.38	961.78	965.17	968.56	971.95	975.34	3.40
247		978.73	982.12	985.51	988.90	992.29	995.68	999.07	*002.46	*005.85	*009.24	3.39
248	14	012.03	016.02	019.41	022.79	026.17	029.55	032.93	036.31	039.69	043.07	3.38
249	249	046.45	049.83	053.21	056.59	059.97	063.35	066.73	070.11	073.49	076.86	3.38
250		080.23	083.60	086.97	090.34	093.71	097.08	100.45	103.82	107.19	110.56	3.37
251	14	113.93	117.30	120.67	124.04	127.41	130.78	134.14	137.50	140.86	144.22	3.37
252	252	147.58	150.94	154.30	157.66	161.02	164.38	167.74	171.10	174.46	177.82	3.36
253	253	181.18	184.54	187.89	191.24	194.59	197.94	201.29	204.64	207.99	211.34	3.35
254	14	214.69	218.04	221.39	224.74	228.09	231.44	234.79	238.14	241.48	244.82	3.35
255	255	248.16	251.50	254.84	258.18	261.52	264.86	268.20	271.54	274.88	278.22	3.34
256	256	281.56	284.90	288.24	291.58	294.91	298.24	301.57	304.90	308.23	311.56	3.33
257	14	314.80	318.15	321.50	324.85	328.21	331.54	334.87	338.20	341.53	344.86	3.33
258	258	348.18	351.52	354.86	358.19	361.53	364.87	368.20	371.53	374.87	378.20	3.32
259		381.38	384.70	388.02	391.34	394.66	397.98	401.30	404.62	407.94	411.26	3.32
260	14	414.58	417.89	421.20	424.51	427.82	431.13	434.44	437.75	441.06	444.37	3.31
261	261	447.68	450.99	454.30	457.61	460.91	464.21	467.51	470.81	474.11	477.41	3.30
262		480.71	484.01	487.31	490.61	493.91	497.20	500.49	503.78	507.07	510.36	3.29
263	14	513.65	516.94	520.23	523.52	526.81	530.10	533.39	536.67	539.95	543.23	3.29
264	264	546.51	549.79	553.07	556.35	559.63	562.91	566.19	569.47	572.75	576.02	3.28
265		579.29	582.56	585.83	589.10	592.37	595.64	598.91	602.18	605.45	608.72	3.27
266	14	611.99	615.26	618.53	621.79	625.05	628.31	631.57	634.83	638.09	641.35	3.26
267	267	644.61	647.87	651.13	654.39	657.65	660.91	664.17	667.43	670.68	673.93	3.25
268	268	677.18	680.43	683.68	686.93	690.18	693.43	696.68	699.92	703.16	706.40	3.25
269	14	709.64	712.88	716.12	719.36	722.60	725.84	729.08	732.32	735.56	738.80	3.24
270	270	742.04	745.28	748.51	751.74	754.97	758.20	761.43	764.66	767.89	771.12	3.23
271	271	774.35	777.58	780.81	784.04	787.27	790.50	793.73	796.96	800.18	803.40	3.23
272	14	806.02	809.24	812.46	815.68	818.90	822.12	825.34	828.56	831.78	835.00	3.22
273	273	842.00	845.21	848.42	851.63	854.84	858.05	861.26	864.47	867.68	870.89	3.21
274		870.89	874.10	877.31	880.51	883.71	886.91	890.11	893.31	896.51	899.71	3.20

Table V—continued.
 Altitude Function, $A(\phi)$.

ϕ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f_{15}											+
275	14 902.91	906.11	909.31	912.51	915.71	918.91	922.10	925.29	928.48	931.67	3.20
276	934.89	938.05	941.24	944.43	947.62	950.81	954.00	957.19	960.38	963.57	3.19
277	960.75	969.93	979.11	976.29	979.47	982.65	985.83	989.01	992.19	995.37	3.18
278	14 908.55	*001.73	*004.91	*008.08	*011.25	*014.42	*017.59	*020.76	*023.93	*027.10	3.17
279	15 030.27	033.44	036.61	039.78	042.95	046.12	049.29	052.45	055.61	058.77	3.17
280	061.93	065.09	068.25	071.41	074.57	077.73	080.89	084.05	087.21	090.37	3.16
281	15 099.53	096.69	099.84	102.99	106.14	109.29	112.44	115.59	118.74	121.89	3.15
282	125.04	128.19	131.34	134.49	137.64	140.79	143.93	147.07	150.21	153.35	3.15
283	156.49	159.63	162.77	165.91	169.05	172.19	175.33	178.47	181.61	184.74	3.14
284	15 187.87	191.00	194.13	197.26	200.39	203.52	206.65	209.78	212.91	216.04	3.13
285	210.17	222.30	225.43	228.56	231.69	234.81	237.93	241.05	244.17	247.29	3.12
286	250.41	253.53	256.65	259.77	262.89	266.01	269.13	272.25	275.38	278.47	3.12
287	15 281.58	284.69	287.80	290.91	294.02	297.13	300.24	303.35	306.46	309.57	3.11
288	312.68	315.79	318.90	322.01	325.11	328.21	331.31	334.41	337.51	340.61	3.10
289	348.71	346.81	349.91	353.01	356.11	359.21	362.31	365.40	368.49	371.58	3.10
290	15 374.67	377.76	380.85	383.94	387.03	390.12	393.21	396.30	399.39	402.48	3.09
291	405.57	408.66	411.75	414.84	417.93	421.01	424.09	427.17	430.25	433.33	3.08
292	486.41	489.49	492.57	495.65	498.73	501.81	504.88	507.95	511.02	514.09	3.08
293	15 467.16	470.23	473.30	476.37	479.44	482.51	485.58	488.65	491.72	494.79	3.07
294	497.86	500.93	503.99	507.05	510.11	513.17	516.23	519.29	522.35	525.41	3.06
295	528.47	531.53	534.59	537.65	540.71	543.77	546.83	549.89	552.94	555.99	3.06
296	15 559.04	562.09	565.14	568.19	571.24	574.29	577.34	580.39	583.44	586.49	3.05
297	589.54	592.59	595.64	598.69	601.73	604.77	607.81	610.85	613.89	616.93	3.04
298	619.97	623.01	626.05	629.09	632.13	635.17	638.21	641.25	644.28	647.31	3.04
299	15 650.34	653.37	656.40	659.43	662.46	665.49	668.52	671.55	674.58	677.61	3.03
300	680.64	683.67	686.70	689.72	692.74	695.76	698.78	701.80	704.82	707.84	3.02
301	710.86	713.88	716.90	719.92	722.94	725.96	728.98	731.99	735.02	738.03	3.02
302	15 741.04	744.05	747.06	750.07	753.08	756.09	759.10	762.11	765.12	768.13	3.01
303	771.14	774.15	777.16	780.17	783.18	786.19	789.20	792.20	795.20	798.20	3.01
304	801.20	804.20	807.20	810.20	813.20	816.20	819.20	822.20	825.20	828.20	3.00

305	15 831 20	884 20	887 19	840 15	846 16	843 17	846 16	840 15	832 14	855 13	858 12	2 90
306	861 11	884 10	887 08	870 08	876 07	873 07	876 06	879 06	882 05	885 04	887 03	2 89
307	880 97	885 95	889 93	889 91	902 89	902 88	905 87	898 85	911 83	914 81	917 79	2 88
308	15 920 77	923 75	926 73	929 71	932 68	932 68	935 65	938 62	941 59	944 56	947 53	2 97
309	950 50	953 47	956 44	959 41	962 38	962 38	965 35	968 32	971 29	974 26	977 22	2 97
310	980 18	983 14	986 10	989 06	992 02	992 02	994 98	997 94	*000 90	*003 86	*006 82	2 86
311	16 009 75	012 74	015 70	018 66	021 62	021 62	024 58	027 54	030 49	033 44	036 39	2 96
312	039 34	042 29	045 24	048 19	051 14	051 14	054 09	057 04	060 00	062 94	065 89	2 95
313	068 84	071 79	074 73	077 67	080 61	080 61	083 55	086 49	089 43	092 37	095 31	2 94
314	16 098 25	101 19	104 13	107 07	110 01	110 01	112 95	115 89	118 83	121 77	124 71	2 94
315	127 64	130 57	133 50	136 43	139 36	139 36	142 29	145 22	148 15	151 08	154 01	2 93
316	156 94	159 87	162 80	165 73	168 66	168 66	171 59	174 51	177 43	180 35	183 27	2 93
317	16 186 19	189 11	192 03	194 95	197 87	197 87	200 79	203 71	206 63	209 55	212 47	2 92
318	215 39	218 31	221 23	224 15	227 06	227 06	229 97	232 88	235 79	238 70	241 61	2 91
319	244 52	247 43	250 34	253 25	256 16	256 16	259 07	261 98	264 89	267 80	270 70	2 91
320	16 273 90	276 50	279 40	282 30	285 20	285 20	288 10	291 00	293 90	296 80	299 70	2 90
321	302 90	305 50	308 40	311 30	314 20	314 20	317 10	320 00	322 90	325 79	328 68	2 90
322	331 57	334 46	337 35	340 24	343 13	343 13	346 02	348 91	351 80	354 69	357 58	2 89
323	16 380 47	383 36	386 25	389 14	392 03	392 03	394 92	397 80	399 68	398 56	396 44	2 89
324	389 32	392 20	395 08	397 96	400 84	400 84	403 72	406 60	409 48	412 36	415 24	2 88
325	418 12	421 00	423 88	426 75	429 62	429 62	432 49	435 36	438 23	441 10	443 97	2 87
326	16 446 84	449 71	452 58	455 45	458 32	458 32	461 19	464 06	466 93	469 80	472 67	2 87
327	475 53	478 39	481 25	484 11	486 97	486 97	489 83	492 69	495 55	498 41	501 27	2 86
328	504 15	506 99	509 85	512 71	515 57	515 57	518 43	521 29	524 14	526 99	529 84	2 86
329	16 532 99	535 54	538 39	541 24	544 09	544 09	546 94	549 79	552 64	555 49	558 34	2 85
330	551 19	554 04	556 89	559 74	562 59	562 59	565 44	568 29	571 13	573 97	576 81	2 84
331	580 65	582 49	585 33	588 17	591 01	591 01	593 85	596 69	600 53	603 37	606 21	2 84
332	16 618 05	620 89	623 73	626 57	629 40	629 40	632 23	635 06	637 89	640 72	643 55	2 83
333	646 38	649 21	652 04	654 87	657 70	657 70	660 53	663 36	666 19	669 02	671 85	2 83
334	674 08	677 51	680 34	683 16	685 98	685 98	688 80	691 62	694 44	697 26	700 08	2 82
335	16 702 90	705 72	708 54	711 36	714 18	714 18	717 00	719 82	722 64	725 46	728 28	2 82
336	731 09	733 90	736 71	739 52	742 33	742 33	745 14	747 95	750 76	753 57	756 38	2 81
337	759 19	762 00	764 81	767 62	770 43	770 43	773 24	776 05	778 86	781 67	784 48	2 81
338	16 767 28	790 08	792 88	795 68	798 48	798 48	801 28	804 08	806 88	809 68	812 48	2 80
339	815 28	818 08	820 88	823 68	826 48	826 48	829 28	832 08	834 88	837 68	840 47	2 80
340	843 26	846 05	848 84	851 63	854 42	854 42	857 21	860 00	862 79	865 58	868 37	2 79

Table V—continued.
Altitude Function, A (v).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	A.
f/λ											
341	16 871.16	873.95	876.74	879.53	882.32	885.11	887.90	890.69	893.48	896.26	2.79
342	899.04	901.82	904.60	907.38	910.16	912.94	915.72	918.50	921.28	924.06	2.78
343	929.84	929.62	932.40	935.18	937.96	940.74	943.52	946.29	949.06	951.83	2.78
344	16 954.40	957.37	960.14	962.91	965.68	968.45	971.22	973.99	976.76	979.53	2.77
345	982.30	985.07	987.84	990.61	993.38	996.15	998.91	*001.67	*004.43	*007.19	2.77
346	17 009.95	012.71	015.47	018.23	020.99	023.75	026.51	029.27	032.03	034.79	2.76
347	17 057.55	040.31	043.07	045.83	048.59	051.35	054.10	056.85	059.60	062.35	2.76
348	065.10	067.85	070.60	073.35	076.10	078.85	081.60	084.35	087.10	089.85	2.75
349	092.60	095.35	098.10	100.85	103.60	106.35	109.09	111.83	114.57	117.31	2.75
350	17 130.05	122.79	125.53	128.27	131.01	133.75	136.49	139.23	141.97	144.71	2.74
351	147.45	150.19	152.93	155.67	158.41	161.14	163.87	166.60	169.33	172.06	2.73
352	174.79	177.32	180.25	182.98	185.71	188.44	191.17	193.90	196.63	199.36	2.73
353	17 202.09	204.82	207.55	210.28	213.01	215.73	218.45	221.17	223.89	226.61	2.72
354	229.33	232.05	234.77	237.49	240.21	242.93	245.65	248.37	251.09	253.81	2.72
355	256.53	259.25	261.97	264.69	267.41	270.12	272.83	275.54	278.25	280.96	2.71
356	17 283.67	286.38	289.09	291.80	294.51	297.22	299.93	302.64	305.35	308.06	2.71
357	310.77	313.48	316.19	318.90	321.61	324.32	327.02	329.72	332.42	335.12	2.71
358	337.82	340.52	343.22	345.92	348.62	351.32	354.02	356.72	359.42	362.12	2.70
359	17 364.82	367.52	370.22	372.92	375.62	378.32	381.01	383.70	386.39	389.08	2.70
360	381.77	384.46	387.15	389.84	402.53	405.22	407.91	410.60	413.29	415.98	2.69
361	418.67	421.36	424.05	426.74	429.43	432.12	434.80	437.48	440.16	442.84	2.69
362	17 445.52	448.20	450.88	453.56	456.24	458.92	461.60	464.28	466.96	469.64	2.68
363	472.32	475.00	477.68	480.36	483.04	485.72	488.40	491.08	493.75	496.42	2.68
364	469.09	501.76	504.43	507.10	509.77	512.44	515.11	517.78	520.45	523.12	2.67
365	17 525.79	528.46	531.13	533.80	536.47	539.14	541.81	544.48	547.14	549.80	2.67
366	552.46	555.12	557.78	560.44	563.10	565.76	568.42	571.08	573.74	576.40	2.66
367	579.06	581.72	584.38	587.04	589.70	592.36	595.02	597.68	600.34	603.00	2.66
368	17 605.65	608.31	610.96	613.61	616.26	618.91	621.56	624.21	626.86	629.51	2.65
369	632.16	634.81	637.46	640.11	642.76	645.41	648.06	650.71	653.36	656.01	2.65
370	658.66	661.31	663.96	666.60	669.24	671.88	674.52	677.16	679.80	682.44	2.64

371	17	685	08	687	72	680	36	693	00	695	04	698	28	700	92	703	56	706	20	708	84	2	64	
372		711	42	716	76	719	40	722	03	724	66	726	46	727	59	729	29	732	55	735	18	2	63	
373		737	81	740	44	743	07	745	40	748	33	750	96	752	29	756	22	758	85	761	48			
374																								
375	17	764	11	766	74	769	37	772	00	774	63	777	26	779	88	782	50	785	12	787	74	2	63	
376		810	56	812	18	815	80	818	43	821	06	824	69	827	32	830	95	833	58	836	21	2	62	
377																								
378	17	842	73	845	34	847	95	850	56	853	17	855	78	858	39	861	00	863	61	866	22	2	61	
379		868	83	871	44	874	05	876	66	879	27	881	88	884	49	887	10	889	71	892	32	2	61	
380		894	92	897	52	900	12	902	72	905	32	907	92	910	52	913	12	915	72	918	32	2	60	
381	17	920	42	923	52	926	12	928	72	931	32	933	92	936	52	939	12	941	72	944	32	2	60	
382		946	92	949	52	952	11	954	70	957	29	959	88	962	47	965	06	967	65	970	24	2	59	
383		972	83	975	42	978	01	980	60	983	19	985	78	988	37	990	96	993	55	996	14	2	59	
384	17	998	73	*001	32	*003	91	*006	50	*009	09	*011	67	*014	25	*016	83	*019	41	*021	90	2	58	
385		18	024	37	027	15	029	73	032	31	034	89	037	47	040	05	042	63	045	21	047	79	2	58
386																								
387	18	070	14	073	71	076	28	079	85	082	42	085	99	089	56	092	13	095	70	098	27	2	57	
388		101	54	104	11	106	68	109	55	112	12	114	63	117	26	119	83	122	40	124	97	2	57	
389		127	53	130	10	132	67	135	23	137	79	140	35	142	91	145	47	148	03	150	59	2	56	
390	18	153	15	155	71	158	27	160	83	163	39	165	95	168	51	171	07	173	63	176	19	2	56	
391		178	75	181	31	183	87	186	43	188	98	191	55	194	10	196	65	199	20	201	75	2	55	
392		204	30	206	85	209	40	211	95	214	50	217	05	219	60	222	15	224	70	227	25	2	55	
393	18	229	80	232	35	234	90	237	45	240	00	242	55	245	10	247	65	250	20	252	74	2	55	
394		250	38	253	82	256	36	259	90	262	44	265	98	268	53	270	08	273	60	276	14	2	54	
395		255	08	258	22	261	76	264	30	267	84	270	38	273	92	276	46	279	00	282	54			
396	18	306	08	309	62	311	15	313	68	316	21	318	74	321	27	323	80	326	33	328	86	2	53	
397		331	39	333	92	336	45	338	98	341	51	344	04	346	57	349	10	351	63	354	16	2	53	
398		356	09	359	22	361	75	364	28	366	81	369	34	371	86	374	38	376	90	379	42			
399	18	381	94	384	46	386	98	389	50	392	02	394	54	397	06	399	58	402	10	404	62	2	52	
400		407	14	409	66	412	18	414	70	417	22	419	74	422	26	424	78	427	30	429	81	2	52	
		432	32																					

TABLE VI.

$$(\theta) = \int \sec^2 \theta / \theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \log_e (\sec \theta + \tan \theta).$$

θ .	(θ) .	$\tan \theta$.	θ .	(θ) .	$\tan \theta$.	θ .	(θ) .	$\tan \theta$.
0 0	0·00000	0·00000	24 0	0·43053	0·44523	48 0	1·30863	1·11061
0 30	0·00873	0·00873	24 30	0·47104	0·45573	48 30	1·33818	1·13029
1 0	0·01746	0·01746	25 0	0·48269	0·46631	49 0	1·36863	1·15037
1 30	0·02619	0·02619	25 30	0·49449	0·47698	49 30	1·40001	1·17085
2 0	0·03493	0·03492	26 0	0·50643	0·48773	50 0	1·43236	1·19175
2 30	0·04367	0·04366	26 30	0·51853	0·49858	50 30	1·46574	1·21810
3 0	0·05243	0·05241	27 0	0·53078	0·50953	51 0	1·50020	1·23490
3 30	0·06120	0·06116	27 30	0·54320	0·52057	51 30	1·53579	1·25717
4 0	0·06998	0·06993	28 0	0·55580	0·53171	52 0	1·57257	1·27994
4 30	0·07878	0·07870	28 30	0·56856	0·54296	52 30	1·61060	1·30323
5 0	0·08760	0·08749	29 0	0·58151	0·55431	53 0	1·64995	1·32704
5 30	0·09644	0·09629	29 30	0·59465	0·56577	53 30	1·69070	1·35142
6 0	0·10530	0·10510	30 0	0·60799	0·57735	54 0	1·73291	1·37688
6 30	0·11418	0·11394	30 30	0·62152	0·58904	54 30	1·77667	1·40195
7 0	0·12309	0·12278	31 0	0·63527	0·60086	55 0	1·82207	1·42815
7 30	0·13203	0·13165	31 30	0·64924	0·61280	55 30	1·86919	1·45501
8 0	0·14100	0·14054	32 0	0·66343	0·62487	56 0	1·91815	1·48256
8 30	0·15001	0·14915	32 30	0·67786	0·63767	56 30	1·9·905	1·51084
9 0	0·15904	0·15838	33 0	0·69253	0·64941	57 0	2·02199	1·53986
9 30	0·16812	0·16734	33 30	0·70744	0·66189	57 30	2·07712	1·56969
10 0	0·17724	0·17633	34 0	0·72263	0·67451	58 0	2·13456	1·60033
10 30	0·18639	0·18534	34 30	0·73808	0·68728	58 30	2·19446	1·63185
11 0	0·19560	0·19438	35 0	0·75382	0·70021	59 0	2·25697	1·66428
11 30	0·20485	0·20345	35 30	0·76984	0·71329	59 30	2·32236	1·69766
12 0	0·21415	0·21256	36 0	0·78617	0·72654	60 0	2·39053	1·73235
12 30	0·22350	0·22169	36 30	0·80280	0·73996	60 30	2·46196	1·76749
13 0	0·23290	0·23087	37 0	0·81977	0·75355	61 0	2·53678	1·80405
13 30	0·24237	0·24008	37 30	0·83707	0·76733	61 30	2·61521	1·84177
14 0	0·25189	0·24933	38 0	0·85473	0·78129	62 0	2·69752	1·88073
14 30	0·26147	0·25862	38 30	0·87275	0·79544	62 30	2·78398	1·92098
15 0	0·27112	0·26795	39 0	0·89114	0·80978	63 0	2·87490	1·96261
15 30	0·28084	0·27732	39 30	0·90994	0·82434	63 30	2·97062	2·00569
16 0	0·29063	0·28675	40 0	0·92914	0·83910	64 0	3·07150	2·05030
16 30	0·30049	0·29621	40 30	0·94877	0·85408	64 30	3·17794	2·09654
17 0	0·31043	0·30573	41 0	0·96884	0·86929	65 0	3·29039	2·14451
17 30	0·32045	0·31530	41 30	0·98937	0·88473	65 30	3·40934	2·19430
18 0	0·33055	0·32492	42 0	1·01039	0·90040	66 0	3·53532	2·24604
18 30	0·34074	0·33460	42 30	1·03191	0·91633	66 30	3·66893	2·29984
19 0	0·35102	0·34433	43 0	1·05395	0·93252	67 0	3·81088	2·35585
19 30	0·36139	0·35412	43 30	1·07653	0·94896	67 30	3·96177	2·41422
20 0	0·37185	0·36397	44 0	1·09968	0·96569	68 0	4·12255	2·47500
20 30	0·38242	0·37388	44 30	1·12343	0·98270	68 30	4·29410	2·53865
21 0	0·39309	0·38386	45 0	1·14779	1·00000	69 0	4·47744	2·60509
21 30	0·40387	0·39391	45 30	1·17280	1·01761	69 30	4·67372	2·67462
22 0	0·41476	0·40403	46 0	1·19849	1·03553	70 0	4·88425	2·74748
22 30	0·42577	0·41421	46 30	1·22483	1·05378			
23 0	0·43690	0·42447	47 0	1·25201	1·07237			
23 30	0·44815	0·43481	47 30	1·27991	1·09131			

TABLE VII.

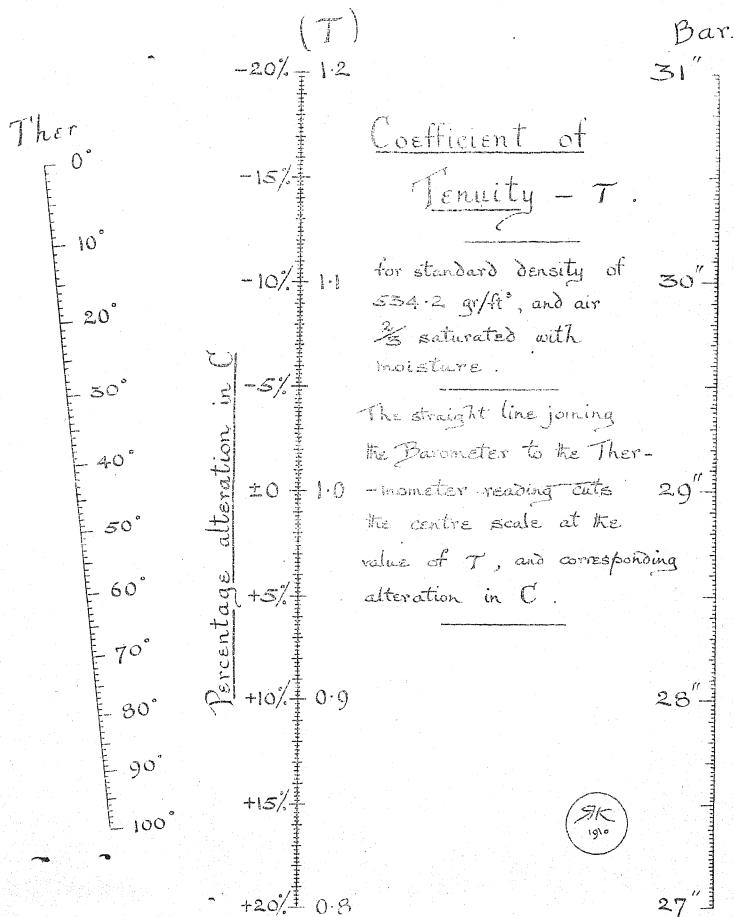
EXTRACT FROM GLAISHER'S HYGROMETRICAL TABLES.

SHOWING the weight in grains of a cubic foot of air at 29-inch barometric height and temperature between 80° and 32° F. Standard density 534.22 grains/ft.³

Difference for 1 inch in Barometer.		Dry Bulb—Degrees F.		Reading of Thermometer—Wet Bulb—Degrees F.																																																								Dry Bulb—Degrees F.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
				80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																		
17.0	80	493.0	493.1	493.2	493.3	493.4	493.5	493.6	493.7	493.8	493.9	494.0	494.1	494.2	494.3	494.4	494.5	494.6	494.7	494.8	494.9	495.0	495.1	495.2	495.3	495.4	495.5	495.6	495.7	495.8	495.9	496.0	496.1	496.2	496.3	496.4	496.5	496.6	496.7	496.8	496.9	497.0	497.1	497.2	497.3	497.4	497.5	497.6	497.7	497.8	497.9	498.0	498.1	498.2	498.3	498.4	498.5	498.6	498.7	498.8	498.9	499.0	499.1	499.2	499.3	499.4	499.5	499.6	499.7	499.8	499.9	500.0	500.1	500.2	500.3	500.4	500.5	500.6	500.7	500.8	500.9	501.0	501.1	501.2	501.3	501.4	501.5	501.6	501.7	501.8	501.9	502.0	502.1	502.2	502.3	502.4	502.5	502.6	502.7	502.8	502.9	503.0	503.1	503.2	503.3	503.4	503.5	503.6	503.7	503.8	503.9	504.0	504.1	504.2	504.3	504.4	504.5	504.6	504.7	504.8	504.9	505.0	505.1	505.2	505.3	505.4	505.5	505.6	505.7	505.8	505.9	506.0	506.1	506.2	506.3	506.4	506.5	506.6	506.7	506.8	506.9	507.0	507.1	507.2	507.3	507.4	507.5	507.6	507.7	507.8	507.9	508.0	508.1	508.2	508.3	508.4	508.5	508.6	508.7	508.8	508.9	509.0	509.1	509.2	509.3	509.4	509.5	509.6	509.7	509.8	509.9	510.0	510.1	510.2	510.3	510.4	510.5	510.6	510.7	510.8	510.9	511.0	511.1	511.2	511.3	511.4	511.5	511.6	511.7	511.8	511.9	512.0	512.1	512.2	512.3	512.4	512.5	512.6	512.7	512.8	512.9	513.0	513.1	513.2	513.3	513.4	513.5	513.6	513.7	513.8	513.9	514.0	514.1	514.2	514.3	514.4	514.5	514.6	514.7	514.8	514.9	515.0	515.1	515.2	515.3	515.4	515.5	515.6	515.7	515.8	515.9	516.0	516.1	516.2	516.3	516.4	516.5	516.6	516.7	516.8	516.9	517.0	517.1	517.2	517.3	517.4	517.5	517.6	517.7	517.8	517.9	518.0	518.1	518.2	518.3	518.4	518.5	518.6	518.7	518.8	518.9	519.0	519.1	519.2	519.3	519.4	519.5	519.6	519.7	519.8	519.9	520.0	520.1	520.2	520.3	520.4	520.5	520.6	520.7	520.8	520.9	521.0	521.1	521.2	521.3	521.4	521.5	521.6	521.7	521.8	521.9	522.0	522.1	522.2	522.3	522.4	522.5	522.6	522.7	522.8	522.9	523.0	523.1	523.2	523.3	523.4	523.5	523.6	523.7	523.8	523.9	524.0	524.1	524.2	524.3	524.4	524.5	524.6	524.7	524.8	524.9	525.0	525.1	525.2	525.3	525.4	525.5	525.6	525.7	525.8	525.9	526.0	526.1	526.2	526.3	526.4	526.5	526.6	526.7	526.8	526.9	527.0	527.1	527.2	527.3	527.4	527.5	527.6	527.7	527.8	527.9	528.0	528.1	528.2	528.3	528.4	528.5	528.6	528.7	528.8	528.9	529.0	529.1	529.2	529.3	529.4	529.5	529.6	529.7	529.8	529.9	530.0	530.1	530.2	530.3	530.4	530.5	530.6	530.7	530.8	530.9	531.0	531.1	531.2	531.3	531.4	531.5	531.6	531.7	531.8	531.9	532.0	532.1	532.2	532.3	532.4	532.5	532.6	532.7	532.8	532.9	533.0	533.1	533.2	533.3	533.4	533.5	533.6	533.7	533.8	533.9	534.0	534.1	534.2	534.3	534.4	534.5	534.6	534.7	534.8	534.9	535.0	535.1	535.2	535.3	535.4	535.5	535.6	535.7	535.8	535.9	536.0	536.1	536.2	536.3	536.4	536.5	536.6	536.7	536.8	536.9	537.0	537.1	537.2	537.3	537.4	537.5	537.6	537.7	537.8	537.9	538.0	538.1	538.2	538.3	538.4	538.5	538.6	538.7	538.8	538.9	539.0	539.1	539.2	539.3	539.4	539.5	539.6	539.7	539.8	539.9	540.0	540.1	540.2	540.3	540.4	540.5	540.6	540.7	540.8	540.9	541.0	541.1	541.2	541.3	541.4	541.5	541.6	541.7	541.8	541.9	542.0	542.1	542.2	542.3	542.4	542.5	542.6	542.7	542.8	542.9	543.0	543.1	543.2	543.3	543.4	543.5	543.6	543.7	543.8	543.9	544.0	544.1	544.2	544.3	544.4	544.5	544.6	544.7	544.8	544.9	545.0	545.1	545.2	545.3	545.4	545.5	545.6	545.7	545.8	545.9	546.0	546.1	546.2	546.3	546.4	546.5	546.6	546.7	546.8	546.9	547.0	547.1	547.2	547.3	547.4	547.5	547.6	547.7	547.8	547.9	548.0	548.1	548.2	548.3	548.4	548.5	548.6	548.7	548.8	548.9	549.0	549.1	549.2	549.3	549.4	549.5	549.6	549.7	549.8	549.9	550.0	550.1	550.2	550.3	550.4	550.5	550.6	550.7	550.8	550.9	551.0	551.1	551.2	551.3	551.4	551.5	551.6	551.7	551.8	551.9	552.0	552.1	552.2	552.3	552.4	552.5	552.6	552.7	552.8	552.9	553.0	553.1	553.2	553.3	553.4	553.5	553.6	553.7	553.8	553.9	554.0	554.1	554.2	554.3	554.4	554.5	554.6	554.7	554.8	554.9	555.0	555.1	555.2	555.3	555.4	555.5	555.6	555.7	555.8	555.9	556.0	556.1	556.2	556.3	556.4	556.5	556.6	556.7	556.8	556.9	557.0	557.1	557.2	557.3	557.4	557.5	557.6	557.7	557.8	557.9	558.0	558.1	558.2	558.3	558.4	558.5	558.6	558.7	558.8	558.9	559.0	559.1	559.2	559.3	559.4	559.5	559.6	559.7	559.8	559.9	560.0	560.1	560.2	560.3	560.4	560.5	560.6	560.7	560.8	560.9	561.0	561.1	561.2	561.3	561.4	561.5	561.6	561.7	561.8	561.9	562.0	562.1	562.2	562.3	562.4	562.5	562.6	562.7	562.8	562.9	563.0	563.1	563.2	563.3	563.4	563.5	563.6	563.7	563.8	563.9	564.0	564.1	564.2	564.3	564.4	564.5	564.6	564.7	564.8	564.9	565.0	565.1	565.2	565.3	565.4	565.5	565.6	565.7	565.8	565.9	566.0	566.1	566.2	566.3	566.4	566.5	566.6	566.7	566.8	566.9	567.0	567.1	567.2	567.3	567.4	567.5	567.6	567.7	567.8	567.9	568.0	568.1	568.2	568.3	568.4	568.5	568.6	568.7	568.8	568.9	569.0	569.1	569.2	569.3	569.4	569.5	569.6	569.7	569.8	569.9	570.0	570.1	570.2	570.3	570.4	570.5	570.6	570.7	570.8	570.9	571.0	571.1	571.2	571.3	571.4	571.5	571.6	571.7	571.8	571.9	572.0	572.1	572.2	572.3	572.4	572.5	572.6	572.7	572.8	572.9	573.0	573.1	573.2	573.3	573.4	573.5	573.6	573.7	573.8	573.9	574.0	574.1	574.2	574.3	574.4	574.5	574.6	574.7	574.8	574.9	575

TABLE VII.^b

CORRECTION FOR T, AND PERCENTAGE CORRECTION
OF BALLISTIC COEFFICIENTS FOR VARIATIONS
OF THERMOMETER AND BAROMETER.



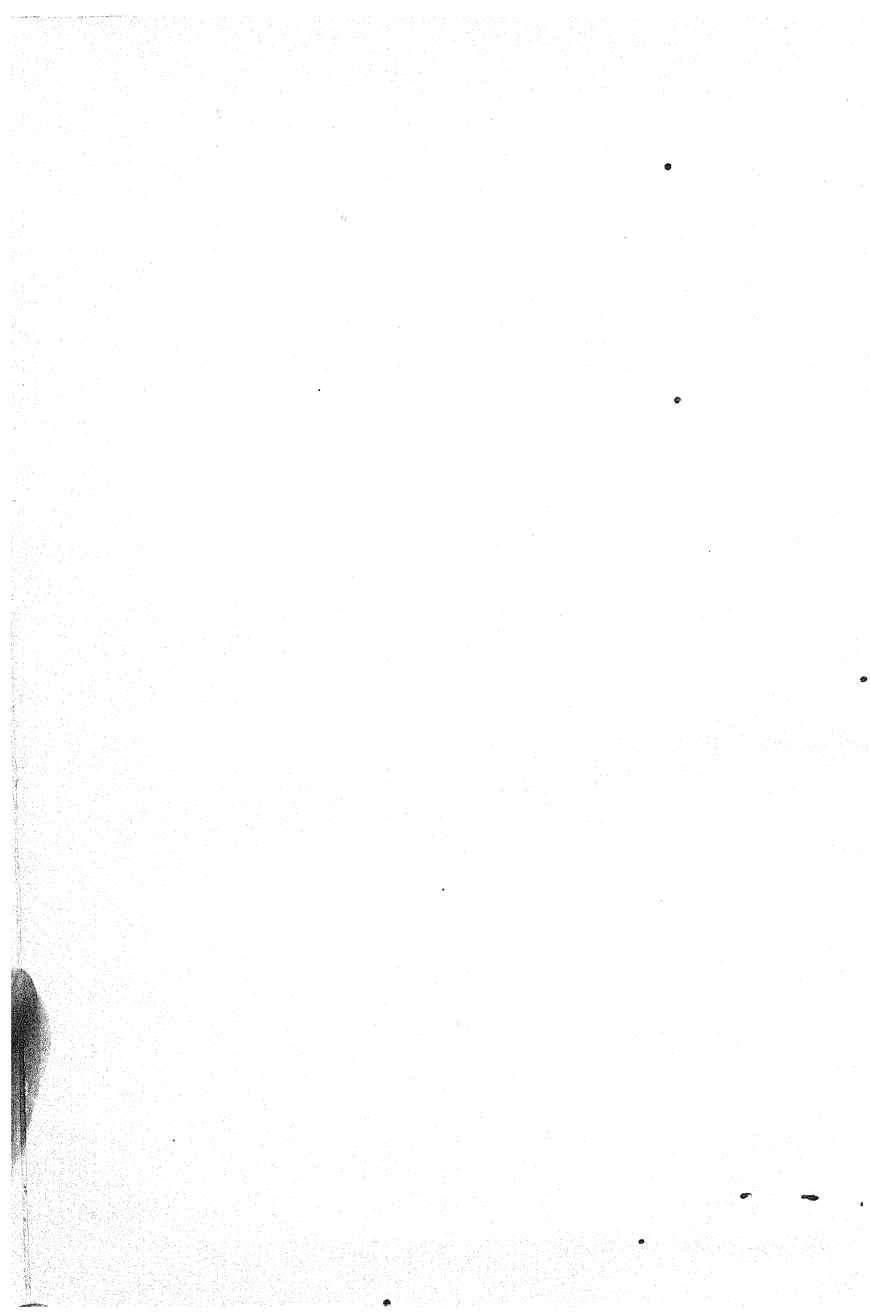


TABLE VIII.

Tenuity correction τ for Temperature and Pressure of Atmosphere
two-thirds saturated with Moisture.

(From the Rev. F. Bashforth's paper, *Proc. R.A.S.*, Vol. XIII, No. 10.)

F.	26 in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +	F.	26 in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +
0	.983	1.021	1.059	1.097	1.134	1.172	38	50	.883	.919	.953	.987	1.021	1.055	34
1	.981	1.019	1.056	1.094	1.132	1.170	38	51	.883	.917	.951	.985	1.019	1.053	34
2	.979	1.017	1.054	1.092	1.130	1.167	38	52	.881	.915	.949	.983	1.017	1.051	34
3	.977	1.015	1.052	1.090	1.127	1.165	38	53	.879	.913	.947	.981	1.015	1.048	34
4	.975	1.012	1.050	1.087	1.125	1.162	38	54	.877	.911	.945	.978	1.012	1.046	34
5	.973	1.010	1.047	1.085	1.122	1.160	37	55	.875	.909	.943	.976	1.010	1.044	34
6	.971	1.008	1.045	1.083	1.120	1.157	37	56	.873	.907	.941	.974	1.008	1.042	34
7	.969	1.006	1.043	1.081	1.118	1.155	37	57	.872	.905	.939	.972	1.006	1.039	34
8	.966	1.004	1.041	1.078	1.115	1.152	37	58	.870	.903	.937	.970	1.004	1.037	34
9	.964	1.001	1.039	1.076	1.113	1.150	37	59	.868	.902	.935	.968	1.002	1.035	33
10	.962	.999	1.036	1.073	1.110	1.147	37	60	.866	.900	.933	.966	1.000	1.033	33
11	.960	.997	1.034	1.071	1.108	1.145	37	61	.864	.898	.931	.964	.998	1.031	33
12	.958	.995	1.032	1.069	1.105	1.142	37	62	.863	.896	.929	.962	.996	1.029	33
13	.956	.993	1.029	1.066	1.103	1.140	37	63	.861	.894	.927	.960	.993	1.027	33
14	.954	.991	1.027	1.064	1.101	1.137	37	64	.859	.892	.925	.958	.991	1.024	33
15	.952	.989	1.025	1.062	1.098	1.135	37	65	.857	.890	.923	.956	.989	1.022	33
16	.950	.986	1.023	1.060	1.096	1.133	37	66	.856	.889	.921	.954	.987	1.020	33
17	.948	.984	1.021	1.057	1.094	1.130	37	67	.854	.887	.919	.952	.985	1.018	33
18	.946	.982	1.019	1.055	1.091	1.128	36	68	.852	.885	.918	.950	.983	1.016	33
19	.944	.980	1.017	1.053	1.089	1.125	36	69	.850	.883	.916	.948	.981	1.014	33
20	.942	.978	1.014	1.051	1.087	1.123	36	70	.849	.881	.914	.946	.979	1.012	33
21	.940	.976	1.012	1.048	1.084	1.121	36	71	.847	.879	.912	.944	.977	1.010	33
22	.938	.974	1.010	1.046	1.082	1.118	36	72	.845	.878	.910	.943	.975	1.008	33
23	.936	.972	1.008	1.044	1.080	1.116	36	73	.843	.876	.908	.941	.973	1.006	32
24	.934	.970	1.006	1.042	1.078	1.114	36	74	.842	.874	.906	.939	.971	1.004	32
25	.932	.968	1.004	1.039	1.075	1.111	36	75	.840	.872	.904	.937	.969	1.002	32
26	.930	.966	1.001	1.037	1.073	1.109	36	76	.838	.870	.902	.935	.967	.999	32
27	.928	.964	.999	1.035	1.071	1.106	36	77	.836	.868	.901	.933	.965	.997	32
28	.926	.962	.997	1.033	1.069	1.104	36	78	.834	.867	.899	.931	.963	.995	32
29	.924	.960	.995	1.031	1.067	1.102	36	79	.833	.865	.897	.929	.961	.993	32
30	.922	.958	.993	1.028	1.064	1.099	36	80	.831	.863	.895	.927	.959	.991	32
31	.920	.956	.991	1.026	1.062	1.097	35	81	.829	.861	.893	.925	.957	.989	32
32	.918	.954	.989	1.024	1.059	1.095	35	82	.827	.859	.891	.923	.955	.987	32
33	.916	.952	.987	1.022	1.057	1.093	35	83	.826	.858	.889	.921	.953	.985	32
34	.914	.950	.985	1.020	1.055	1.090	35	84	.824	.856	.887	.919	.951	.983	32
35	.913	.948	.983	1.018	1.053	1.088	35	85	.822	.854	.885	.917	.949	.980	32
36	.911	.946	.981	1.016	1.051	1.086	35	86	.821	.852	.884	.915	.947	.978	32
37	.909	.944	.979	1.013	1.048	1.083	35	87	.819	.850	.882	.912	.945	.976	32
38	.907	.942	.977	1.011	1.046	1.081	35	88	.817	.848	.880	.911	.943	.974	31
39	.905	.940	.974	1.009	1.044	1.079	35	89	.815	.847	.878	.909	.941	.972	31
40	.903	.938	.972	1.007	1.042	1.077	35	90	.814	.845	.876	.908	.939	.970	31
41	.901	.936	.971	1.005	1.040	1.075	35	91	.812	.843	.874	.905	.937	.968	31
42	.899	.934	.969	1.003	1.038	1.072	35	92	.810	.841	.872	.903	.935	.966	31
43	.898	.932	.967	1.001	1.036	1.070	35	93	.808	.839	.870	.902	.933	.964	31
44	.896	.930	.964	0.999	1.033	1.068	34	94	.806	.837	.868	.900	.931	.962	31
45	.894	.928	.963	0.997	1.031	1.066	34	95	.805	.836	.867	.898	.929	.960	31
46	.892	.926	.960	0.995	1.029	1.063	34	96	.803	.834	.865	.896	.927	.957	31
47	.890	.924	.958	0.993	1.027	1.061	34	97	.801	.832	.863	.893	.924	.955	31
48	.888	.923	.957	.991	1.025	1.059	34	98	.799	.830	.861	.891	.922	.953	31
49	.886	.920	.955	.989	1.023	1.057	34	99	.797	.828	.859	.889	.920	.951	31
50	.884	.919	.953	.987	1.021	1.055	34	100	.796	.826	.857	.887	.918	.949	31

TABLE VIII.

Ballistic Table for Spherical Shot.

(Recalculated by Mr. Hadeock, late R.A., from Bashforth's data, and extended to low velocities.)

For lower velocities this table is provisional, pending the results of further experiments.

v	ΔT	T	AS	S	ΔD	D
300	1.2232	0.0000	366.91	0.00	7.5191	0.0000
310	1.1505	1.2232	356.67	366.91	6.8464	7.5191
320	1.0824	2.3737	346.37	723.58	6.2357	14.3645
330	1.0217	3.4561	337.22	1069.95	5.7113	20.6082
340	0.9647	4.4778	328.01	1407.17	5.2355	26.3145
350	0.9137	5.4425	319.78	1733.18	4.8148	31.5480
360	0.8653	6.3562	311.51	2054.96	4.4333	36.3628
370	0.8218	7.2215	304.07	2366.47	4.0967	40.7961
380	0.7805	8.0423	296.60	2670.54	3.7884	44.8928
390	0.7432	8.8238	289.94	2967.14	3.5147	48.6812
400	0.7076	9.5670	283.05	3256.98	3.2629	52.1960
410	0.6743	10.2746	276.88	3540.03	3.0380	55.4588
420	0.6445	10.9499	270.69	3816.91	2.8303	58.4663
430	0.6161	11.5944	264.51	4087.60	2.6385	61.3271
440	0.5763	12.2095	258.59	4352.11	2.4150	63.9666
450	0.5508	12.7858	247.86	4605.70	2.2575	66.3815
460	0.5265	13.3366	242.20	4853.56	2.1111	68.6390
470	0.5035	13.8631	236.64	5096.76	1.9758	70.7501
480	0.4816	14.3666	231.19	5322.40	1.8506	72.7279
490	0.4609	14.8482	225.94	5543.58	1.7349	74.5765
500	0.4413	15.3091	220.63	5760.42	1.6277	76.3114
510	0.4227	15.7564	215.55	6010.05	1.5285	77.9391
520	0.4060	16.1791	210.61	6225.60	1.4366	79.4676
530	0.3893	16.5781	205.80	6436.21	1.3513	80.9042
540	0.3725	16.9664	201.14	6642.01	1.2722	82.2556
550	0.3575	17.3369	196.61	6843.15	1.1988	83.5277
560	0.3429	17.6964	192.01	7029.76	1.1293	84.7265
570	0.3291	18.0393	187.57	7231.77	1.0648	85.8568
580	0.3157	18.3684	183.11	7419.54	1.0059	86.9206
590	0.3028	18.6841	178.64	7602.45	0.9455	87.9243
600	0.2903	18.9869	174.19	7781.06	0.8925	88.8710
610	0.2796	19.2772	169.95	7955.28	0.8424	89.7665
620	0.2673	19.5568	165.75	8125.23	0.7953	90.6039
630	0.2567	19.8251	161.74	8290.98	0.7516	91.4012
640	0.2467	20.0798	157.92	8452.72	0.7111	92.1528
650	0.2371	20.3255	154.14	8610.64	0.6729	92.8689
660	0.2281	20.5636	150.53	8764.78	0.6374	93.5303
670	0.2196	20.7917	147.09	8915.31	0.6044	94.1742
680	0.2115	21.0112	143.80	9062.40	0.5736	94.7786
690	0.2038	21.2227	140.65	9206.20	0.5449	95.3522
700	0.1966	21.4265	137.63	9346.85	0.5180	95.8971
710	0.1898	21.6231	134.73	9484.48	0.4930	96.4161
720	0.1832	21.8129	131.88	9619.21	0.4692	96.9081
730	0.1770	21.9961	129.22	9751.09	0.4472	97.3773
740	0.1711	22.1731	126.69	9880.61	0.4264	97.8246
750	0.1653	22.3442	123.99	10006.90	0.4066	98.2509
760	0.1600	22.5095	121.67	10130.89	0.3882	98.6575
770	0.1547	22.6696	119.12	10252.46	0.3706	99.0457
780	0.1496	22.8242	116.72	10371.68	0.3539	99.4163
790	0.1447	22.9738	114.36	10488.30	0.3379	99.7702
800	0.1399	23.1185	111.99	10602.60	0.3226	100.1050
810	0.1352	23.2584	109.50	10714.49	0.3078	100.4305
820	0.1308	23.3936	107.07	10823.99	0.2937	100.7383
830	0.1261	23.5242	104.08	10931.06	0.2803	101.0320

Table VIII—continued.
Ballistic Table for Spherical Shot.

v	ΔT	T	ΔS	S	ΔD	D
<i>f/s.</i>						
840	0.1218	23.6603	102.33	11035.74	0.2675	101.3123
850	0.1177	23.7721	100.01	11138.07	0.2553	101.5798
860	0.1137	23.8808	97.76	11238.08	0.2438	101.8361
870	0.1098	24.0035	95.53	11336.34	0.2328	102.0739
880	0.1062	24.1123	93.44	11431.37	0.2225	102.3117
890	0.1026	24.2195	91.35	11524.81	0.2127	102.5342
900	0.0989	24.3221	89.33	11616.16	0.2034	102.7408
910	0.0960	24.4214	87.32	11705.40	0.1945	102.9303
920	0.0928	24.5173	85.37	11792.81	0.1860	103.1148
930	0.0896	24.6101	83.48	11878.18	0.1780	103.2908
940	0.0869	24.6999	81.65	11961.06	0.1704	103.5088
950	0.0840	24.7868	79.88	12043.31	0.1631	103.6792
960	0.0812	24.8708	78.01	12123.14	0.1561	103.8423
970	0.0785	24.9521	76.19	12201.15	0.1493	103.9984
980	0.0759	25.0306	74.43	12277.34	0.1429	104.1477
990	0.0734	25.1065	72.67	12351.77	0.1368	104.2906
1000	0.0709	25.1799	70.87	12424.44	0.1307	104.4274
1010	0.0684	25.2508	69.08	12495.31	0.1249	104.5581
1020	0.0660	25.3192	67.31	12564.39	0.1193	104.6830
1030	0.0636	25.3852	65.56	12631.70	0.1140	104.8023
1040	0.0614	25.4498	63.81	12697.25	0.1088	104.9163
1050	0.0591	25.5102	62.08	12761.06	0.1039	105.0251
1060	0.0570	25.5693	60.42	12823.14	0.0992	105.1290
1070	0.0550	25.6263	58.82	12883.56	0.0948	105.2282
1080	0.0531	25.6813	57.31	12942.38	0.0906	105.3230
1090	0.0513	25.7344	55.89	12999.69	0.0868	105.4136
1100	0.0496	25.7857	54.59	13055.58	0.0832	105.5004
1110	0.0481	25.8353	53.36	13110.17	0.0799	105.5836
1120	0.0466	25.8834	52.21	13163.53	0.0768	105.6635
1130	0.0453	25.9300	51.15	13215.74	0.0739	105.7403
1140	0.0440	25.9753	50.16	13266.89	0.0712	105.8142
1150	0.0428	26.0193	49.23	13317.05	0.0687	105.8854
1160	0.0417	26.0621	48.36	13366.25	0.0663	105.9541
1170	0.0406	26.1038	47.53	13414.63	0.0640	106.0204
1180	0.0396	26.1444	46.73	13462.16	0.0619	106.0844
1190	0.0386	26.1840	45.97	13508.89	0.0600	106.1463
1200	0.0377	26.2226	45.27	13554.86	0.0580	106.2062
1210	0.0369	26.2603	44.61	13600.13	0.0562	106.2642
1220	0.0361	26.2972	44.00	13644.74	0.0545	106.3204
1230	0.0353	26.3333	43.43	13688.74	0.0529	106.3749
1240	0.0346	26.3686	42.87	13732.17	0.0514	106.4278
1250	0.0339	26.4032	42.36	13775.04	0.0500	106.4792
1260	0.0332	26.4371	41.85	13817.40	0.0486	106.5292
1270	0.0326	26.4703	41.39	13859.25	0.0473	106.5778
1280	0.0320	26.5029	40.94	13900.64	0.0461	106.6251
1290	0.0314	26.5349	40.49	13941.58	0.0449	106.6712
1300	0.0308	26.5663	40.04	13982.07	0.0437	106.7161
1310	0.0302	26.5971	39.59	14022.11	0.0425	106.7596
1320	0.0297	26.6273	39.13	14061.70	0.0415	106.8023
1330	0.0291	26.6570	38.75	14100.88	0.0404	106.8438
1340	0.0286	26.6861	38.33	14139.63	0.0394	106.8842
1350	0.0281	26.7147	37.92	14177.96	0.0384	106.9236
1360	0.0276	26.7428	37.52	14215.88	0.0374	106.9620
1370	0.0271	26.7704	37.15	14253.40	0.0365	106.9994
1380	0.0267	26.7975	36.80	14290.55	0.0356	107.0360
1390	0.0262	26.8242	36.45	14327.35	0.0348	107.0715
1400	0.0258	26.8504	36.11	14363.80	0.0340	107.1063
1410	0.0254	26.8762	35.77	14399.91	0.0332	107.1403
1420	0.0250	26.9016	35.43	14435.68	0.0324	107.1735
1430	0.0246	26.9266	35.16	14471.16	0.0317	107.2059

Table VIII—continued.

Ballistic Table for Spherical Shot.

v	ΔT	T	ΔS	S	ΔD	D
<i>f</i> / ₃						
1440	0·0242	26·9512	34·85	14506·32	0·0310	107·2376
1450	0·0239	26·9754	34·84	14541·17	0·0303	107·2636
1460	0·0235	26·9992	34·84	14575·71	0·0296	107·2899
1470	0·0231	27·0227	33·95	14609·95	0·0290	107·3285
1480	0·0228	27·0458	33·89	14643·93	0·0284	107·3575
1490	0·0224	27·0686	33·81	14677·62	0·0278	107·3859
1500	0·0221	27·0910	33·14	14711·03	0·0272	107·4137
1510	0·0218	27·1131	32·85	14744·17	0·0266	107·4409
1520	0·0214	27·1349	32·59	14777·02	0·0260	107·4675
1530	0·0211	27·1563	32·34	14809·61	0·0255	107·4935
1540	0·0208	27·1774	32·06	14841·95	0·0249	107·5190
1550	0·0205	27·1982	31·82	14874·01	0·0244	107·5439
1560	0·0202	27·2187	31·58	14905·83	0·0239	107·5682
1570	0·0200	27·2389	31·33	14937·41	0·0234	107·5922
1580	0·0197	27·2589	31·10	14968·74	0·0230	107·6156
1590	0·0194	27·2786	30·86	14999·84	0·0225	107·6386
1600	0·0191	27·2980	30·64	15030·70	0·0221	107·6611
1610	0·0189	27·3171	30·42	15061·34	0·0216	107·6832
1620	0·0186	27·3360	30·19	15091·76	0·0212	107·7048
1630	0·0184	27·3546	29·99	15121·95	0·0208	107·7260
1640	0·0182	27·3730	29·79	15151·94	0·0204	107·7468
1650	0·0179	27·3912	29·60	15181·73	0·0201	107·7672
1660	0·0177	27·4091	29·38	15211·33	0·0197	107·7873
1670	0·0175	27·4268	29·20	15240·71	0·0193	107·8070
1680	0·0173	27·4443	29·02	15269·91	0·0190	107·8263
1690	0·0171	27·4616	28·84	15298·93	0·0186	107·8453
1700	0·0168	27·4787	28·64	15327·77	0·0183	107·8639
1710	0·0167	27·4955	28·47	15356·41	0·0180	107·8822
1720	0·0165	27·5122	28·31	15384·88	0·0176	107·9002
1730	0·0163	27·5287	28·13	15413·19	0·0173	107·9178
1740	0·0161	27·5450	27·97	15441·32	0·0170	107·9351
1750	0·0159	27·5611	27·81	15469·29	0·0168	107·9521
1760	0·0157	27·5770	27·64	15497·10	0·0165	107·9689
1770	0·0155	27·5927	27·49	15524·74	0·0162	107·9854
1780	0·0154	27·6082	27·33	15552·23	0·0159	108·0016
1790	0·0152	27·6236	27·16	15579·56	0·0156	108·0175
1800	0·0150	27·6388	27·03	15606·72	0·0154	108·0331
1810	0·0148	27·6538	26·87	15633·75	0·0151	108·0485
1820	0·0147	27·6686	26·72	15660·62	0·0149	108·0636
1830	0·0145	27·6833	26·54	15687·34	0·0146	108·0786
1840	0·0143	27·6978	26·40	15713·88	0·0144	108·0931
1850	0·0142	27·7121	26·25	15740·28	0·0141	108·1075
1860	0·0140	27·7263	26·09	15766·53	0·0139	108·1216
1870	0·0139	27·7403	25·93	15792·62	0·0137	108·1355
1880	0·0137	27·7542	25·79	15818·55	0·0135	108·1492
1890	0·0136	27·7679	25·64	15844·34	0·0132	108·1627
1900	0·0134	27·7815	25·48	15869·98	0·0130	108·1759

TABLE IX.

BALLISTIC TABLE
for Flat-Headed Projectiles.

The laws upon which the S Table for flat-headed projectiles has been constructed are deduced from experiments carried out at Shoeburyness in 1904-6, and are as follows:—

$$5000 \text{ f/s} > v > 1556 \text{ f/s,}$$

$$Cr = [6 \cdot 4683473 - 10] v^2.$$

$$1556 \text{ f/s} > v > 1130 \text{ f/s,}$$

$$Cr = [3 \cdot 2763377 - 10] v^3.$$

$$1130 \text{ f/s} > v > 1000 \text{ f/s,}$$

$$Cr = [0 \cdot 2232593 - 10] v^4.$$

$$1000 \text{ f/s} > v > 800 \text{ f/s,}$$

$$Cr = [3 \cdot 2232593 - 10] v^3.$$

$$800 \text{ f/s} > v > 0 \text{ f/s,}$$

$$Cr = [6 \cdot 1263493 - 10] v^2.$$

In the above equations, $Cr = pg = K \left(\frac{v}{1000} \right)^3$. The figures in the brackets are the logarithms of the coefficients of v .

Table IX—continued.
 $s = C[S(V) - S(v)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											+
50	19 080.1	105.0	119.9	134.8	149.6	164.4	179.2	194.0	208.7	223.4	14.8
51	228.1	237.7	267.3	281.9	296.5	311.0	325.5	340.0	354.5	368.9	14.5
52	383.3	397.7	412.0	426.3	440.6	454.8	469.0	483.2	497.4	511.5	14.3
53	19 525.6	529.7	558.8	567.8	581.8	595.8	609.8	623.7	637.6	651.5	14.0
54	54.4	679.2	695.0	706.8	720.6	734.3	748.0	761.7	775.3	788.9	13.7
55	602.5	816.1	820.7	843.2	856.7	870.2	883.7	897.1	910.5	923.9	13.5
56	19 937.3	950.0	968.9	977.2	990.5	*003.7	*016.9	*030.1	*043.3	*056.5	13.2
57	20 069.6	082.7	095.8	108.9	121.9	134.9	147.9	160.9	173.8	186.7	13.0
58	139.6	212.5	225.4	238.2	251.0	263.8	276.6	289.3	302.0	314.7	12.8
59	20 327.4	340.1	352.7	365.3	377.9	390.5	403.0	415.5	428.0	440.5	12.6
60	453.0	468.5	477.9	490.3	502.7	515.1	527.4	539.7	552.0	564.3	12.4
61	576.6	588.8	601.0	613.2	625.4	637.6	649.8	661.9	673.0	686.1	12.2
62	20 698.2	710.2	722.2	734.2	746.2	758.2	770.2	782.1	794.0	805.9	12.0
63	817.8	829.7	841.5	853.3	865.1	876.9	888.7	900.4	912.1	923.8	11.8
64	935.5	947.2	958.9	970.5	982.1	993.7	*005.3	*016.9	*028.4	*039.9	11.6
65	21 051.4	062.9	074.4	085.9	097.3	108.7	120.1	131.5	142.9	154.2	11.4
66	165.5	176.8	188.1	199.4	210.7	221.9	233.1	244.3	255.5	266.7	11.2
67	277.9	289.1	300.2	311.3	322.4	333.5	344.6	355.7	366.7	377.7	11.1
68	21 838.7	399.7	421.7	431.7	432.6	443.5	454.4	465.3	476.2	487.0	10.9
69	497.8	508.6	519.4	530.2	541.0	551.8	562.6	573.3	584.0	594.7	10.8
70	605.4	616.1	626.8	637.4	648.0	658.6	669.2	679.8	690.4	700.9	10.6
71	21 711.4	721.9	732.4	742.9	753.4	763.9	774.4	784.8	795.2	805.6	10.5
72	816.0	826.4	836.8	847.1	857.4	867.7	878.0	888.3	898.6	908.9	10.3
73	919.1	929.3	939.5	949.7	959.9	970.1	980.3	990.5	*000.6	*010.7	10.2
74	22 030.8	080.9	041.0	051.1	061.2	071.2	081.2	091.2	101.2	111.2	10.0
75	121.2	131.2	141.2	151.1	161.0	170.9	180.7	190.6	200.6	210.4	9.9
76	220.2	230.0	239.6	249.6	259.4	269.2	279.0	288.8	298.5	308.2	9.8
77	22 317.9	327.6	337.3	347.0	356.7	366.3	376.0	385.6	395.2	404.8	9.6
78	424.0	434.0	443.6	452.6	462.1	471.6	481.1	490.6	500.1	509.6	9.5
79	509.6	519.1	528.6	538.0	547.4	556.8	566.2	575.6	585.0	594.4	9.4

80	22	603.7	613.0	622.3	631.6	640.9	650.1	659.3	668.5	677.7	686.8	9.2
81		695.9	705.0	714.1	723.2	732.2	741.2	750.2	759.2	768.2	777.1	9.0
82		736.0	734.9	503.8	812.6	821.4	830.2	839.0	847.8	856.5	865.2	8.8
83	22	878.9	889.6	891.2	899.8	908.4	917.0	925.6	934.1	942.6	951.1	8.6
84		936.6	908.1	976.6	985.0	993.4	1001.8	1010.2	1018.5	1026.8	1035.1	8.4
85	23	045.4	061.7	059.9	008.1	076.3	084.5	092.7	100.9	109.0	117.1	8.2
86	23	135.2	133.3	141.4	149.4	157.4	165.4	173.4	181.4	189.3	197.2	8.0
87		205.1	213.0	220.9	228.8	236.6	244.4	252.2	260.0	267.8	275.6	7.8
88		288.3	291.0	298.7	306.4	314.0	321.6	329.2	336.8	344.4	352.0	7.6
89	23	359.6	367.2	374.7	382.2	389.7	397.2	404.7	412.1	419.5	426.9	7.5
90		434.3	441.7	449.1	456.4	463.7	471.0	478.3	485.6	492.9	500.1	7.3
91		507.3	514.5	521.7	528.9	536.1	543.2	550.3	557.4	564.5	571.6	7.1
92	23	578.7	585.8	592.9	599.9	606.9	613.9	620.9	627.9	634.8	641.7	7.0
93		648.6	655.5	662.4	669.3	676.2	683.0	689.8	696.6	703.4	710.2	6.8
94		717.0	723.8	730.6	737.3	744.0	750.7	757.4	764.1	770.8	777.4	6.7
95	23	784.0	790.6	797.2	803.8	810.4	817.0	823.6	830.1	836.6	843.1	6.6
96		849.6	856.1	862.6	869.1	875.5	881.9	888.3	894.7	901.1	907.5	6.4
97		913.8	920.2	926.5	932.8	939.1	945.4	951.7	958.0	964.3	970.5	6.3
98	23	976.7	982.9	989.1	995.3	1001.5	1007.7	1013.9	1020.0	1026.1	1032.2	6.2
99	24	008.8	014.4	020.5	026.6	032.7	038.7	044.8	050.8	056.8	062.8	6.0
100			044.4	050.5	056.6	062.7	068.7	074.8	080.8	086.8	092.8	5.9
101	24	157.7	163.5	169.3	175.1	180.8	186.5	192.2	197.9	203.6	209.3	5.7
102		214.9	220.5	226.1	231.7	237.3	242.9	248.4	253.9	259.4	264.9	5.6
103		270.4	275.9	281.4	286.8	292.2	297.6	303.0	308.4	313.8	319.1	5.4
104	24	324.4	329.7	335.0	340.3	345.6	350.8	356.0	361.2	366.4	371.6	5.2
105		376.8	382.0	387.1	392.2	397.3	402.4	407.5	412.6	417.7	422.7	5.1
106		437.7	442.7	447.7	452.7	457.7	462.7	467.7	472.7	477.7	482.7	5.0
107	24	477.2	482.1	487.0	491.9	496.7	501.5	506.3	511.1	515.9	520.7	4.8
108		535.4	540.3	545.2	549.9	554.6	559.3	563.9	568.6	573.3	577.9	4.7
109		572.2	576.8	581.4	586.0	590.6	595.1	599.7	604.3	608.8	613.3	4.6
110	24	617.8	622.3	626.8	631.3	635.7	640.1	644.5	648.9	653.3	657.7	4.4
111		662.1	666.5	670.9	675.2	679.5	683.8	688.1	692.4	696.7	701.0	4.3
112		705.2	709.5	713.8	718.0	722.2	726.4	730.6	734.8	739.0	743.1	4.2
113	24	747.2	751.4	755.5	759.6	763.7	767.8	771.9	776.0	780.1	784.2	4.1
114		798.3	802.4	806.5	810.5	814.6	818.6	822.7	826.7	830.7	834.7	4.0
115		838.7	842.7	846.7	850.7	854.7	858.7	862.6	866.6	870.5	874.4	4.0

Table IX—continued.
 $= C[S(V) - S(\phi)]$ (for flat-headed projectiles).

v	ϕ	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f_s											+
116	24 868.4	872.3	876.2	880.1	884.0	887.9	891.8	895.7	899.6	903.5	3.9
117	907.4	911.3	915.1	919.0	922.8	926.6	930.5	934.3	938.1	941.9	3.8
118	945.7	949.5	953.3	957.1	960.9	964.7	968.4	972.2	975.9	979.6	3.8
119	24 983.4	987.1	990.8	994.5	998.2	1001.9	1005.6	1009.3	1013.0	1016.7	3.7
120	25 020.4	1023.5	1027.3	1031.0	1034.7	1038.5	1042.2	1046.0	1049.7	1053.4	3.7
121	060.9	060.5	061.1	061.7	062.3	062.9	063.5	064.0	064.6	065.2	3.6
122	093.7	096.3	098.8	101.4	103.4	105.9	108.4	110.4	112.9	114.5	3.5
123	128.0	131.5	135.0	138.5	142.0	145.4	148.9	152.4	155.8	159.3	3.5
124	162.7	166.2	169.6	173.0	176.5	179.9	183.3	186.7	190.1	193.5	3.4
125	25 190.9	200.3	203.7	207.1	210.4	213.8	217.1	220.5	223.8	227.1	3.4
126	230.5	233.8	237.1	240.4	243.7	247.0	250.3	253.6	256.9	260.2	3.3
127	265.5	268.7	272.0	275.3	278.5	281.8	285.1	288.3	291.6	294.9	3.3
128	25 296.1	299.3	302.5	305.7	308.9	312.1	315.3	318.5	321.7	324.9	3.2
129	325.1	331.8	337.6	343.4	349.1	354.9	360.7	366.4	372.1	377.8	3.2
130	359.7	362.8	365.9	369.0	372.1	375.2	378.3	381.4	384.5	387.6	3.1
131	25 390.8	393.0	397.0	400.0	403.1	406.1	409.2	412.2	415.3	418.3	3.1
132	421.4	424.4	427.4	430.4	433.4	436.4	439.5	442.5	445.5	448.5	3.0
133	451.5	454.5	457.5	460.5	463.5	466.5	469.4	472.4	475.3	478.3	3.0
134	25 481.2	484.2	487.1	490.0	492.9	495.9	498.8	501.7	504.6	507.6	2.9
135	510.5	513.4	516.3	519.2	522.1	525.0	527.9	530.8	533.6	536.5	2.9
136	530.3	532.2	535.0	537.9	540.7	543.5	546.4	549.2	552.0	554.9	2.8
137	25 567.7	570.5	573.3	576.1	578.9	581.7	584.5	587.3	590.1	592.9	2.8
138	595.7	598.5	601.3	604.0	606.8	609.5	612.3	615.0	617.8	620.5	2.8
139	625.3	628.0	630.7	633.4	636.1	638.8	641.5	644.2	646.9	649.6	2.7
140	25 650.5	653.2	655.9	658.6	661.3	664.0	666.7	669.4	672.0	674.7	2.7
141	677.7	680.3	682.9	685.5	688.1	690.7	693.3	695.8	698.4	701.0	2.6
142	703.7	706.3	708.9	711.5	714.1	716.7	719.3	721.8	724.4	726.9	2.6
143	25 730.8	733.4	735.0	737.6	740.2	742.8	745.3	747.9	750.4	753.0	2.6
144	755.5	758.1	760.6	763.2	765.7	768.2	770.8	773.3	775.8	778.4	2.5
145	780.9	783.4	785.9	788.4	790.9	793.4	795.9	798.4	800.9	803.4	2.5

146	25 805.9	808.3	810.8	813.3	815.8	818.2	820.7	825.2	825.6	828.1	2.5
147	880.5	883.0	885.4	887.9	890.3	892.7	895.2	897.6	899.0	901.4	2.4
148	854.9	857.3	859.7	862.1	864.5	866.9	869.3	871.7	874.1	876.5	2.4
149	25 578.9	581.2	583.6	586.0	588.3	590.7	593.1	595.4	597.8	900.1	2.4
150	802.5	804.8	807.2	809.5	911.8	914.2	916.6	918.9	921.2	923.6	2.3
151	925.9	928.2	930.6	932.9	935.2	937.5	939.8	942.1	944.4	946.7	2.3
152	25 949.0	951.3	953.6	955.9	958.1	960.4	962.7	965.0	967.3	969.5	2.3
153	971.8	974.1	976.3	978.5	980.8	983.0	985.3	987.5	989.7	991.9	2.3
154	894.2	896.4	898.6	900.8	903.1	905.3	907.5	909.7	911.9	914.1	2.2
155	26 016.3	018.5	020.7	022.9	025.1	027.3	029.5	031.6	033.8	036.0	2.2
156	038.2	040.4	042.5	044.7	046.9	049.1	051.3	053.4	055.6	057.8	2.2
157	060.0	062.1	064.3	066.5	068.6	070.8	072.9	075.1	077.2	079.4	2.2
158	26 081.6	083.7	085.9	088.0	090.2	092.3	094.4	096.6	098.7	100.8	2.1
159	103.0	105.1	107.2	109.4	111.5	113.6	115.8	117.9	120.0	122.1	2.1
160	124.3	126.4	128.5	130.6	132.8	134.9	137.0	139.1	141.2	143.4	2.1
161	26 145.5	147.6	149.7	151.8	153.9	156.1	158.2	160.3	162.4	164.5	2.1
162	166.6	168.7	170.8	172.9	175.0	177.1	179.2	181.3	183.4	185.5	2.1
163	187.6	189.6	191.7	193.8	195.9	198.0	200.1	202.1	204.2	206.3	2.1
164	26 208.4	210.4	212.5	214.6	216.7	218.7	220.8	222.9	224.9	227.0	2.1
165	230.1	231.1	233.2	235.2	237.3	239.3	241.4	243.4	245.5	247.5	2.0
166	249.6	251.6	253.7	255.7	257.8	259.8	261.9	263.9	265.9	268.0	2.0
167	26 270.0	272.0	274.1	276.1	278.1	280.2	282.2	284.2	286.2	288.3	2.0
168	280.3	282.3	284.3	286.4	288.4	290.4	292.4	294.4	296.5	298.5	2.0
169	310.5	312.5	314.5	316.5	318.6	320.6	322.6	324.6	326.6	328.6	2.0
170	26 330.6	332.6	334.6	336.6	338.6	340.6	342.6	344.6	346.6	348.6	2.0
171	350.6	352.6	354.6	356.6	358.6	360.6	362.6	364.6	366.6	368.6	2.0
172	370.4	372.3	374.3	376.3	378.3	380.2	382.2	384.2	386.1	388.1	2.0
173	26 390.1	392.0	394.0	396.0	397.9	399.9	401.8	403.8	405.8	407.7	2.0
174	409.7	411.6	413.6	415.5	417.5	419.4	421.4	423.3	425.3	427.2	1.9
175	429.2	431.1	433.1	435.0	437.0	438.9	440.8	442.8	444.7	446.6	1.9
176	26 448.6	450.5	452.4	454.3	456.3	458.2	460.1	462.0	463.9	465.8	1.9
177	467.8	469.7	471.6	473.5	475.4	477.3	479.2	481.1	483.0	484.9	1.9
178	486.9	488.8	490.7	492.6	494.5	496.5	498.4	500.3	502.2	504.1	1.9
179	26 506.0	507.9	509.8	511.7	513.6	515.5	517.4	519.3	521.2	523.1	1.9
180	528.8	530.7	532.6	534.5	536.4	538.3	540.2	542.1	544.0	545.9	1.9
181	543.9	545.7	547.6	549.5	551.4	553.2	555.1	557.0	558.8	560.7	1.9

Table IX—continued.
 $s = C[S(V) - S(v)]$ (for flat-headed projectiles).

θ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
$f_{1/2}$											
182	26 562.6	564.4	566.3	568.2	570.0	571.9	573.8	575.6	577.5	579.3	1.9
183	581.2	583.0	584.0	586.7	588.6	590.4	592.3	594.1	596.0	597.8	1.8
184	599.7	601.5	603.4	605.2	607.1	608.9	610.8	612.6	614.4	616.3	1.8
185	26 618.1	619.9	621.8	623.6	625.4	627.3	629.1	630.9	632.8	634.6	1.8
186	636.4	638.3	640.1	641.9	643.8	645.6	647.4	649.3	651.1	652.9	1.8
187	654.7	656.6	658.4	660.2	662.0	663.8	665.7	667.5	669.3	671.1	1.8
188	26 672.9	674.7	676.5	678.4	680.2	682.0	683.8	685.6	687.4	689.2	1.8
189	691.0	692.8	694.6	696.4	698.2	700.0	701.8	703.6	705.3	707.1	1.8
190	708.3	710.7	712.5	714.3	716.1	717.8	719.6	721.4	723.2	725.0	1.8
191	26 726.7	728.5	730.3	732.1	733.8	735.6	737.4	739.2	740.9	742.7	1.8
192	744.5	746.3	748.0	749.8	751.6	753.3	755.1	756.8	758.6	760.4	1.8
193	762.2	763.9	765.7	767.5	769.2	771.0	772.8	774.5	776.3	778.0	1.8
194	26 779.8	781.6	783.3	785.1	786.8	788.6	790.3	792.1	793.8	795.6	1.8
195	797.3	799.1	800.8	802.6	804.3	806.0	807.8	809.5	811.2	813.0	1.7
196	814.7	816.4	818.2	819.9	821.6	823.4	825.1	826.8	828.5	830.3	1.7
197	26 829.0	831.7	833.4	835.2	836.9	838.6	840.3	842.0	843.8	845.5	1.7
198	849.2	850.9	852.6	854.3	856.1	857.8	859.5	861.2	862.9	864.6	1.7
199	866.3	868.1	869.8	871.5	873.2	874.9	876.6	878.3	880.0	881.7	1.7
200	26 883.4	885.1	886.8	888.5	890.2	891.9	893.6	895.3	896.9	898.6	1.7
201	900.3	902.0	903.7	905.4	907.1	908.7	910.4	912.1	913.8	915.5	1.7
202	917.2	918.8	920.5	922.2	923.9	925.6	927.2	928.9	930.6	932.3	1.7
203	26 924.0	925.6	927.3	929.0	930.7	932.3	934.0	935.7	937.3	939.0	1.7
204	940.3	942.0	943.7	945.4	947.0	948.7	950.4	952.0	953.7	955.4	1.7
205	960.7	962.3	963.9	965.5	967.1	968.7	970.3	971.9	973.5	975.1	1.7
206	26 963.9	965.6	967.2	968.9	970.5	972.2	973.8	975.5	977.1	978.8	1.7
207	980.4	982.1	983.7	985.4	987.0	988.7	990.3	992.0	993.6	995.3	1.7
208	27 016.8	018.4	020.1	021.7	023.3	025.0	026.6	028.2	029.8	031.5	1.6
209	27 033.1	034.7	036.4	038.0	039.6	041.2	042.9	044.5	046.1	047.7	1.6
210	049.3	051.0	052.6	054.2	055.8	057.4	059.0	060.7	062.3	063.9	1.6
211	065.5	067.1	068.7	070.3	072.0	073.6	075.2	076.8	078.4	080.0	1.6

212	27	081.6	088.2	084.8	086.4	088.0	089.6	091.2	092.8	094.4	096.0	1.6
213	27	067.6	099.2	100.8	102.4	104.0	105.6	107.2	108.7	110.3	111.9	1.6
214	27	113.5	115.1	116.7	118.3	119.8	121.4	123.0	124.6	126.2	127.8	1.6
215	27	129.3	130.9	132.5	134.1	135.7	137.2	138.8	140.4	142.0	143.5	1.6
216	27	145.1	146.7	148.3	149.8	151.4	153.0	154.5	156.1	157.7	159.2	1.6
217	27	160.8	162.4	163.9	165.5	167.1	168.6	170.2	171.8	173.3	174.9	1.6
218	27	176.5	178.0	179.6	181.1	182.7	184.2	185.8	187.3	188.9	190.4	1.5
219	27	192.0	193.5	195.1	196.6	198.2	199.7	201.3	202.8	204.4	205.9	1.5
220	27	207.5	209.0	210.6	212.1	213.7	215.2	216.8	218.3	219.9	221.4	1.5
221	27	222.9	224.5	226.0	227.6	229.1	230.6	232.2	233.7	235.2	236.8	1.5
222	27	238.3	239.8	241.4	242.9	244.4	245.9	247.5	249.0	250.6	252.1	1.5
223	27	253.6	255.1	256.7	258.2	259.7	261.2	262.8	264.3	265.8	267.3	1.5
224	27	268.8	270.4	271.9	273.4	274.9	276.4	277.9	279.5	281.0	282.5	1.5
225	27	284.0	285.5	287.0	288.5	290.1	291.6	293.1	294.6	296.1	297.6	1.5
226	27	289.1	290.6	292.1	293.6	295.1	296.6	298.1	299.6	301.1	302.6	1.5
227	27	314.1	315.6	317.1	318.6	320.1	321.6	323.1	324.6	326.1	327.5	1.5
228	27	329.0	330.5	332.0	333.5	335.0	336.5	337.9	339.4	340.9	342.4	1.5
229	27	343.9	345.4	346.9	348.3	349.8	351.3	352.8	354.3	355.8	357.2	1.5
230	27	358.7	360.2	361.7	363.2	364.6	366.1	367.6	369.1	370.5	372.0	1.5
231	27	373.5	375.0	376.4	377.9	379.4	380.8	382.3	383.8	385.2	386.7	1.5
232	27	388.2	389.6	391.1	392.6	394.0	395.5	396.9	398.4	399.9	401.3	1.5
233	27	402.8	404.2	405.7	407.2	408.6	410.1	411.5	413.0	414.5	415.9	1.5
234	27	417.4	418.8	420.3	421.7	423.2	424.6	426.1	427.5	429.0	430.4	1.4
235	27	431.9	433.3	434.8	436.2	437.7	439.1	440.6	442.0	443.5	444.9	1.4
236	27	446.3	447.8	449.2	450.7	452.1	453.5	455.0	456.4	457.9	459.3	1.4
237	27	451.7	452.2	453.6	455.0	456.5	457.9	459.3	460.7	462.2	463.6	1.4
238	27	457.0	457.5	458.9	459.3	460.7	462.1	463.5	464.9	466.3	467.7	1.4
239	27	462.3	462.8	464.2	465.6	467.0	468.4	469.8	471.2	472.6	474.0	1.4
240	27	467.6	468.1	469.5	470.9	472.3	473.7	475.1	476.5	477.9	479.3	1.4
241	27	472.9	473.4	474.8	476.2	477.6	479.0	480.4	481.8	483.2	484.6	1.4
242	27	478.2	478.7	479.1	480.5	481.9	483.3	484.7	486.1	487.5	488.9	1.4
243	27	483.5	484.0	485.4	486.8	488.2	489.6	491.0	492.4	493.8	495.2	1.4
244	27	488.8	489.3	490.7	492.1	493.5	494.9	496.3	497.7	499.1	500.5	1.4
245	27	494.1	494.6	496.0	497.4	498.8	500.2	501.6	503.0	504.4	505.8	1.4
246	27	499.4	499.9	501.3	502.7	504.1	505.5	506.9	508.3	509.7	511.1	1.4
247	27	504.7	505.2	506.6	508.0	509.4	510.8	512.2	513.6	515.0	516.4	1.4

Table IX—continued.
 $s = C[S(V) - S(\theta)]$ (for flat-headed projectiles).

v	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											+
248	27 615.0	616.4	617.8	619.1	620.5	621.9	623.3	624.6	626.0	627.4	1.4
249	628.7	629.1	630.5	632.9	634.2	635.6	637.0	638.3	639.7	641.1	1.4
250	642.3	643.8	645.2	646.5	647.9	649.2	650.6	651.9	653.3	654.6	1.4
251	27 656.0	657.3	658.7	660.0	661.4	662.7	664.1	665.4	666.8	668.1	1.3
252	662.5	663.8	665.2	667.5	668.8	670.1	671.5	672.8	674.1	675.5	1.3
253	679.0	680.3	681.6	683.0	684.3	685.6	687.0	688.3	689.6	691.0	1.3
254	27 696.3	697.7	699.0	700.3	701.7	703.0	704.3	705.7	707.0	708.3	1.3
255	709.7	711.0	712.3	713.7	715.0	716.3	717.7	719.0	720.3	721.7	1.3
256	723.0	724.3	725.7	727.0	728.3	729.6	731.0	732.3	733.6	735.0	1.3
257	27 736.3	737.6	738.9	740.3	741.6	742.9	744.2	745.6	746.9	748.2	1.3
258	749.5	750.9	752.2	753.5	754.8	756.1	757.5	758.8	760.1	761.4	1.3
259	762.7	764.0	765.3	766.7	768.0	769.3	770.6	771.9	773.2	774.5	1.3
260	27 775.8	777.1	778.4	779.7	781.0	782.3	783.6	784.9	786.2	787.5	1.3
261	788.8	790.1	791.4	792.7	794.0	795.3	796.6	797.9	799.2	800.5	1.3
262	801.8	803.1	804.4	805.7	807.0	808.3	809.6	810.9	812.2	813.5	1.3
263	27 814.8	816.1	817.4	818.7	820.0	821.3	822.6	823.9	825.2	826.5	1.3
264	827.7	829.0	830.3	831.6	832.9	834.2	835.5	836.8	838.0	839.3	1.3
265	840.6	841.9	843.2	844.4	845.7	847.0	848.3	849.5	850.8	852.1	1.3
266	27 853.4	854.6	855.9	857.2	858.4	859.7	861.0	862.3	863.5	864.8	1.3
267	866.6	867.9	869.2	870.5	871.8	873.1	874.4	875.7	876.9	878.2	1.3
268	879.8	880.1	881.3	882.6	883.9	885.1	886.4	887.7	888.9	890.2	1.3
269	27 891.5	892.7	894.0	895.3	896.5	897.8	899.1	900.3	901.6	902.9	1.3
270	904.1	905.4	906.7	907.9	909.2	910.5	911.7	913.0	914.2	915.5	1.3
271	916.7	918.0	919.2	920.5	921.7	923.0	924.2	925.5	926.7	928.0	1.3
272	27 929.2	930.5	931.7	933.0	934.2	935.5	936.7	938.0	939.2	940.5	1.3
273	942.0	943.2	944.5	945.7	946.9	948.1	949.4	950.6	951.7	952.9	1.2
274	954.1	955.4	956.6	957.9	959.1	960.4	961.6	962.8	964.1	965.3	1.2
275	27 966.5	967.8	969.0	970.2	971.5	972.7	973.9	975.2	976.4	977.6	1.2
276	978.9	980.1	981.3	982.6	983.8	985.0	986.2	987.4	988.7	990.0	1.2
277	991.2	992.4	993.6	994.9	996.1	997.3	998.5	999.8	*001.0	*002.2	1.2

275	28	003.4	004.7	005.9	007.1	008.3	009.5	010.8	012.0	013.2	014.4	1.2
276	28	013.6	016.9	018.1	019.3	020.5	021.8	023.0	024.2	025.4	026.6	1.2
280	28	027.8	029.1	030.3	031.5	032.7	033.9	035.1	036.3	037.5	038.7	1.2
281	28	039.9	041.2	042.4	043.6	044.8	046.0	047.2	048.4	049.6	050.8	1.2
282	28	052.0	053.3	054.5	055.7	056.9	058.1	059.3	060.5	061.7	062.9	1.2
283	28	064.1	065.3	066.5	067.7	068.9	070.1	071.3	072.5	073.7	074.9	1.2
284	28	076.1	077.3	078.5	079.7	080.9	082.1	083.3	084.5	085.7	086.9	1.2
285	28	088.0	089.2	090.4	091.6	092.8	094.0	095.2	096.4	097.6	098.8	1.2
286	28	099.9	101.1	102.3	103.5	104.7	105.9	107.1	108.3	109.5	110.7	1.2
287	28	111.8	113.0	114.2	115.4	116.6	117.7	118.9	120.1	121.3	122.5	1.2
288	28	123.6	124.8	126.0	127.2	128.4	129.6	130.7	131.9	133.1	134.3	1.2
289	28	135.4	136.6	137.8	139.0	140.1	141.3	142.5	143.7	144.9	146.0	1.2
290	28	147.2	148.4	149.6	150.7	151.9	153.1	154.3	155.4	156.6	157.8	1.2
291	28	158.9	160.1	161.3	162.4	163.6	164.8	166.0	167.1	168.3	169.5	1.2
292	28	170.6	171.8	172.9	174.1	175.2	176.4	177.6	178.7	179.9	181.0	1.2
293	28	182.2	183.3	184.5	185.7	186.8	188.0	189.1	190.3	191.4	192.6	1.2
294	28	184.5	185.7	186.8	188.0	189.1	190.3	191.4	192.6	193.7	194.9	1.1
295	28	205.3	206.4	207.6	208.7	209.9	211.0	212.2	213.3	214.5	215.6	1.1
296	28	216.8	217.9	219.1	220.2	221.4	222.5	223.7	224.8	225.9	227.1	1.1
297	28	228.3	229.4	230.6	231.7	232.9	234.0	235.2	236.3	237.5	238.6	1.1
298	28	239.7	240.9	242.0	243.2	244.3	245.4	246.6	247.7	248.9	250.0	1.1
299	28	251.1	252.3	253.4	254.6	255.7	256.8	258.0	259.1	260.3	261.4	1.1
300	28	262.5	263.7	264.8	265.9	267.1	268.2	269.3	270.5	271.6	272.7	1.1
301	28	273.8	275.0	276.1	277.2	278.4	279.5	280.6	281.7	282.9	284.0	1.1
302	28	285.1	286.2	287.4	288.5	289.6	290.7	291.9	293.0	294.1	295.2	1.1
303	28	296.4	297.5	298.6	299.7	300.9	302.0	303.1	304.2	305.4	306.5	1.1
304	28	307.6	308.7	309.8	311.0	312.1	313.2	314.3	315.4	316.5	317.7	1.1
305	28	318.8	319.9	321.0	322.1	323.2	324.3	325.5	326.6	327.7	328.8	1.1
306	28	329.9	331.0	332.1	333.2	334.3	335.4	336.5	337.6	338.7	339.9	1.1
307	28	341.0	342.1	343.2	344.3	345.4	346.5	347.6	348.7	349.8	350.9	1.1
308	28	352.0	353.1	354.2	355.3	356.4	357.5	358.6	359.7	360.8	361.9	1.1
309	28	363.0	364.1	365.2	366.3	367.4	368.5	369.6	370.7	371.8	372.9	1.1
310	28	374.0	375.1	376.2	377.3	378.4	379.5	380.6	381.7	382.8	383.9	1.1
311	28	385.0	386.1	387.2	388.3	389.4	390.5	391.6	392.6	393.7	394.8	1.1
312	28	395.9	397.0	398.1	399.2	400.3	401.4	402.5	403.6	404.7	405.7	1.1
313	28	406.8	407.9	409.0	410.0	411.1	412.2	413.3	414.4	415.5	416.5	1.1

Table IX—continued.
 $s = C[S(V) - S(v)]$ (for flat-headed projectiles).

μ	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Δ
f/s											
314	28 417.6	418.7	419.8	420.9	422.0	423.0	424.1	425.2	426.3	427.4	1.1
315	428.4	429.5	430.6	431.7	432.8	433.8	434.9	436.0	437.1	438.1	1.1
316	439.2	440.3	441.4	442.4	443.5	444.6	445.7	446.7	447.8	448.9	1.1
317	28 450.0	451.0	452.1	453.2	454.2	455.3	456.4	457.5	458.5	459.6	1.1
318	460.7	461.7	462.8	463.9	464.9	466.0	467.1	468.2	469.2	470.3	1.1
319	471.3	472.4	473.4	474.5	475.6	476.7	477.7	478.8	479.9	480.9	1.1
320	28 482.0	483.0	484.1	485.2	486.2	487.3	488.4	489.4	490.5	491.6	1.1
321	492.6	493.7	494.8	495.9	496.9	497.9	499.0	500.1	501.1	502.2	1.1
322	503.2	504.3	505.4	506.4	507.5	508.5	509.6	510.6	511.7	512.7	1.1
323	28 513.8	514.8	515.9	516.9	518.0	519.0	520.1	521.1	522.2	523.2	1.0
324	524.3	525.3	526.4	527.4	528.5	529.5	530.6	531.6	532.7	533.7	1.0
325	534.8	535.8	536.9	537.9	539.0	540.0	541.1	542.1	543.2	544.2	1.0
326	28 545.2	546.3	547.3	548.4	549.4	550.4	551.5	552.5	553.6	554.6	1.0
327	556.6	557.7	558.8	559.8	560.9	561.9	563.0	564.0	565.0	566.0	1.0
328	567.0	568.1	569.1	570.2	571.2	572.2	573.3	574.3	575.3	576.3	1.0
329	28 576.4	577.4	578.4	579.5	580.5	581.5	582.6	583.6	584.6	585.7	1.0
330	586.7	587.7	588.8	589.8	590.8	591.9	592.9	593.9	594.9	596.0	1.0
331	597.0	598.0	599.1	600.1	601.1	602.1	603.1	604.2	605.2	606.2	1.0
332	28 607.2	608.3	609.3	610.3	611.4	612.4	613.4	614.4	615.5	616.5	1.0
333	617.5	618.5	619.6	620.6	621.6	622.6	623.7	624.7	625.7	626.7	1.0
334	627.7	628.8	629.8	630.8	631.8	632.8	633.8	634.9	635.9	636.9	1.0
335	28 637.9	638.9	639.9	640.9	642.0	643.0	644.0	645.0	646.0	647.0	1.0
336	648.0	649.0	650.1	651.1	652.1	653.1	654.1	655.1	656.1	657.1	1.0
337	658.1	659.2	660.2	661.2	662.2	663.2	664.2	665.2	666.2	667.2	1.0
338	28 668.2	669.2	670.2	671.2	672.2	673.2	674.2	675.2	676.2	677.2	1.0
339	678.2	679.2	680.2	681.2	682.2	683.2	684.2	685.2	686.2	687.2	1.0
340	688.2	689.2	690.2	691.2	692.2	693.2	694.2	695.2	696.2	697.2	1.0
341	28 698.2	699.2	700.2	701.2	702.2	703.2	704.2	705.2	706.2	707.2	1.0
342	708.2	709.2	710.2	711.2	712.2	713.2	714.2	715.2	716.2	717.2	1.0
343	718.1	719.1	720.1	721.1	722.1	723.1	724.1	725.1	726.1	727.0	1.0

344	28	728	0	730	0	731	0	732	0	733	0	734	0	734	9	735	9	736	9	1	0
345		737	9	739	9	740	8	741	8	742	8	743	8	744	8	745	8	746	7	1	0
346		747	7	749	7	750	7	751	7	752	6	753	6	754	6	755	6	756	6	1	0
347	28	757	5	759	5	760	5	761	5	762	4	763	4	764	4	765	4	766	4	1	0
348		767	3	769	3	770	3	771	2	772	2	773	2	774	2	775	1	776	1	1	0
349		777	1	779	0	780	0	781	0	782	0	783	0	784	0	785	9	786	9	1	0
350	28	786	8	788	8	789	8	790	7	791	7	792	7	793	6	794	6	795	6	1	0
351		796	5	798	5	799	4	800	4	801	4	802	3	803	3	804	3	805	2	1	0
352		806	2	808	1	809	1	810	1	811	0	812	0	813	0	814	9	815	9	1	0
353	28	815	9	817	8	818	8	819	7	820	7	821	7	822	6	823	6	824	5	1	0
354		825	5	827	4	828	4	829	3	830	3	831	3	832	2	833	2	834	1	1	0
355		835	1	837	0	838	0	839	0	840	8	841	8	842	7	843	7	844	7	1	0
356	28	844	6	846	6	847	5	848	5	849	4	850	4	851	3	852	3	853	2	1	0
357		852	2	854	1	855	1	856	1	857	0	858	0	859	8	860	8	861	7	0	9
358		863	7	865	6	866	5	867	5	868	4	869	4	870	3	871	3	872	2	0	9
359	28	873	2	874	1	875	1	876	0	877	9	878	9	879	8	880	8	881	7	0	9
360		882	6	884	5	885	4	886	4	887	4	888	3	889	3	890	2	891	1	0	9
361		892	1	893	0	894	0	895	8	896	8	897	7	898	7	899	6	900	5	0	9
362	28	901	5	903	4	904	3	905	2	906	2	907	1	908	0	909	0	910	9	0	9
363		910	1	911	8	912	7	913	7	914	6	915	5	916	4	917	4	918	3	0	9
364		920	2	921	2	922	1	923	1	924	0	925	9	926	8	927	7	928	7	0	9
365	28	929	6	930	5	931	5	932	4	933	3	934	3	935	2	936	1	937	1	0	9
366		938	9	939	9	940	8	941	7	942	6	943	6	944	5	945	4	946	3	0	9
367		948	2	949	1	950	0	951	9	952	8	953	7	954	7	955	6	956	5	0	9
368	28	957	4	958	3	959	3	960	2	961	1	962	0	963	0	964	8	965	7	0	9
369		966	6	967	6	968	5	969	4	970	3	971	2	972	2	973	0	974	9	0	9
370		975	8	976	8	977	7	978	6	979	5	980	4	981	4	982	3	983	2	0	9
371	28	985	0	985	9	986	9	987	8	988	7	989	6	990	5	991	4	992	3	0	9
372		994	2	995	1	996	0	997	8	998	7	999	6	1000	5	1001	5	1002	4	0	9
373	29	1003	3	1004	2	1005	1	1006	9	1007	8	1008	7	1009	6	1010	5	1011	5	0	9
374	29	1012	4	1013	3	1014	2	1015	1	1016	0	1017	8	1018	7	1019	6	1020	6	0	9
375		1021	5	1022	4	1023	3	1024	2	1025	1	1026	0	1027	8	1028	7	1029	6	0	9
376		1030	5	1031	4	1032	3	1033	2	1034	1	1035	0	1036	9	1037	8	1038	7	0	9
377	29	1039	6	1040	5	1041	4	1042	3	1043	2	1044	1	1045	0	1046	8	1047	7	0	9
378		1049	6	1050	4	1051	3	1052	2	1053	1	1054	0	1055	9	1056	8	1057	7	0	9
379		1057	6	1058	5	1059	4	1060	3	1061	2	1062	1	1063	0	1064	8	1065	7	0	9

Table IX—continued.

[illegible]

(9263)

2 E

TABLE X.

Four Figure Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	Fourth Figure.								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	34
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	37
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	2	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	7	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	5	6	7

Table X—continued.

Four Figure Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	Fourth Figure.								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2	2	3	4	5	6	7		
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2	2	3	4	5	6	7		
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2	2	3	4	5	6	7		
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1	2	3	4	4	5	6	7	
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1	2	3	4	4	5	6	7	
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1	2	3	4	4	5	6	7	
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1	2	3	4	4	5	6	7	
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1	2	3	3	4	4	5	6	7
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1	2	3	3	4	5	5	6	7
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1	2	3	3	4	5	5	6	7
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1	2	3	3	4	5	5	6	7
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1	2	3	3	4	5	5	6	7
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1	2	3	3	4	5	5	6	7
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1	2	3	3	4	4	5	6	7
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1	2	3	3	4	4	5	6	7
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1	2	2	3	4	4	5	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1	2	2	3	4	4	5	5	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1	2	2	3	4	4	5	5	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1	2	2	3	4	4	5	5	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1	2	2	3	4	4	5	5	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1	2	2	3	3	4	5	5	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1	2	2	3	3	4	5	5	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1	2	2	3	3	4	4	5	6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1	2	2	3	3	4	4	5	6
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1	2	2	3	3	4	4	5	6
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1	2	2	3	3	4	4	5	6
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1	2	2	3	3	4	4	5	6
82	9139	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1	2	2	3	3	4	4	5	6
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1	2	2	3	3	4	4	5	6
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1	2	2	3	3	4	4	5	6
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1	2	2	3	3	4	4	5	6
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1	2	2	3	3	4	4	5	6
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1	1	2	2	3	3	4	4	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1	1	2	2	3	3	4	4	5
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1	1	2	2	3	3	4	4	5
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1	1	2	2	3	3	4	4	5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1	1	2	2	3	3	4	4	5
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1	1	2	2	3	3	4	4	5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1	1	2	2	3	3	4	4	5
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1	1	2	2	3	3	4	4	5
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1	1	2	2	3	3	4	4	5
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1	1	2	2	3	3	4	4	5
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1	1	2	2	3	3	4	4	5
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1	1	2	2	3	3	4	4	5
99	9956	9961	9965	9969	9975	9978	9983	9987	9991	9996	0 1	1	2	2	3	3	4	4	5

TABLE XI.

Anti-logarithms or Numbers to Logarithms.

logs.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1235	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	1	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	1	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	1	2	2	2	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	1	2	2	2	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	1	2	2	2	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	1	2	2	2	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	1	2	2	2	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	1	2	2	2	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	1	2	2	2	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	1	2	2	2	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	1	1	2	2	2	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	1	1	2	2	2	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	1	1	2	2	2	3
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	1	1	2	2	2	3
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	1	1	2	2	2	3
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	1	1	2	2	2	3
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	1	1	2	2	2	3
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	1	1	2	2	2	3
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	1	1	2	2	2	3
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	1	1	2	2	2	3
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	1	1	2	2	2	3
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	1	1	2	2	2	3
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	1	1	2	2	2	3
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	0	1	1	1	1	2	2	2	3
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	1	1	2	2	2	2	3
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	1	1	2	2	2	2	3
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	1	1	2	2	2	2	3
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	1	1	2	2	2	2	3
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	1	1	2	2	2	2	3
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	1	1	2	2	2	2	3
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	1	1	2	2	2	2	3
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	1	1	2	2	2	2	3
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	1	1	2	2	2	2	3
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	1	1	2	2	2	2	3
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	1	1	2	2	2	2	3
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	1	1	2	2	2	2	3
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	1	1	2	2	2	2	3
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	1	1	2	2	2	2	3
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	1	1	2	2	2	2	3

Table XI—continued.

Anti-logarithms or Numbers to Logarithms.

logs.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
'50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	5	6	7	8
'51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	6	7	8
'52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	6	7	8
'53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	7	8
'54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	7	8
'55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
'56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
'57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
'58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
'59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
'60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	7	8
'61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
'62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
'63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
'64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
'65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
'66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
'67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	6	7	8	9
'68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
'69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
'70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
'71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
'72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
'73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
'74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
'75	5623	5636	5649	5662	5675	5688	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
'76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
'77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
'78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
'79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
'80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
'81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	13
'82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
'83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
'84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
'85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
'86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
'87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
'88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
'89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
'90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
'91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
'92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	17
'93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
'94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
'95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
'96	9120	9141	9162	9183	9204	9226	9247	9268	9289	9311	2	4	6	8	11	13	15	17	19
'97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
'98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
'99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

TABLE XII.

Natural Sines.

	0'	10'	20'	30'	40'	50'	1	2	3	4	5	6	7	8	9
0°	0000	0029	0058	0087	0116	0145	3	6	9	12	15	17	20	23	26
1	0175	0204	0233	0262	0291	0320	3	6	9	12	15	17	20	23	26
2	0349	0378	0407	0436	0465	0494	3	6	9	12	15	17	20	23	26
3	0523	0552	0581	0610	0640	0669	3	6	9	12	15	17	20	23	26
4	0698	0727	0756	0785	0814	0843	3	6	9	12	15	17	20	23	26
5	0872	0901	0929	0958	0987	1016	3	6	9	12	14	17	20	23	26
6	1045	1074	1103	1132	1161	1190	3	6	9	12	14	17	20	23	26
7	1219	1248	1276	1305	1334	1363	3	6	9	12	14	17	20	23	26
8	1392	1421	1449	1478	1507	1536	3	6	9	12	14	17	20	23	26
9	1564	1593	1622	1650	1679	1708	3	6	9	12	14	17	20	23	26
10	1736	1765	1794	1822	1851	1880	3	6	9	12	14	17	20	23	26
11	1908	1937	1965	1994	2022	2051	3	6	9	11	14	17	20	23	26
12	2079	2108	2136	2164	2193	2221	3	6	9	11	14	17	20	23	26
13	2250	2278	2306	2334	2363	2391	3	6	8	11	14	17	20	23	25
14	2419	2447	2476	2504	2532	2560	3	6	8	11	14	17	20	23	25
15	2588	2616	2644	2672	2700	2728	3	6	8	11	14	17	20	22	25
16	2756	2784	2812	2840	2868	2896	3	6	8	11	14	17	20	22	25
17	2924	2952	2979	3007	3035	3062	3	6	8	11	14	17	19	22	25
18	3090	3118	3145	3173	3201	3228	3	6	8	11	14	17	19	22	25
19	3256	3283	3311	3338	3365	3393	3	5	8	11	14	16	19	22	25
20	3420	3448	3475	3502	3529	3557	3	5	8	11	14	16	19	22	25
21	3584	3611	3638	3665	3692	3719	3	5	8	11	14	16	19	22	24
22	3746	3773	3800	3827	3854	3881	3	5	8	11	14	16	19	21	24
23	3907	3934	3961	3987	4014	4041	3	5	8	11	14	16	19	21	24
24	4067	4094	4120	4147	4173	4200	3	5	8	11	13	16	19	21	24
25	4226	4253	4279	4305	4331	4358	3	5	8	11	13	16	18	21	24
26	4384	4410	4436	4462	4488	4514	3	5	8	10	13	16	18	21	23
27	4540	4566	4592	4617	4643	4669	3	5	8	10	13	15	18	21	23
28	4695	4720	4746	4772	4797	4823	3	5	8	10	13	15	18	20	23
29	4848	4874	4899	4924	4950	4975	3	5	8	10	13	15	18	20	23
30	5000	5025	5050	5075	5100	5125	3	5	8	10	13	15	18	20	23
31	5150	5175	5200	5225	5250	5275	2	5	7	9	12	15	17	20	22
32	5299	5324	5348	5373	5398	5422	2	5	7	9	12	15	17	20	22
33	5446	5471	5495	5519	5544	5568	2	5	7	9	12	15	17	19	22
34	5592	5616	5640	5664	5688	5712	2	5	7	9	12	14	17	19	22
35	5736	5760	5783	5807	5831	5854	2	5	7	10	12	14	17	19	21
36	5878	5901	5925	5948	5972	5995	2	5	7	9	12	14	16	19	21
37	6018	6041	6065	6088	6111	6134	2	5	7	9	12	14	16	18	21
38	6157	6180	6202	6225	6248	6271	2	5	7	9	11	14	16	18	20
39	6293	6316	6338	6361	6383	6406	2	4	7	9	11	13	16	18	20
40	6428	6450	6472	6494	6517	6539	2	4	7	9	11	13	15	18	20
41	6561	6583	6604	6626	6648	6670	2	4	7	9	11	13	15	17	20
42	6691	6713	6734	6756	6777	6799	2	4	6	9	11	13	15	17	19
43	6820	6841	6862	6884	6905	6926	2	4	6	8	11	13	15	17	19
44	6947	6967	6988	7009	7030	7050	2	4	6	8	10	12	15	17	19

Table XII—continued.

Natural Sines.

	0'	10'	20'	30'	40'	50'	1 2 3	4 5 6	7 8 9
45°	7071	7092	7112	7133	7153	7173	2 4 6	8 10 12	14 16 18
46	7193	7214	7234	7254	7274	7294	2 4 6	8 10 12	14 16 18
47	7314	7335	7355	7375	7392	7412	2 4 6	8 10 12	14 16 18
48	7431	7451	7470	7490	7509	7528	2 4 6	8 10 12	13 15 17
49	7547	7566	7585	7604	7623	7642	2 4 6	8 9 11	13 15 17
50	7660	7679	7698	7716	7735	7753	2 4 6	7 9 11	13 15 17
51	7771	7790	7808	7826	7844	7862	2 4 5	7 9 11	13 14 16
52	7880	7898	7916	7934	7951	7969	2 4 5	7 9 11	12 14 16
53	7986	8004	8021	8039	8056	8073	2 3 5	7 9 10	12 14 16
54	8090	8107	8124	8141	8158	8175	2 3 5	7 8 10	12 14 15
55	8192	8208	8225	8241	8258	8274	2 3 5	7 8 10	12 13 15
56	8290	8307	8323	8339	8355	8371	2 3 5	6 8 10	11 13 14
57	8387	8403	8418	8434	8450	8465	2 3 5	6 8 9	11 12 14
58	8480	8496	8511	8526	8542	8557	2 3 5	6 8 9	11 12 14
59	8572	8587	8601	8616	8631	8646	1 3 4	6 7 9	10 12 13
60	8660	8675	8689	8704	8718	8732	1 3 4	6 7 9	10 11 13
61	8746	8760	8774	8788	8802	8816	1 3 4	6 7 8	10 11 12
62	8829	8843	8857	8870	8884	8897	1 3 4	5 7 8	9 11 12
63	8910	8923	8936	8949	8962	8975	1 3 4	5 6 8	9 10 12
64	8988	9001	9013	9026	9038	9051	1 3 4	5 6 8	9 10 11
65	9063	9075	9088	9100	9112	9124	1 2 4	5 6 7	8 10 11
66	9135	9147	9159	9171	9182	9194	1 2 3	5 6 7	8 9 10
67	9205	9216	9228	9239	9250	9261	1 2 3	4 6 7	8 9 10
68	9272	9283	9293	9304	9315	9325	1 2 3	4 5 6	7 9 10
69	9336	9346	9356	9367	9377	9387	1 2 3	4 5 6	7 8 9
70	9397	9407	9417	9426	9436	9446	1 2 3	4 5 6	7 8 9
71	9455	9465	9474	9483	9492	9502	1 2 3	4 5 6	6 7 8
72	9511	9520	9528	9537	9546	9555	1 2 3	4 4 5	6 7 8
73	9563	9572	9580	9588	9596	9605	1 2 2	3 4 5	6 7 7
74	9613	9621	9628	9636	9644	9652	1 2 2	3 4 5	5 6 7
75	9659	9667	9674	9681	9689	9696	1 1 2	3 4 4	5 6 7
76	9703	9710	9717	9724	9730	9737	1 1 2	3 3 4	5 5 6
77	9744	9750	9757	9763	9769	9775	1 1 2	3 3 4	4 5 6
78	9781	9787	9793	9799	9805	9811	1 1 2	2 3 3	4 5 5
79	9816	9822	9827	9833	9838	9843	1 1 2	2 3 3	4 4 5
80	9848	9853	9858	9863	9868	9872	0 1 1	2 2 3	3 4 4
81	9877	9881	9886	9890	9894	9899	0 1 1	2 2 3	3 3 4
82	9903	9907	9911	9914	9918	9922	0 1 1	2 2 2	3 3 3
83	9925	9929	9932	9936	9939	9942	0 1 1	1 2 2	3 3 3
84	9945	9948	9951	9954	9957	9959	0 1 1	1 1 2	2 2 2
85	9962	9964	9967	9969	9971	9974	0 0 1	1 1 1	2 2 2
86	9976	9978	9980	9981	9983	9985	0 0 1	1 1 1	1 1 2
87	9986	9988	9989	9990	9992	9993	0 0 0	1 1 1	1 1 1
88	9994	9995	9996	9997	9997	9998	0 0 0	0 0 0	1 1 1
89	9998	9999	9999	1-0000	1-0000	1-0000	0 0 0	0 0 0	0 0 0

TABLE XIII.

Natural Cosines.

Deg.	0'	10'	20'	30'	40'	50'	1 2 3	4 5 6	7 8 9
0	1.0000	1.0000	1.0000	1.0000	9999	9999	0 0 0	0 0 0	0 0 0
1	9998	9998	9997	9997	9996	9995	0 0 0	0 0 0	0 0 0
2	9994	9993	9992	9990	9989	9988	0 0 0	0 0 0	1 1 1
3	9986	9985	9983	9981	9980	9978	0 0 1	1 1 1	1 1 1
4	9976	9974	9971	9969	9967	9964	0 0 1	1 1 1	1 1 2
5	9962	9959	9957	9954	9951	9948	0 1 1	1 1 1	2 2 2
6	9945	9942	9939	9936	9932	9929	0 1 1	1 2 2	2 2 2
7	9925	9922	9918	9914	9911	9907	0 1 1	2 2 2	3 3 3
8	9903	9899	9894	9890	9886	9881	0 1 1	2 2 2	3 3 3
9	9877	9872	9868	9863	9858	9853	0 1 1	2 2 3	3 3 4
10	9848	9843	9838	9833	9827	9822	1 1 2	2 2 3	3 4 4
11	9816	9811	9805	9799	9793	9787	1 1 2	2 3 3	4 4 5
12	9781	9775	9769	9763	9757	9750	1 1 2	2 3 3	4 5 5
13	9744	9737	9730	9724	9717	9710	1 1 2	3 3 4	4 5 6
14	9708	9696	9689	9681	9674	9667	1 1 2	3 4 4	5 5 6
15	9659	9652	9644	9636	9628	9621	1 2 2	3 4 4	5 6 7
16	9613	9605	9596	9588	9580	9572	1 2 2	3 4 5	5 6 7
17	9563	9555	9546	9537	9528	9520	1 2 3	3 4 5	6 7 7
18	9511	9502	9492	9483	9474	9465	1 2 3	4 4 5	6 7 8
19	9455	9446	9436	9426	9417	9407	1 2 3	4 5 6	6 7 8
20	9397	9387	9377	9367	9356	9346	1 2 3	4 5 6	7 8 9
21	9386	9375	9365	9354	9343	9332	1 2 3	4 5 6	7 8 9
22	9372	9361	9350	9339	9328	9316	1 2 3	4 6 6	7 9 10
23	9356	9344	9332	9321	9309	9297	1 2 3	5 6 7	8 9 10
24	9335	9324	9312	9300	9288	9275	1 2 4	5 6 7	8 9 10
25	9063	9051	9038	9026	9013	9001	1 3 4	5 6 7	8 10 11
26	8988	8975	8962	8949	8936	8923	1 3 4	5 6 8	9 10 11
27	8910	8897	8884	8870	8857	8843	1 3 4	5 7 8	9 11 12
28	8829	8816	8802	8788	8774	8760	1 3 4	6 7 8	9 11 12
29	8746	8732	8718	8704	8689	8675	1 3 4	6 7 8	10 11 12
30	8660	8646	8631	8616	8601	8587	1 3 4	6 7 9	10 11 13
31	8572	8557	8542	8526	8511	8496	2 3 5	6 8 9	10 12 13
32	8480	8465	8450	8434	8418	8403	2 3 5	6 8 9	11 12 14
33	8387	8371	8355	8339	8323	8307	2 3 5	6 8 9	11 12 14
34	8290	8274	8258	8241	8225	8208	2 3 5	7 8 10	11 13 14
35	8192	8175	8158	8141	8124	8107	2 3 5	7 8 10	12 13 15
36	8090	8073	8056	8039	8021	8004	2 3 5	7 9 10	12 14 15
37	7986	7969	7951	7934	7916	7898	2 4 5	7 9 10	12 14 16
38	7880	7862	7844	7826	7808	7790	2 4 5	7 9 11	12 14 16
39	7771	7753	7735	7716	7698	7679	2 4 6	7 9 11	13 14 16
40	7660	7642	7623	7604	7585	7566	2 4 6	8 9 11	13 15 17
41	7547	7528	7509	7490	7470	7451	2 4 6	8 10 11	13 15 17
42	7431	7412	7392	7373	7353	7333	2 4 6	8 10 12	13 15 17
43	7314	7294	7274	7254	7234	7214	2 4 6	8 10 12	14 16 18
44	7193	7173	7153	7132	7112	7092	2 4 6	8 10 12	14 16 18

Table XIII.—continued.

Natural Cosines.

Deg.	0'	10'	20'	30'	40'	50'	1 2 3	4 5 6	7 8 9
45	7071	7050	7030	7009	6988	6967	2 4 6	8 10 12	15 17 19
46	0947	0926	0905	6884	6862	6841	2 4 6	8 11 13	15 17 19
47	0820	0799	0777	6756	6734	6713	2 4 6	9 11 13	15 17 19
48	6691	6670	6648	6626	6604	6583	2 4 7	9 11 13	15 17 19
49	6561	6539	6517	6494	6472	6450	2 4 7	9 11 13	15 17 20
50	6128	6106	6083	6061	6038	6016	2 4 7	9 11 13	15 18 20
51	0293	0271	0248	0225	0202	0180	2 5 7	9 11 13	16 18 20
52	0157	0134	0111	0088	0065	0041	2 5 7	9 12 14	16 18 20
53	6018	5995	5972	5948	5925	5901	2 5 7	9 12 14	16 18 21
54	5878	5854	5831	5807	5783	5760	2 5 7	9 12 14	16 19 21
55	5736	5712	5688	5664	5640	5616	2 5 7	10 12 14	17 19 21
56	5592	5568	5544	5519	5495	5471	2 5 7	10 12 14	17 19 22
57	5446	5422	5398	5373	5348	5324	2 5 7	10 12 15	17 19 22
58	5299	5275	5250	5225	5200	5175	2 5 7	10 12 15	17 20 22
59	5150	5125	5100	5075	5050	5025	3 5 8	10 13 15	17 20 22
60	5000	4975	4950	4924	4899	4874	3 5 8	10 13 15	18 20 23
61	4848	4823	4797	4772	4746	4720	3 5 8	10 13 15	18 20 23
62	4695	4669	4643	4617	4592	4566	3 5 8	10 13 15	18 20 23
63	4540	4514	4488	4462	4436	4410	3 5 8	10 13 15	18 21 23
64	4384	4358	4331	4305	4279	4253	3 5 8	11 13 16	18 21 23
65	4226	4200	4173	4147	4120	4094	3 5 8	11 13 16	18 21 24
66	4067	4041	4014	3987	3961	3934	3 5 8	11 14 16	19 21 24
67	3907	3881	3854	3827	3800	3773	3 5 8	11 14 16	19 21 24
68	3746	3719	3692	3665	3638	3611	3 5 8	11 14 16	19 21 24
69	3584	3557	3529	3502	3475	3448	3 5 8	11 14 16	19 22 24
70	3420	3393	3365	3338	3311	3283	3 5 8	11 14 16	19 22 25
71	3256	3228	3201	3173	3145	3118	3 6 8	11 14 16	19 22 25
72	3090	3062	3035	3007	2979	2952	3 6 8	11 14 17	19 22 25
73	2924	2896	2868	2840	2812	2784	3 6 8	11 14 17	19 22 25
74	2756	2728	2700	2672	2644	2616	3 6 8	11 14 17	20 22 25
75	2588	2560	2532	2504	2476	2447	3 6 8	11 14 17	20 22 25
76	2419	2391	2363	2334	2306	2278	3 6 8	11 14 17	20 23 25
77	2250	2221	2193	2164	2136	2108	3 6 9	11 14 17	20 23 25
78	2079	2051	2022	1994	1965	1937	3 6 9	11 14 17	20 23 26
79	1908	1880	1851	1822	1794	1765	3 6 9	12 14 17	20 23 26
80	1736	1708	1679	1650	1622	1593	3 6 9	12 14 17	20 23 26
81	1564	1536	1507	1478	1449	1421	3 6 9	12 14 17	20 23 26
82	1392	1363	1334	1305	1276	1248	3 6 9	12 14 17	20 23 26
83	1219	1190	1161	1132	1103	1074	3 6 9	12 14 17	20 23 26
84	1045	1016	0987	0958	0929	0901	3 6 9	12 14 17	20 23 26
85	0872	0843	0814	0785	0756	0727	3 6 9	12 15 17	20 23 26
86	0698	0669	0640	0610	0581	0552	3 6 9	12 15 17	20 23 26
87	0523	0494	0465	0436	0407	0378	3 6 9	12 15 17	20 23 26
88	0349	0320	0291	0262	0233	0204	3 6 9	12 15 17	20 23 26
89	0175	0145	0116	0087	0058	0029	3 6 9	12 15 17	20 23 26

TABLE XIV.

Natural Tangents.

	0'	5'	10'	15'	20'	25'	30'	35'	40'	45'	50'	55'	1 2	3 4
0°	0000	0015	0029	0044	0058	0073	0087	0102	0116	0131	0145	0160	3 6	9 12
1	0175	0189	0204	0218	0233	0247	0262	0276	0291	0306	0320	0335	3 6	9 12
2	0349	0364	0378	0393	0407	0422	0437	0451	0466	0480	0495	0509	3 6	9 12
3	0524	0539	0553	0568	0582	0597	0612	0626	0641	0655	0670	0685	3 6	9 12
4	0699	0714	0729	0743	0758	0772	0787	0802	0816	0831	0846	0860	3 6	9 12
5	0875	0890	0904	0919	0934	0948	0963	0978	0992	1007	1022	1036	3 6	9 12
6	1051	1066	1080	1095	1110	1125	1139	1154	1169	1184	1198	1213	3 6	9 12
7	1228	1243	1257	1272	1287	1302	1317	1331	1346	1361	1376	1391	3 6	9 12
8	1405	1420	1435	1450	1465	1480	1495	1509	1524	1539	1554	1569	3 6	9 12
9	1584	1599	1614	1629	1644	1658	1673	1688	1703	1718	1733	1748	3 6	9 12
10	1763	1778	1793	1808	1823	1838	1853	1868	1883	1899	1914	1929	3 6	9 12
11	1944	1959	1974	1989	2004	2019	2035	2050	2065	2080	2095	2110	3 6	9 12
12	2136	2141	2156	2171	2186	2202	2217	2232	2247	2263	2278	2293	3 6	9 12
13	2309	2324	2339	2355	2370	2385	2401	2416	2432	2447	2462	2478	3 6	9 12
14	2493	2509	2524	2540	2555	2571	2586	2602	2617	2633	2648	2664	3 6	9 12
15	2679	2695	2711	2726	2742	2758	2773	2789	2805	2820	2836	2852	3 6	9 13
16	2867	2883	2899	2915	2931	2946	2962	2978	2994	3010	3026	3041	3 6	9 13
17	3057	3073	3089	3106	3121	3137	3153	3169	3185	3201	3217	3233	3 6	10 13
18	3249	3265	3281	3298	3314	3330	3346	3362	3378	3395	3411	3427	3 6	10 13
19	3443	3460	3476	3492	3508	3525	3541	3558	3574	3590	3607	3623	3 6	10 13
20	3640	3656	3673	3689	3706	3722	3739	3755	3772	3789	3805	3822	3 7	10 13
21	3839	3855	3872	3889	3906	3922	3939	3956	3973	3990	4006	4023	3 7	10 13
22	4040	4057	4074	4091	4108	4125	4142	4159	4176	4193	4210	4228	3 7	10 14
23	4245	4262	4279	4296	4314	4331	4348	4365	4383	4400	4417	4435	3 7	10 14
24	4452	4470	4487	4505	4522	4540	4557	4575	4592	4610	4628	4645	4 7	10 14
25	4668	4681	4699	4716	4734	4752	4770	4788	4806	4823	4841	4859	4 7	11 14
26	4877	4895	4913	4931	4950	4968	4986	5004	5022	5040	5059	5077	4 7	11 15
27	5095	5114	5132	5150	5169	5187	5206	5224	5243	5261	5280	5298	4 7	11 15
28	5317	5336	5355	5373	5392	5411	5430	5448	5467	5486	5505	5524	4 8	11 15
29	5543	5562	5581	5600	5619	5639	5658	5677	5696	5715	5735	5754	4 8	12 15
30	5774	5793	5812	5832	5851	5871	5890	5910	5930	5949	5969	5989	4 8	12 16
31	6009	6028	6048	6068	6088	6108	6128	6148	6168	6188	6208	6228	4 8	12 16
32	6249	6269	6289	6310	6330	6350	6371	6391	6412	6432	6453	6473	4 8	12 16
33	6494	6515	6536	6556	6577	6598	6619	6640	6661	6682	6703	6724	4 8	13 17
34	6745	6766	6787	6809	6830	6851	6873	6894	6916	6937	6959	6980	4 9	13 17
35	7002	7024	7046	7067	7089	7111	7133	7155	7177	7199	7221	7243	4 9	13 18
36	7265	7288	7310	7332	7355	7377	7400	7422	7445	7467	7490	7513	5 9	15 18
37	7536	7558	7581	7604	7627	7650	7673	7696	7720	7743	7766	7789	5 9	14 18
38	7813	7836	7860	7883	7907	7931	7954	7978	8002	8026	8050	8074	5 10	14 19
39	8098	8122	8146	8170	8195	8219	8243	8268	8292	8317	8342	8366	5 10	15 20
40	8391	8416	8441	8466	8491	8516	8541	8566	8591	8617	8642	8667	5 10	15 20
41	8693	8718	8744	8770	8796	8821	8847	8873	8899	8925	8952	8978	5 10	16 21
42	9004	9030	9057	9083	9110	9137	9163	9190	9217	9244	9271	9298	5 11	16 21
43	9325	9352	9380	9407	9435	9462	9490	9517	9545	9573	9601	9629	6 11	17 22
44	9657	9685	9713	9742	9770	9798	9827	9856	9884	9913	9942	9971	6 11	17 23

Table XIV—continued.

Natural Tangents.

	0'	5'	10'	15'	20'	25'	30'	35'	40'	45'	50'	55'	1	2	3	4
45°	1.000	0029	0058	0088	0117	0147	0176	0206	0235	0265	0295	0325	6 12	18	24	
46°	1.035	0385	0416	0446	0477	0507	0538	0569	0599	0630	0661	0692	6 12	18	25	
47°	1.072	0755	0786	0818	0850	0881	0913	0945	0977	1003	1041	1074	6 13	19	25	
48°	1.111	1139	1171	1204	1237	1270	1303	1336	1369	1403	1436	1470	7 13	20	26	
49°	1.150	1538	1571	1606	1640	1674	1708	1743	1778	1812	1847	1882	7 14	21	28	
50°	1.192	1953	1988	2024	2059	2095	2131	2167	2203	2239	2276	2312	7 14	22	29	
51°	1.235	2386	2423	2460	2497	2534	2572	2609	2647	2685	2723	2761	8 15	23	30	
52°	1.280	2835	2876	2915	2954	2993	3032	3072	3111	3151	3190	3230	8 16	23	31	
53°	1.327	3311	3351	3392	3432	3473	3514	3555	3597	3638	3679	3722	8 16	25	33	
54°	1.376	3806	3848	3891	3934	3976	4019	4063	4106	4150	4193	4237	9 17	26	34	
55°	1.428	4326	4370	4415	4460	4505	4550	4596	4641	4687	4733	4779	9 18	27	36	
56°	1.483	4872	4919	4966	5013	5061	5108	5156	5204	5253	5301	5350	10 19	29	38	
57°	1.540	5448	5497	5547	5597	5647	5697	5747	5798	5849	5900	5952	10 20	30	40	
58°	1.600	6055	6107	6160	6212	6265	6319	6372	6426	6479	6534	6588	11 21	32	43	
59°	1.664	6698	6753	6808	6864	6920	6977	7033	7090	7147	7205	7262	11 23	34	45	
60°	1.732	7379	7437	7496	7556	7615	7675	7735	7796	7856	7917	7979	12 24	36	48	
61°	1.804	8103	8165	8228	8291	8354	8418	8482	8546	8611	8676	8741	13 26	38	51	
62°	1.881	8873	8940	9007	9074	9142	9210	9278	9347	9416	9486	9556	14 27	41	55	
63°	1.963	9697	9768	9840	9912	9984	0057	0130	0204	0278	0353	0428	15 29	44	58	
64°	2.050	0579	0655	0732	0809	0887	0965	1044	1123	1203	1283	1364	16 31	47	63	
65°	2.144	1527	1600	1692	1775	1859	1943	2028	2113	2199	2286	2373	17 34	51	68	
66°	2.246	2549	2637	2727	2817	2907	2998	3090	3183	3276	3369	3464	18 37	55	74	
67°	2.356	3654	3750	3847	3945	4043	4142	4242	4342	4443	4545	4648	20 40	60	79	
68°	2.475	4855	4960	5065	5172	5279	5386	5495	5605	5715	5826	5938	22 43	65	87	
69°	2.605	6165	6279	6395	6511	6628	6746	6865	6985	7106	7228	7351	24 47	71	95	
70°	2.747	2.760	2.773	2.785	2.798	2.811	2.824	2.837	2.850	2.864	2.877	2.891	3 5	8	10	
71°	2.904	2.918	2.932	2.946	2.960	2.974	2.989	3.003	3.018	3.033	3.047	3.063	3 6	9	11	
72°	3.078	3.093	3.108	3.124	3.140	3.156	3.172	3.188	3.204	3.221	3.237	3.254	3 6	10	13	
73°	3.271	3.288	3.305	3.323	3.340	3.358	3.376	3.394	3.412	3.431	3.450	3.468	4 7	11	14	
74°	3.487	3.507	3.526	3.546	3.566	3.585	3.606	3.626	3.647	3.668	3.689	3.710	4 8	12	16	
75°	3.732	3.754	3.776	3.798	3.821	3.844	3.867	3.890	3.914	3.938	3.962	3.986	5 9	14	19	
76°	4.011	4.036	4.061	4.087	4.113	4.139	4.165	4.192	4.219	4.247	4.275	4.303	5 11	16	21	
77°	4.331	4.360	4.390	4.419	4.449	4.480	4.511	4.542	4.574	4.606	4.638	4.671	6 12	19	25	
78°	4.705	4.739	4.773	4.808	4.843	4.879	4.915	4.952	4.989	5.027	5.066	5.105	7 15	22	29	
79°	5.143	5.185	5.226	5.267	5.309	5.352	5.396	5.440	5.485	5.530	5.576	5.623	9 17	26	35	
80°	5.671	5.720	5.769	5.820	5.871	5.923	5.976	6.030	6.084	6.140	6.197	6.255				
81°	6.314	6.374	6.435	6.497	6.561	6.625	6.691	6.758	6.827	6.897	6.968	7.041				
82°	7.115	7.191	7.269	7.348	7.429	7.511	7.596	7.682	7.770	7.861	7.953	8.048				
83°	8.144	8.248	8.345	8.449	8.556	8.665	8.777	8.892	9.010	9.131	9.255	9.383				
84°	9.514	9.649	9.788	9.931	10.08	10.23	10.39	10.55	10.71	10.88	11.06	11.24				
85°	11.43	11.62	11.83	12.03	12.25	12.47	12.71	12.95	13.20	13.46	13.73	14.01				
86°	14.30	14.61	14.92	15.26	15.60	15.97	16.35	16.75	17.17	17.61	18.07	18.56				
87°	19.08	19.63	20.21	20.82	21.47	22.16	22.90	23.69	24.54	25.45	26.43	27.49				
88°	28.64	29.88	31.24	32.73	34.37	36.18	38.19	40.44	42.96	45.83	49.10	52.88				
89°	57.29	62.50	68.75	76.39	85.94	98.22	114.6	137.5	171.9	229.2	343.8	687.5				

Difference
columns
cease to be
useful.

TABLE XV.

Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Diff.	Cosec.	Tan.	Diff.	Cotan.	Secant.	Diff.	Cosine.	Angle.
0° 0'	Infra. neg.	Infra.	Infra.	Infra. neg.	Infra.	Infra.	10-0000	0	10-0000	90° 0'
0 1	6-4637	3010	13-5363	6-4637	3010	13-5363	10-0000	0	10-0000	89 59
0 2	6-7648	1761	13-2352	6-7648	1761	13-2352	10-0000	0	10-0000	89 58
0 3	6-9408	1249	13-0592	6-9408	1249	13-0592	10-0000	0	10-0000	89 57
0 4	7-0658	969	12-9342	7-0658	969	12-9342	10-0000	0	10-0000	89 56
0 5	7-1627	792	12-8373	7-1627	792	12-8373	10-0000	0	10-0000	89 55
0 6	7-2419	669	12-7581	7-2419	669	12-7581	10-0000	0	10-0000	89 54
0 7	7-3068	512	12-6832	7-3068	512	12-6832	10-0000	0	10-0000	89 53
0 8	7-4637	414	12-5963	7-4637	414	12-5363	10-0000	0	10-0000	89 52
0 9	7-5429	348	12-4571	7-5429	348	12-4571	10-0000	0	10-0000	89 51
0 10	7-6099	300	12-3901	7-6099	300	12-3901	10-0000	0	10-0000	89 50
0 11	7-6678	263	12-3322	7-6678	263	12-3322	10-0000	0	10-0000	89 49
0 12	7-7190	235	12-2810	7-7190	235	12-2810	10-0000	0	10-0000	89 48
0 13	7-7648	212	12-2352	7-7648	212	12-2352	10-0000	0	10-0000	89 47
0 14	7-8061	193	12-1939	7-8061	193	12-1939	10-0000	0	10-0000	89 46
0 15	7-8439	177	12-1561	7-8439	177	12-1561	10-0000	0	10-0000	89 45
0 16	7-8787	164	12-1213	7-8787	164	12-1213	10-0000	0	10-0000	89 44
0 17	7-9109	152	12-0891	7-9109	152	12-0891	10-0000	0	10-0000	89 43
0 18	7-9408	137	12-0592	7-9408	137	12-0591	10-0000	0	10-0000	89 42
0 19	8-0078	118	12-0222	8-0078	118	12-0222	10-0000	0	10-0000	89 41
0 20	8-0658	104	11-9842	8-0658	104	11-9842	10-0000	0	10-0000	89 40
0 21	8-1169	93	11-9431	8-1169	93	11-9431	10-0000	0	10-0000	89 39
0 22	8-1627	84	11-8973	8-1627	84	11-8973	10-0000	0	10-0000	89 38
0 23	8-2041	77	11-8559	8-2041	77	11-8559	10-0000	0	10-0000	89 37
0 24	8-2419	70	11-8181	8-2419	70	11-8181	10-0000	0	10-0000	89 36
0 25	8-2766	65	11-7834	8-2766	65	11-7834	10-0000	0	10-0000	89 35
0 26	8-3089	60	11-7512	8-3089	60	11-7512	10-0000	0	10-0000	89 34
0 27	8-3388	56	11-7212	8-3388	56	11-7212	10-0000	0	10-0000	89 33
0 28	8-3668	53	11-6932	8-3668	53	11-6932	10-0000	0	10-0000	89 32
0 29	8-3931	50	11-6689	8-3931	50	11-6689	10-0000	0	10-0000	89 31
0 30	8-4179	46	11-6481	8-4179	46	11-6481	10-0000	0	10-0000	89 30
0 31	8-4637	42	11-6302	8-4637	42	11-6302	10-0000	0	10-0000	89 29
0 32	8-5000	39	11-6150	8-5000	39	11-6150	10-0000	0	10-0000	89 28
0 33	8-5429	35	11-5572	8-5429	35	11-5572	10-0000	0	10-0000	89 27
0 34	8-5776	32	11-5224	8-5776	32	11-5224	10-0000	0	10-0000	89 26
0 35	8-6097	30	11-5003	8-6097	30	11-5003	10-0000	0	10-0000	89 25
0 36	8-6397	28	11-4603	8-6397	28	11-4603	10-0000	0	10-0000	89 24
0 37	8-6677	26	11-4323	8-6677	26	11-4323	10-0000	0	10-0000	89 23
0 38	8-6940	25	11-4069	8-6940	25	11-4069	10-0000	0	10-0000	89 22
0 39	8-7188	24	11-3812	8-7188	24	11-3812	10-0000	0	10-0000	89 21
0 40	8-7423	22	11-3577	8-7423	22	11-3577	10-0000	0	10-0000	89 20
0 41	8-7645	21	11-3355	8-7645	21	11-3355	10-0000	0	10-0000	89 19
0 42	8-7857	20	11-3143	8-7857	20	11-3143	10-0000	0	10-0000	89 18
0 43	8-8059	19	11-2941	8-8059	19	11-2941	10-0000	0	10-0000	89 17
0 44	8-8251	18	11-2749	8-8251	18	11-2749	10-0000	0	10-0000	89 16
0 45	8-8446	18	11-2564	8-8446	18	11-2564	10-0000	0	10-0000	89 15
0 46	8-8613	17	11-2387	8-8613	17	11-2387	10-0000	0	10-0000	89 14
0 47	8-8759	16	11-2217	8-8759	16	11-2217	10-0000	0	10-0000	89 13
0 48	8-8946	16	11-2054	8-8946	16	11-2054	10-0000	0	10-0000	89 12
0 49	8-9104	15	11-1896	8-9104	15	11-1896	10-0000	0	10-0000	89 11
0 50	8-9256	15	11-1744	8-9256	15	11-1744	10-0000	0	10-0000	89 10
0 51	8-9403	14	11-1597	8-9403	14	11-1597	10-0000	0	10-0000	89 09
0 52	8-9545	14	11-1455	8-9545	14	11-1455	10-0000	0	10-0000	89 08
0 53	8-9682	13	11-1318	8-9682	13	11-1318	10-0000	0	10-0000	89 07
0 54	8-9816	13	11-1184	8-9816	13	11-1184	10-0000	0	10-0000	89 06
0 55	8-9949	13	11-1055	8-9949	13	11-1055	10-0000	0	10-0000	89 05
0 56	9-0070	12	11-0930	9-0070	12	11-0930	10-0000	0	10-0000	89 04
0 57							10-0000	0	10-0000	89 03
0 58							10-0000	0	10-0000	89 02
0 59							10-0000	0	10-0000	89 01
0 60							10-0000	0	10-0000	89 00
	Cosine.	Diff. for 1'	Secant.	Cotan.	Diff. for 1'	Tan.	Cosec.	Diff. for 1'	Sine.	Angle.

Table XV.—continued.

Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Dif.	Cosec.	Tan.	Dif.	Cotan.	Secant.	Dif.	Cosine.	...
6° 0'	9°0192	12	10°9808	9°0216	12	10°9784	10°0024	0	9°9976	84° 0'
6 10	9°0311	12	10°9689	9°0336	12	10°9664	10°0025	0	9°9975	83 50
6 20	9°0426	11	10°9574	9°0452	11	10°9547	10°0027	0	9°9973	83 40
6 30	9°0539	11	10°9461	9°0567	11	10°9433	10°0028	0	9°9972	83 30
6 40	9°0648	11	10°9352	9°0678	11	10°9322	10°0029	0	9°9971	83 20
6 50	9°0755	10	10°9245	9°0786	11	10°9214	10 0031	0	9°9969	83 10
7 0	9°0859	10	10°9141	9°0891	10	10°9109	10°0032	0	9°9968	83 0
7 10	9°0961	10	10°9039	9°0995	10	10°9005	10°0034	0	9°9966	82 50
7 20	9°1060	10	10°8940	9°1096	10	10°8904	10°0036	0	9°9964	82 40
7 30	9°1157	10	10°8843	9°1194	10	10°8806	10°0037	0	9°9963	82 30
7 40	9°1252	9	10°8748	9°1291	9	10°8709	10°0039	0	9°9961	82 20
7 50	9°1345	9	10°8655	9°1385	9	10°8615	10°0041	0	9°9959	82 10
8 0	9°1436	9	10°8564	9°1478	9	10°8522	10°0042	0	9°9958	82 0
8 10	9°1525	9	10°8475	9°1569	9	10°8431	10°0044	0	9°9956	81 50
8 20	9°1612	9	10°8388	9°1658	9	10°8342	10°0046	0	9°9954	81 40
8 30	9°1697	83	10°8303	9°1745	85	10°8255	10°0048	2	9°9952	81 30
8 40	9°1783	78	10°8207	9°1831	80	10°8203	10°0054	2	9°9946	81 0
8 50	9°1876	74	10°8224	9°1926	76	10°8176	10°0060	2	9°9940	80 30
9 0	9°1967	70	10°8139	9°2019	72	10°8137	10°0066	2	9°9934	80 0
10 0	9°2056	67	10°8054	9°2110	69	10°8054	10°0073	2	9°9927	79 30
10 10	9°2143	64	10°7971	9°2200	66	10°7971	10°0081	2	9°9919	79 0
11 0	9°2229	61	10°7889	9°2288	63	10°7889	10°0088	3	9°9912	78 30
11 10	9°2313	58	10°7808	9°2375	61	10°7808	10°0096	3	9°9904	78 0
11 20	9°2396	56	10°7728	9°2461	59	10°7728	10°0104	3	9°9896	77 30
11 30	9°2478	54	10°7649	9°2546	57	10°7649	10°0113	3	9°9887	77 0
11 40	9°2559	52	10°7571	9°2630	55	10°7571	10°0122	3	9°9878	76 30
11 50	9°2639	50	10°7494	9°2713	53	10°7494	10°0131	3	9°9869	76 0
12 0	9°2718	48	10°7418	9°2795	51	10°7418	10°0141	3	9°9859	75 30
12 10	9°2796	46	10°7343	9°2876	50	10°7343	10°0151	3	9°9849	75 0
12 20	9°2873	45	10°7269	9°2956	49	10°7269	10°0161	3	9°9839	74 30
12 30	9°2949	43	10°7196	9°3035	47	10°7196	10°0172	4	9°9828	74 0
12 40	9°3024	42	10°7124	9°3113	46	10°7124	10°0184	4	9°9805	73 0
12 50	9°3098	41	10°7053	9°3190	45	10°7053	10°0218	4	9°9782	72 0
13 0	9°3171	40	10°6983	9°3266	44	10°6983	10°0243	4	9°9757	71 0
13 10	9°3243	39	10°6914	9°3341	43	10°6914	10°0270	5	9°9730	70 0
13 20	9°3314	38	10°6846	9°3415	42	10°6846	10°0298	5	9°9702	69 0
13 30	9°3384	37	10°6779	9°3488	41	10°6779	10°0328	5	9°9672	68 0
13 40	9°3453	36	10°6713	9°3560	40	10°6713	10°0360	5	9°9640	67 0
13 50	9°3521	35	10°6648	9°3631	39	10°6648	10°0393	6	9°9607	66 0
14 0	9°3588	34	10°6584	9°3701	38	10°6584	10°0427	6	9°9573	65 0
14 10	9°3654	33	10°6521	9°3770	37	10°6521	10°0463	6	9°9537	64 0
14 20	9°3719	32	10°6459	9°3838	36	10°6459	10°0501	7	9°9499	63 0
14 30	9°3783	31	10°6398	9°3905	35	10°6398	10°0541	7	9°9459	62 0
14 40	9°3846	30	10°6338	9°3971	34	10°6338	10°0582	7	9°9418	61 0
14 50	9°3908	29	10°6279	9°4036	33	10°6279	10°0625	7	9°9375	60 0
15 0	9°3969	28	10°6221	9°4100	32	10°6221	10°0669	8	9°9331	59 0
15 10	9°4029	27	10°6164	9°4163	31	10°6164	10°0715	8	9°9284	58 0
15 20	9°4088	26	10°6108	9°4225	30	10°6108	10°0764	8	9°9236	57 0
15 30	9°4146	25	10°6053	9°4286	29	10°6053	10°0814	9	9°9188	56 0
15 40	9°4203	24	10°6000	9°4346	28	10°6000	10°0866	9	9°9134	55 0
15 50	9°4259	23	10°5948	9°4405	27	10°5948	10°0920	9	9°9080	54 0
16 0	9°4314	22	10°5897	9°4463	26	10°5897	10°0977	10	9°9023	53 0
16 10	9°4368	21	10°5847	9°4520	25	10°5847	10°1036	10	9°8965	52 0
16 20	9°4421	20	10°5798	9°4576	24	10°5798	10°1096	10	9°8905	51 0
16 30	9°4473	19	10°5750	9°4631	23	10°5750	10°1157	11	9°8843	50 0
16 40	9°4524	18	10°5703	9°4685	22	10°5703	10°1222	11	9°8778	49 0
16 50	9°4574	17	10°5657	9°4738	21	10°5657	10°1289	12	9°8711	48 0
17 0	9°4623	16	10°5612	9°4790	20	10°5612	10°1359	12	9°8641	47 0
17 10	9°4671	15	10°5568	9°4841	19	10°5568	10°1431	12	9°8569	46 0
17 20	9°4718	14	10°5525	9°4891	18	10°5525	10°1505	13	9°8495	45 0
17 30	9°4764	13	10°5483	9°4940	17	10°5483				
17 40	9°4809	12	10°5442	9°4988	16	10°5442				
17 50	9°4853	11	10°5402	9°5035	15	10°5402				
18 0	9°4896	10	10°5363	9°5081	14	10°5363				
18 10	9°4938	9	10°5325	9°5126	13	10°5325				
18 20	9°4979	8	10°5288	9°5170	12	10°5288				
18 30	9°5019	7	10°5252	9°5213	11	10°5252				
18 40	9°5058	6	10°5217	9°5255	10	10°5217				
18 50	9°5096	5	10°5183	9°5296	9	10°5183				
19 0	9°5133	4	10°5150	9°5336	8	10°5150				
19 10	9°5169	3	10°5118	9°5375	7	10°5118				
19 20	9°5204	2	10°5087	9°5413	6	10°5087				
19 30	9°5238	1	10°5057	9°5450	5	10°5057				
19 40	9°5271	0	10°5028	9°5486	4	10°5028				
19 50	9°5303	0	10°5000	9°5521	3	10°5000				
20 0	9°5334	0	10°4973	9°5555	2	10°4973				
20 10	9°5364	0	10°4947	9°5588	1	10°4947				
20 20	9°5393	0	10°4922	9°5620	0	10°4922				
20 30	9°5421	0	10°4898	9°5651	0	10°4898				
20 40	9°5448	0	10°4875	9°5681	0	10°4875				
20 50	9°5474	0	10°4853	9°5710	0	10°4853				
21 0	9°5500	0	10°4832	9°5738	0	10°4832				
21 10	9°5525	0	10°4812	9°5765	0	10°4812				
21 20	9°5549	0	10°4793	9°5791	0	10°4793				
21 30	9°5572	0	10°4775	9°5816	0	10°4775				
21 40	9°5594	0	10°4758	9°5840	0	10°4758				
21 50	9°5616	0	10°4742	9°5863	0	10°4742				
22 0	9°5637	0	10°4727	9°5885	0	10°4727				
22 10	9°5657	0	10°4713	9°5906	0	10°4713				
22 20	9°5676	0	10°4700	9°5926	0	10°4700				
22 30	9°5694	0	10°4688	9°5945	0	10°4688				
22 40	9°5711	0	10°4677	9°5963	0	10°4677				
22 50	9°5728	0	10°4667	9°5980	0	10°4667				
23 0	9°5744	0	10°4658	9°5996	0	10°4658				
23 10	9°5759	0	10°4650	9°6011	0	10°4650				
23 20	9°5773	0	10°4643	9°6025	0	10°4643				
23 30	9°5786	0	10°4637	9°6038	0	10°4637				
23 40	9°5798	0	10°4632	9°6050	0	10°4632				
23 50	9°5809	0	10°4628	9°6061	0	10°4628				
24 0	9°5819	0	10°4625	9°6071	0	10°4625				
24 10	9°5828	0	10°4623	9°6080	0	10°4623				
24 20	9°5836	0	10°4622	9°6088	0	10°4622				
24 30	9°5843	0	10°4622	9°6095	0	10°4622				
24 40	9°5849	0	10°4623	9°6101	0	10°4623				
24 50	9°5854	0	10°4625	9°6106	0	10°4625				
25 0	9°5858	0	10°4628	9°6110	0	10°4628				
25 10	9°5861	0	10°4632	9°6113	0	10°4632				
25 20	9°5863	0	10°4637	9°6115	0	10°4637				
25 30	9°5864	0	10°4643	9°6116	0	10°4643				
25 40	9°5864	0	10°4650	9°6116	0	10°4650				
25 50	9°5863	0	10°4658	9°6115	0	10°4658				
26 0	9°5861	0	10°4667	9°6113	0	10°4667				
26 10	9°5858	0	10°4677	9°6110	0	10°4677				
26 20	9°5854	0	10°4688	9°6106	0	10°4688				
26 30	9°5849	0	10°4700	9°6101	0	10°4700				
26 40	9°5843	0	10°4713	9°6095	0	10°4713				
26 50	9°5836	0	10°4727	9°6088	0	10°4727				
27 0	9°5828	0	10°4742	9°6080	0	10°4742				
27 10	9°5819	0	10°4758	9°6071	0	10°4758				
27 20	9°5809	0</								

TABLE XVI.

Conversion of Measures.

(Chiefly based on data contained in Col. Noble's Useful Tables.)

Length.

Metric to British.			
Mètres.	Yards.	Feet.	Inches.
1	1.0936	3.2809	39.37
2	2.1873	6.5618	78.74
3	3.2809	9.8427	118.11
4	4.3745	13.1236	157.48
5	5.4682	16.4045	196.85
6	6.5618	19.6854	236.22
7	7.6554	22.9663	275.60
8	8.7491	26.2472	314.97
9	9.8427	29.5281	354.34

British to Metric.					
Yds.	Mètres.	Ft.	Mètres.	Ins.	Centimètres.
1	0.91438	1	0.30479	1	2.5400
2	1.82877	2	0.60959	2	5.0799
3	2.74315	3	0.91438	3	7.6199
4	3.65753	4	1.21918	4	10.1598
5	4.57192	5	1.52397	5	12.6998
6	5.48630	6	1.82877	6	15.2397
7	6.40068	7	2.13356	7	17.7797
8	7.31507	8	2.43836	8	20.3196
9	8.22946	9	2.74315	9	22.8596

Metric Table of Length.

Milli-mètres.	10 = 1 centimètre.
100 = 1 décimètre.	1000 = 1 mètre.
Mètres.	10 = 1 décamètre.
100 = 1 hectomètre.	1000 = 1 kilomètre.

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of yards in 2354 mètres (see cols. 1 and 2) mètres. yards. 2000 = 2187.3 300 = 328.09 50 = 54.68 4 = 4.37 ∴ 2354 = 2574.44	of feet in 12.4 mètres (see cols. 1 and 3). mètres. feet. 10 = 32.809 2 = 6.562 0.4 = 1.312 ∴ 12.4 = 40.083	of inches in 30.5 centimètres (see cols. 1 and 4). Note, 1 m. = 100 cm. cm. inches. 30.5 = 11.911 0.5 = 0.197 ∴ 30.5 = 12.008	of mètres in 1026 yards (see cols. 5 and 6). yards. mètres. 1000 = 914.38 20 = 18.28 6 = 5.49 ∴ 1026 = 938.16	of mètres in 1742 feet (see cols. 7 and 8). feet. mètres. 1000 = 304.79 700 = 213.36 40 = 12.19 2 = 0.61 ∴ 1742 = 530.95	of centimètres in 17.72 ins. (see cols. 9 and 10). inches. cms. 10 = 25.400 7 = 17.780 0.7 = 1.778 0.02 = 0.051 ∴ 17.72 = 45.009
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NOTE.—If a table of conversion is not at hand, an approximation to the equivalent in inches of a distance measured in centimètres may be obtained by multiplying by 0.4: thus, 30.5 cm. multiply by 0.4, and we have 12.2 inches; the real equivalent as shown above is 12.008 inches.

Weight.

Metric to British.			
Kilo-grammes.	Tons.	Pounds Avoirdupois.	Grains Troy.
1	.000984	2.2046	15.4323
2	.001968	4.4092	30.8647
3	.002953	6.6139	46.2970
4	.003937	8.8185	61.7294
5	.004921	11.0231	77.1617
6	.005905	13.2277	92.5941
7	.006889	15.4323	108.0264
8	.007874	17.6370	123.4589
9	.008858	19.8416	138.8911

British to Metric.					
Tons.	Metric tons or milliers.	Pounds Avoirdupois.	Kilo-grammes.	Grains Troy.	Grammes.
1	1.016	1	0.4536	1	.6648
2	2.032	2	0.9072	2	1.296
3	3.048	3	1.3608	3	1.944
4	4.064	4	1.8144	4	2.592
5	5.080	5	2.2680	5	3.240
6	6.096	6	2.7216	6	3.888
7	7.112	7	3.1752	7	4.536
8	8.128	8	3.6288	8	5.184
9	9.144	9	4.0824	9	5.832

Metric Table of Weight.

Milli-grammes.	10 = 1 centigramme.
100 = 1 décigramme.	1000 = 1 gramme.
Grammes.	10 = 1 décigramme.
100 = 1 hectogramme.	1000 = 1 kilogramme.
Kilo-grammes.	10 = 1 myriagramme.
100 = 1 quintal.	1000 = 1 tonne, or metric ton.

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of tons in 35 tonnes. (see cols. 1 and 2). tonnes. tons. 30 = 29.53 5 = 4.92 ∴ 35 = 34.45	of pounds in 56.3 kgs. (see cols. 1 and 3). kgs. lbs. 50 = 110.231 6 = 13.228 0.3 = 0.661 ∴ 56.3 = 124.120	of grains in 120 grammes (see cols. 1 and 4). grammes. grains. 100 = 1543.23 20 = 308.65 ∴ 120 = 1851.88	of tonnes in 38 tonnes. (see cols. 5 and 6). tonnes. tonnes. 30 = 30.48 8 = 8.13 ∴ 38 = 38.61	of kilogrammes in 68 pounds. (see cols. 7 and 8). lbs. kgs. 60 = 27.216 8 = 3.629 ∴ 68 = 30.845	of grammes in 85 grains. (see cols. 9 and 10). grains. grammes 80 = 5.184 5 = 0.324 ∴ 85 = 5.508
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NOTE.—7000 grains troy = 1 pound avoirdupois.

Table XVI—continued.

Pressure.

*Metric and Atmospheric
to British.*

Kilo- grammes per sq. cm.	Pounds per square inch.	Tons per square inch.	Atmo- spheres.	Pounds per square inch.	Tons per square inch.
1	14.223	0.0635	1	14.7	0.0656
2	28.446	0.1270	2	29.4	0.1313
3	42.668	0.1905	3	44.1	0.1969
4	56.891	0.2540	4	58.8	0.2625
5	71.114	0.3175	5	73.5	0.3281
6	85.337	0.3810	6	88.2	0.3938
7	99.560	0.4445	7	102.9	0.4594
8	113.783	0.5080	8	117.6	0.5250
9	128.006	0.5715	9	132.3	0.5906

*British to
Metric and Atmospheric.*

Pounds per square inch.	Kilo- grammes per sq. cm.	Atmo- spheres.	Tons per square inch.	Kilo- grammes per sq. cm.	Atmo- spheres.
1	0.07031	0.068	1	157.49	152.38
2	0.14062	0.136	2	314.99	304.76
3	0.21093	0.204	3	472.48	457.14
4	0.28124	0.272	4	629.97	609.52
5	0.35155	0.340	5	787.47	761.91
6	0.42186	0.408	6	944.96	914.29
7	0.49217	0.476	7	1102.45	1069.67
8	0.56248	0.544	8	1259.95	1219.05
9	0.63279	0.612	9	1417.44	1371.43

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of pounds per square inch in 32.1 kilo- grammes per square centimetre (see cols. 1 and 2).	of tons per square inch in 3210 kilo- grammes per square centimetre (see cols. 1 and 3).	of tons per square inch in 3254 atmo- spheres (see cols. 4 and 6).	of kilogrammes per square centimetre in 15 lbs. on the square inch (see cols. 7 and 8).	of kilogrammes per square centimetre in 18.2 tons per square inch (see cols. 10 and 11).	of atmospheres in 14.6 tons per square inch (see cols. 10 and 12).
1 = 14.223 10 = 142.23 2 = 28.446 0.1 = 1.42	1 = 0.0635 10 = 0.635 200 = 12.7 10 = 0.06	1 = 0.0656 10 = 0.656 3254 = 13.09 50 = 0.33 4 = 0.03	1 = 0.07031 10 = 0.7031 5 = 0.35156	1 = 0.068 10 = 0.68 8 = 0.42186 0.3 = 0.21093	1 = 152.38 10 = 1523.8 4 = 609.5 0.6 = 91.4
∴ 32.1 = 456.55	∴ 3210 = 20.38	∴ 3254 = 21.36	∴ 15 = 1.0547	∴ 18.2 = 2882.1	∴ 14.6 = 2224.7

Energy.

Metric to British.

Mètre-tons.	Foot-tons.
1	3.2291
2	6.4581
3	9.6872
4	12.9162
5	16.1453
6	19.3743
7	22.6034
8	25.8324
9	29.0615

British to Metric.

Foot-tons.	Mètre tons.
1	0.3097
2	0.6194
3	0.9291
4	1.2388
5	1.5484
6	1.8581
7	2.1678
8	2.4775
9	2.7872

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of foot-tons in 4367 mètre-tons (see cols. 1 and 2).	of mètre-tons in 3592 foot-tons (see cols. 3 and 4).
1 = 3.2291 10 = 32.291 4000 = 12916.2 300 = 968.72 60 = 193.74 7 = 22.603	1 = 0.3097 10 = 3.097 3000 = 929.1 800 = 154.84 90 = 27.87 2 = 0.62
∴ 4367 = 14101.26	∴ 3592 = 1112.43

NOTE.—1000 mètre-tons is called a *dinamode* in Italy.

TABLE XVII.

Double Entry for α , $\frac{R}{C}$, and n .

R. C.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	
Velocity f/s. 500	03909	07920	12032	16250	20579	25022	29583	34266	39072	44010	49086	54300	59661	65175	70845	76689	82688	88854	95228																		
510	03757	07611	11555	15594	19780	24039	28369	32896	37529	42288	47118	52134	57275	62561	67995	73584	79332	85250	91345	97620																	
520	03613	07318	11106	14990	18994	23085	27290	31595	36064	40645	45365	50250	55319	60590	66071	71767	77693	83854	90250	96880																	
530	03477	07042	10684	14429	18276	22211	26252	30389	34682	39080	43522	48150	52886	57754	62755	67903	73217	78714	84413	90320																	
540	03340	06782	10296	13903	17602	21392	25279	29261	33375	37594	41881	46322	50872	55548	60350	65285	70355	75572	80944	86472																	
550	03220	06538	09929	13406	16968	20619	24363	28202	32151	36184	40334	44594	48960	53463	58078	62820	67690	72700	77858	83164																	
560	03116	06309	09580	12933	16367	19886	23495	27202	30974	34847	38871	42961	47171	51493	55931	60491	65174	69990	74945	80042																	
570	03008	06089	09246	12481	15793	19187	22668	26251	29971	33755	37482	41418	45471	49632	53904	58292	62798	67439	72215	77094																	
580	02905	05879	08927	12043	15245	18520	21879	25339	28827	32401	36164	39950	43864	47873	51965	56153	60452	64871	69506	74300																	
590	02807	05679	08623	11637	14723	17885	21130	24460	27835	31299	34913	38577	42343	46236	50172	54245	58428	62723	67139	71676																	
600	02714	05480	08333	11245	14227	17281	20409	23616	26901	30270	33721	37267	40901	44628	48458	52382	56416	60557	64812	69188																	
610	02626	05300	08057	10873	13757	16707	19720	22811	25987	29251	32594	36022	39531	43130	46822	50614	54505	58509	62634	66881																	
620	02542	05137	07795	10520	13310	16163	19073	22060	25154	28284	31516	34835	38225	41702	45272	48929	52684	56549	60530	64635																	
630	02461	04974	07547	10185	12885	15647	18464	21356	24327	27369	30492	33690	36960	40304	43724	47233	50839	54552	58385	62349																	
640	02384	04819	07312	09808	12482	15157	17859	20691	23570	26506	29523	32625	35794	39043	42382	45792	49293	52896	56605	60430																	
650	02311	04672	07060	09506	12100	14691	17343	20053	22838	25697	28630	31643	34739	37916	41268	44693	47211	51198	54709	58420																	
660	02242	04532	06881	09278	11739	14246	16821	19451	22143	24905	27736	30617	33585	36627	39750	42943	46221	49583	53033	56575																	
670	02175	04397	06680	09001	11380	13820	16319	18886	21475	24154	26903	29822	32857	35953	38929	41917	44900	47943	51031	54160																	
680	02111	04268	06486	08736	11053	13411	15835	18304	20824	23433	26099	28791	31577	34441	37391	40352	43327	46376	49507	52625																	
690	02050	04145	06299	08482	10728	13018	15367	17764	20220	22740	25321	27989	30640	33407	36244	39143	42125	45175	48305	51518																	
700	01992	04028	06119	08239	10415	12642	14917	17247	19632	22072	24570	27126	29746	32429	35176	37998	40898	43871	46915	50036																	
710	01936	03917	05946	08006	10113	12275	14487	16752	19069	21430	23851	26348	28900	31494	34154	36891	39693	42560	45501	48519																	
720	01883	03809	05779	07783	09829	11929	14079	16280	18530	20820	23168	25563	28011	30508	33177	35836	38555	41337	44189	47171																	
730	01832	03705	05619	07569	09560	11601	13691	15830	18015	20241	22521	24858	27256	29709	32242	34824	37464	40164	42922	45772																	
740	01783	03605	05466	07364	09304	11289	13321	15400	17523	19691	21910	24201	26580	28943	31406	33852	36414	38938	41525	44180																	
750	01736	03559	05319	07168	09059	10991	12967	14988	17055	19167	21323	23530	25802	28118	30488	32917	35404	40564	43247	45977																	
760	01691	03416	05178	06980	08832	10735	12676	14652	16611	18655	20708	22808	24950	27134	29365	32630	34947	39255	42469	44711																	
770	01647	03327	05043	06798	08594	10428	12296	14210	16178	18177	20224	22301	24413	26567	28854	31174	33524	35908	39243	43501																	
780	01604	03242	04913	06623	08373	10159	11978	13841	15756	17702	19694	21715	23830	25965	28114	30348	32634	34975	37373	39828																	
790	01563	03160	04789	06455	08159	09898	11671	13486	15346	17241	19178	21151	23161	25259	27381	29553	31777	34054	36380	38773																	
800	01524	03081	04670	06293	07952	09646	11374	13144	14950	16794	18678	20603	22568	24560	26575	28635	30752	32916	35136	37406																	
810	01486	03005	04556	06137	07752	09399	11088	12814	14570	16361	18198	20086	22012	23982	25994	28052	30159	32316	34523	36782																	
820	01449	02932	04445	05986	07560	09166	10814	12497	14207	15950	17741	19585	21462	23380	25341	27345	29397	31498	33646	35845																	
830	01414	02862	04338	05841	07377	08945	10551	12193	13860	15561	17307	19104	20934	22803	24714	26666	28666	30713	32805	34946																	
840	01381	02794	04235	05702	07202	08734	10300	11901	13529	15193	16896	18644	20427	22240	24112	26016	27966	29961	31998	34084																	
850	01349	02729	04193	05659	07153	08682	10250	11851	13483	15146	16840	18572	20347	22159	24012	25900	27828	29800	31816	33874																	
860	01318	02666	04041	05442	06875	08338	09830	11355	12910	14495	16129	17800	19512	21265	23058	24897	26784	28716	30690	32706																	
870	01288	02605	03949	05320	06721	08150	09609	11099	12619	14179	15765	17376	19004	20727	22485	24280	26111	27985	29900	31856																	
880	01259	02547	03861	05202	06572	07969	09396	10853	12340	13864	15416	16990	18611	20295	21950	23686	25467	27298	29175	31095																	
890	01231	02491	03776	05058	06388	07765	09191	10616	12072	13561	15080	16623	18207	19824	21477	23169	24900	26677	28500	30368																	
900	01204	02437	03694	04978	06289	07627	08994	10388	11813	13269	14756	16273	17822	19405	21022	22678	24371	26104	27871	29680																	
910	01178	02384	03615	04872	06154	07465	08804	10168	11562	12988	14444	15932	17452	18994	20568	22176	23819	25504	27228	28990																	
920	01153	02333	03588	04879	06194	07538	08900	10290																													

Table XVII—continued.

R C	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.							
Velocity f/s.																																																		
960	01060	02144	03255	04387	05551	06787	07946	09183	10447	11740	13055	14404	15778	17188	18617	20064	21583	23112	24678	26270	27915	29587	31300	33062	34855	36688	38562	40481	42447	44461	46522	48634	50790	52989	55258	57588														
970	01038	02101	03190	04299	05441	06604	07790	08995	10227	11485	12768	14076	15408	16764	18144	19547	20983	22451	23951	25483	27047	28642	30268	31925	33613	35332	37082	38863	40685	42548	44451	46394	48376	50397	52457	54556	56684	58841	61027	63242	65486	67759	70061	72392	74752	77141	79559			
980	01017	02059	03126	04214	05333	06474	07638	08821	10050	11295	12565	13858	15183	16538	17921	19335	20779	22253	23761	25303	26880	28490	30134	31812	33525	35272	37053	38868	40718	42603	44524	46481	48474	50502	52564	54660	56790	58944	61122	63324	65550	67801	70076	72375	74698	77045	79416			
990	00997	02018	03063	04132	05228	06347	07490	08661	09857	11079	12326	13607	14922	16271	17654	19071	20521	22003	23517	25063	26641	28251	29893	31567	33273	35011	36781	38583	40416	42281	44178	46107	48068	50060	52083	54137	56221	58335	60478	62650	64851	67082	69343	71634	73955	76306				
1000	00977	01978	03002	04052	05125	06223	07345	08494	09667	10867	12092	13341	14614	15921	17261	18634	20041	21481	22953	24457	25993	27561	29161	30793	32457	34153	35881	37641	39432	41254	43107	45000	46932	48903	50914	52955	55026	57127	59258	61419	63610	65831	68082	70363	72674	75015				
1010	00958	01980	02943	03972	05024	06101	07203	08330	09480	10653	11853	13081	14336	15618	16928	18267	19635	21033	22461	23919	25407	26925	28473	30051	31659	33297	34965	36663	38391	40149	41937	43755	45603	47481	49389	51327	53295	55292	57318	59372	61454	63565	65696	67847	70028	72239	74480			
1020	00940	01901	02886	03895	04927	05984	07067	08173	09302	10459	11643	12853	14089	15351	16639	17954	19297	20669	22061	23483	24935	26417	27929	29471	31043	32645	34277	35939	37631	39353	41105	42887	44699	46541	48413	50315	52247	54200	56173	58166	60179	62211	64262	66333	68424	70535	72666	74817		
1030	00922	01865	02831	03822	04835	05872	06936	08022	09132	10269	11433	12632	13864	15129	16427	17759	19125	20516	21931	23371	24835	26323	27835	29371	30931	32515	34123	35755	37411	39091	40794	42520	44268	46038	47830	49643	51477	53331	55205	57099	58992	60905	62837	64789	66761	68752	70763			
1040	00905	01830	02779	03752	04748	05766	06811	07878	08970	10088	11232	12407	13601	14827	16077	17353	18656	19989	21351	22743	24164	25615	27098	28613	30159	31736	33344	34983	36653	38354	40086	41849	43643	45467	47321	49204	51116	53058	55020	56992	58984	60995	63026	65077	67148	69239	71350	73481		
1050	00888	01797	02729	03685	04665	05668	06692	07740	08816	09915	11040	12192	13368	14569	15799	17053	18337	19648	20989	22351	23735	25150	26595	28071	29578	31105	32653	34231	35839	37477	39145	40843	42571	44329	46107	47905	49723	51561	53419	55297	57195	59113	61051	63009	64987	66985	68993	71011		
1060	00872	01785	02681	03620	04585	05571	06578	07607	08661	09739	10841	11967	13117	14291	15489	16711	17957	19227	20521	21839	23185	24559	25961	27391	28849	30335	31849	33391	34961	36559	38185	39839	41511	43219	44953	46713	48499	50311	52143	53995	55867	57759	59671	61603	63555	65527	67519	69531		
1070	00855	01763	02634	03553	04508	05479	06468	07481	08527	09592	10683	11797	12941	14117	15327	16571	17849	19151	20485	21851	23249	24679	26141	27633	29145	30677	32239	33831	35453	37105	38787	40499	42241	43993	45765	47557	49369	51201	53053	54925	56817	58729	60661	62613	64585	66567	68569	70591		
1080	00840	01703	02589	03499	04433	05389	06363	07361	08391	09441	10517	11615	12742	13899	15065	16269	17500	18755	20040	21353	22694	24065	25466	26897	28359	29851	31373	32925	34507	36119	37761	39433	41135	42867	44619	46391	48183	50005	51847	53709	55591	57493	59415	61357	63319	65291	67273	69265		
1090	00825	01674	02546	03441	04361	05302	06263	07247	08261	09297	10358	11442	12553	13688	14845	16033	17248	18487	19754	21051	22375	23728	25111	26523	27965	29437	30939	32471	34033	35625	37237	38869	40521	42193	43885	45597	47329	49081	50853	52645	54457	56289	58141	60013	61905	63817	65749	67691		
1100	00811	01646	02505	03386	04291	05218	06167	07139	08137	09160	10206	11277	12373	13493	14636	15809	17000	18222	19483	20774	22097	23451	24835	26249	27693	29167	30671	32205	33769	35363	36987	38641	40325	42039	43783	45557	47361	49195	51049	52923	54817	56731	58665	60619	62593	64587	66591	68605		
1110	00797	01618	02464	03333	04224	05136	06074	07036	08019	09030	10061	11119	12201	13307	14437	15595	16780	17989	19222	20487	21785	23106	24458	25841	27255	28699	30173	31677	33201	34745	36319	37923	39557	41211	42895	44609	46353	48127	49931	51755	53599	55463	57347	59251	61175	63119	65083	67047		
1120	00783	01591	02424	03281	04159	05058	05985	06937	07916	08924	09961	11027	12123	13241	14387	15561	16763	17994	19255	20547	21871	23227	24615	26037	27491	28977	30495	32045	33627	35241	36885	38559	40263	41997	43761	45555	47369	49203	51057	52931	54825	56739	58673	60627	62591	64565	66549	68543		
1130	00770	01565	02395	03231	04095	04984	05900	06841	07797	08783	09799	10824	11880	12961	14066	15197	16355	17539	18748	19981	21237	22515	23815	25137	26491	27877	29295	30745	32227	33741	35285	36859	38463	40097	41761	43465	45199	46963	48757	50581	52435	54309	56203	58117	60051	61995	63959	65933	67917	
1140	00757	01540	02348	03182	04038	04914	05819	06747	07693	08667	09665	10685	11729	12799	13893	15012	16157	17329	18526	19747	20991	22258	23549	24863	26201	27573	28977	30413	31881	33381	34913	36477	38073	39691	41331	42993	44687	46413	48169	49955	51771	53617	55483	57369	59275	61191	63127	65083	67049	
1150	00744	01515	02313	03134	03977	04847	05741	06659	07593	08556	09541	10551	11585	12644	13727	14834	15965	17128	18313	19529	20767	22038	23341	24677	26047	27451	28889	30361	31865	33401	34969	36569	38191	39835	41501	43199	44929	46691	48485	50309	52163	54037	55931	57845	59779	61733	63707	65691	67685	
1160	00732	01491	02279	03089	03920	04783	05666	06568	07497	08450	09425	10423	11445	12491	13561	14654	15771	16913	18079	19269	20491	21745	23031	24349	25699	27081	28495	29941	31419	32931	34475	36051	37659	39291	40945	42621	44319	46039	47781	49545	51331	53147	54983	56839	58715	60611	62527	64453	66389	
1170	00720	01468	02245	03045	03865	04719	05592	06483	07408	08347	09313	10299	11313	12351	13414	14500	15613	16751	17913	19100	20311	21545	22801	24089	25409	26761	28145	29561	31019	32501	34015	35561	37139	38741	40373	42029	43701	45399	47123	48871	50633	52417	54221	56045	57889	59753	61637	63541	65465	
1180	00708	01445	02211	03002	03812	04656	05519	06401	07311	08246	09203	10180	11184	12213	13266	14344	15445	16567	17711	18877	20065	21275	22507	23761	25037	26345	27695	29077	30491	31937	33415	34925	36467	38041	39647	41285	42947	44631	46339	48071	49825	51599	53393	55207	57041	58895	60769	62663	64577	66501
1190	00697	01423	02178	02959	03761	04594	05448	06321	07221	08148	09096	10065	11059	12079	13123	14191	15283	16397	17533	18691	19871	21073	22297	23543	24811	26111	27443	28807	30203	31631	33091	34581	36101	37651	39231	40841	42481	44151	45851	475										

Table XVII—continued.

R G	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	5100.	5200.	5300.																								
Velocity f/s.	00458	00944	01457	01997	02564	03161	03788	04441	05126	05839	06579	07343	08124	08924	09743	10591	11468	12374	13309	14274	15269	16294	17349	18434	19549	20694	21869	23074	24309	25574	26869	28194	29549	30914	32309	33724	35159	36624	38119	39644	41199	42784	44399	46044	47719	49424	51159	52924	54719	56544	58409																										
1470	00458	00944	01457	01997	02564	03161	03788	04441	05126	05839	06579	07343	08124	08924	09743	10591	11468	12374	13309	14274	15269	16294	17349	18434	19549	20694	21869	23074	24309	25574	26869	28194	29549	30914	32309	33724	35159	36624	38119	39644	41199	42784	44399	46044	47719	49424	51159	52924	54719	56544	58409																										
1480	00458	00944	01457	01997	02564	03161	03788	04441	05126	05839	06579	07343	08124	08924	09743	10591	11468	12374	13309	14274	15269	16294	17349	18434	19549	20694	21869	23074	24309	25574	26869	28194	29549	30914	32309	33724	35159	36624	38119	39644	41199	42784	44399	46044	47719	49424	51159	52924	54719	56544	58409																										
1490	00458	00944	01457	01997	02564	03161	03788	04441	05126	05839	06579	07343	08124	08924	09743	10591	11468	12374	13309	14274	15269	16294	17349	18434	19549	20694	21869	23074	24309	25574	26869	28194	29549	30914	32309	33724	35159	36624	38119	39644	41199	42784	44399	46044	47719	49424	51159	52924	54719	56544	58409																										
1500	00440	00936	01401	01920	02468	03044	03649	04282	04946	05639	06359	07104	07874	08669	09490	10333	11200	12092	13009	13950	14915	15905	16921	17962	19027	20119	21238	22384	23554	24758	25981	27233	28522	29838	31184	32562	33972	35415	36894	38407	39953	41543	43169	44831	46534	48279	50066	51896	53769	55684	57600																										
1510	00434	00896	01382	01890	02430	03000	03594	04201	04828	05473	06144	06840	07561	08306	09074	98874	10674	11494	12339	13209	14104	15024	15969	16939	17934	18954	19994	21054	22138	23244	24374	25529	26709	27924	29174	30454	31769	33109	34474	35869	37284	38729	40199	41694	43219	44769	46344	47944	49569	51219	52899	54609	56344																								
1520	00428	00884	01364	01872	02403	02967	03559	04180	04830	05508	06214	06945	07700	08488	09309	10161	11034	11929	12844	13779	14734	15709	16704	17719	18754	19809	20884	21979	23094	24229	25384	26559	27754	28979	30229	31504	32809	34134	35479	36844	38229	39634	41069	42529	44009	45509	47029	48569	50129	51709	53309	54929	56569																								
1530	00422	00872	01346	01848	02374	02930	03515	04129	04773	05444	06142	06867	07620	08398	09199	10024	10874	11752	12651	13574	14519	15484	16469	17474	18509	19564	20639	21734	22849	23984	25139	26309	27494	28699	29924	31174	32449	33744	35059	36389	37734	39099	40489	41899	43329	44779	46249	47739	49239	50759	52289	53829	55389																								
1540	00416	00861	01329	01824	02344	02894	03472	04079	04716	05380	06071	06790	07537	08306	09104	99224	10760	11640	12534	13447	14384	15339	16314	17309	18324	19359	20409	21474	22559	23664	24789	25929	27084	28259	29449	30654	31879	33124	34389	35674	36979	38299	39634	40989	42369	43769	45189	46629	48089	49569	51059	52559	54069	55589																							
1550	00410	00850	01312	01800	02315	02858	03423	04020	04650	05317	06001	06714	07456	08221	09010	98223	10663	11528	12415	13324	14259	15218	16194	17194	18219	19259	20319	21394	22484	23589	24709	25844	26994	28159	29339	30534	31749	32979	34224	35489	36769	38069	39389	40729	42089	43469	44869	46279	47699	49129	50569	52019	53479	54949	56429	57919																					
1560	00405	00839	01295	01777	02286	02823	03387	03981	04604	05255	05933	06639	07374	08134	08917	09724	10558	11417	12298	13202	14131	15085	16066	17070	18100	19156	20230	21324	22438	23564	24709	25869	27044	28234	29439	30659	31894	33144	34409	35689	36979	38279	39589	40909	42239	43579	44929	46289	47659	49029	50409	51799	53189	54589	55989	57389	58789																				
1570	00400	00828	01278	01755	02258	02788	03346	03933	04549	05194	05865	06565	07294	08048	08825	09625	10454	11306	12182	13081	14005	14953	15928	16925	17950	18990	20077	21177	22290	23419	24564	25729	26909	28099	29309	30534	31774	33029	34299	35579	36869	38169	39479	40799	42129	43469	44809	46159	47509	48859	50209	51559	52909	54259	55609	56959	58309	59659																			
1580	00395	00818	01262	01733	02230	02754	03305	03885	04485	05114	05769	06449	07154	07884	08639	09419	10224	11054	11909	12789	13694	14624	15574	16544	17534	18544	19574	20624	21694	22784	23894	25019	26154	27309	28479	29664	30864	32079	33304	34539	35784	37039	38299	39569	40839	42119	43399	44679	45959	47239	48519	49799	51079	52359	53639	54919	56199	57479	58759	59999																	
1590	00389	00808	01246	01711	02203	02720	03265	03838	04434	05053	05694	06359	07049	07764	08504	09264	10044	10844	11664	12504	13364	14244	15144	16064	16994	17944	18904	19884	20884	21894	22919	23954	25009	26079	27154	28244	29349	30469	31599	32739	33889	35039	36199	37369	38539	39709	40879	42049	43219	44389	45559	46729	47899	49069	50239	51409	52579	53749	54919	56089	57259	58429	59599														
1600	00387	00798	01231	01690	02176	02687	03225	03792	04388	05014	05655	06314	06994	07694	08414	09154	99914	10674	11454	12254	13074	13914	14764	15634	16514	17404	18304	19224	20154	21094	22044	22994	23954	24924	25904	26884	27874	28864	29864	30864	31864	32864	33864	34864	35864	36864	37864	38864	39864	40864	41864	42864	43864	44864	45864	46864	47864	48864	49864	50864	51864	52864	53864	54864	55864	56864	57864	58864	59864								
1610	00382	00758	01215	01663	02149	02654	03186	03746	04336	04950	05590	06254	06944	07654	08384	09134	99894	10664	11444	12244	13064	13904	14754	15624	16504	17394	18294	19204	20124	21054	21994	22944	23894	24844	25794	26744	27694	28644	29594	30544	31494	32444	33394	34344	35294	36244	37194	38144	39094	40044	40994	41944	42894	43844	44794	45744	46694	47644	48594	49544	50494	51444	52394	53344	54294	55244	56194	57144	58094	59044	59994						
1620	00377	00778	01200	01648	02122	02621	03147	03701	04285	04898	05536	06200	06884	07584	08304	09044	99784	10644	11424	12224	13044	13884	14734	15594	16464	17344	18234	19134	20044	20954	21864	22774	23684	24594	25504	26414	27324	28234	29144	30054	30964	31874	32784	33694	34594	35504	36414	37324	38234	39144	40054	40964	41874	42784	43694	44594	45504	46414	47324	48234	49144	50054	50964	51874	52784	53694	54594	55504	56414	57324	58234	59144	59994				
1630	00373	00778	01195	01628	02094	02589	03109	03657	04234	04841	05474	06134	06814	07514	08234	08964	99694	10634	11404	12194	13004	13824	14654	15494	16344	17194	18054	18924	19794	20664	21534	22404	23274	24144	25014	25884	26744	27614	28484	29344	30214	31084	31954	32824	33694	34564	35434	36304	37174	38044	38914	39784	40654	41524	42394	43264	44134	45004	45874	46744	47614	48484	49354	50224	51094	51964	52834	53704	54574	55444	56314	57184	58054	58924	59794	59994	
1640	00369	00759	01171	01608	02069	02557	03071	03613	04184	04784	05414	06064	06734	07424	08134	08864	99594	10614	11384	12174	12984	13804	14634	15474	16324	17174	18024	18874	19724	20574	21424	22274	23124	23974	24824	25674	26524	27374	28224	29074	29924	30774	31624	32474	33324	34174	35024	35874	36724	37574	38424	39274	40124	40974	41824	42674	43524	44374	45224	46074	46924	47774	48624	49474	50324	51174	52024	52874	53724	54574	55424	56274	57124	57974	58824	59674	59994
1650	00365	00750	01157	01588	02044	02526	03034	03570	04134	04728	05349	05989	06649	07324	08014	08714	99414	10544	11314	12104	12914	13724	14534																																																						

Table XVII—continued.

R. C.	100.	200.	300.	400.	500.	600.	700.	800.	900.	1000.	1100.	1200.	1300.	1400.	1500.	1600.	1700.	1800.	1900.	2000.	2100.	2200.	2300.	2400.	2500.	2600.	2700.	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	5100.	5200.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
Velocity f/s.	00252	00510	00801	01098	01412	01744	02096	02467	02862	03280	03721	04180	04653	05150	05680	06232	06807	07405	08026	08680	09357	10058	10783	11532	12305	13102	13922	14764	15628	16514	17421	18350	19300	20271	21262	22274	23306	24359	25442	26555	27698	28870	29971	31099	32254	33436	34645	35881	37144	38434	39750	41091	42457	43848	45264	46695	48150	49630	51134	52662																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
1980	00250	00514	00798	01087	01398	01726	02074	02442	02833	03246	03683	04140	04615	05110	05635	06200	06787	07396	08026	08680	09357	10058	10783	11532	12305	13102	13922	14764	15628	16514	17421	18350	19300	20271	21262	22274	23306	24359	25442	26555	27698	28870	29971	31099	32254	33436	34645	35881	37144	38434	39750	41091	42457	43848	45264	46695	48150	49630	51134	52662																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
2000	00248	00509	00785	01076	01384	01708	02058	02426	02818	03231	03665	04119	04588	05078	05593	06143	06717	07314	07934	08586	09261	99947	10700	11426	12175	12947	13742	14560	15401	16264	17150	18058	18989	19941	20914	21908	22922	23955	24998	26061	27143	28254	29395	30566	31766	32995	34252	35537	36849	38188	39554	40947	42368	43816	45290	46789	48312	49859	51430	52934																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
2010	00246	00504	00777	01065	01370	01691	02032	02394	02775	03180	03607	04062	04541	05047	05580	06142	06731	07347	07990	08657	09351	10070	10814	11580	12373	13191	14034	14902	15794	16711	17654	18621	19613	20633	21678	22750	23849	24975	26129	27314	28527	29767	31042	32348	33685	35053	36457	37897	39371	40879	42428	44010	45624	47269	48945	50653	52394	54168	55975	57796	59640	61516	63424	65364	67335	69336	71368	73431	75524	77646	79797	81978	84188	86427	88695	90992	93318	95672	98054	100465	102917	105398	107908	110447	113015	115611	118234	120884	123561	126264	128992	131744	134519	137316	140135	142976	145838	148721	151624	154547	157489	160450	163430	166429	169446	172480	175531	178598	181681	184780	187894	191023	194166	197323	200494	203679	206877	210088	213311	216546	219792	223049	226316	229593	232880	236177	239484	242801	246128	249464	252810	256165	259529	262902	266284	269675	273075	276484	279901	283326	286759	290199	293646	297099	300558	304022	307491	310965	314444	317928	321416	324908	328404	331904	335408	338916	342428	345944	349464	352988	356516	360048	363584	367124	370668	374216	377767	381322	384881	388443	392008	395576	399147	402721	406298	410000	413706	417416	421129	424844	428561	432280	436000	439722	443446	447172	450899	454627	458356	462085	465814	469543	473272	477001	480730	484458	488186	491914	495642	499370	503097	506824	510551	514278	518004	521730	525456	529181	532906	536630	540354	544078	547801	551524	555247	558969	562691	566412	570133	573854	577574	581294	585013	588732	592450	596168	599885	603601	607316	611030	614744	618457	622169	625881	629592	633302	637011	640720	644428	648135	651841	655546	659250	662953	666655	670356	674056	677755	681453	685150	688846	692541	696235	699928	703620	707311	711001	714690	718378	722065	725751	729436	733120	736803	740485	744166	747846	751524	755201	758877	762552	766226	769899	773571	777242	780912	784581	788249	791916	795582	799247	802911	806574	810236	813897	817557	821216	824873	828528	832182	835835	839487	843138	846788	850436	854083	857728	861372	865014	868655	872295	875934	879572	883209	886844	890478	894110	897741	901371	905000	908628	912255	915881	919506	923130	926753	930375	933996	937615	941233	944850	948466	952081	955694	959306	962917	966527	970136	973743	977349	980954	984558	988161	991763	995364	998964	1002500	1005354	1008207	1011058	1013908	1016756	1019603	1022448	1025291	1028132	1030972	1033811	1036648	1039483	1042316	1045147	1047977	1050805	1053631	1056456	1059279	1062100	1064919	1067736	1070551	1073364	1076175	1078984	1081791	1084596	1087399	1090200	1092999	1095796	1098591	1101384	1104175	1106964	1109751	1112536	1115319	1118100	1120879	1123656	1126431	1129204	1131975	1134744	1137511	1140276	1143039	1145800	1148559	1151316	1154071	1156824	1159575	1162324	1165071	1167816	1170559	1173300	1176039	1178776	1181511	1184244	1186975	1189704	1192431	1195156	1197879	1200600	1203319	1206036	1208751	1211464	1214175	1216884	1219591	1222296	1225000	1227702	1230403	1233102	1235800	1238496	1241191	1243884	1246575	1249264	1251951	1254636	1257319	1260000	1262679	1265356	1268031	1270704	1273375	1276044	1278711	1281376	1284039	1286700	1289359	1292016	1294671	1297324	1300000	1302654	1305307	1307958	1310607	1313254	1315900	1318544	1321186	1323827	1326466	1329103	1331738	1334371	1337002	1339631	1342258	1344883	1347506	1350127	1352746	1355363	1357978	1360591	1363202	1365811	1368418	1371023	1373626	1376227	1378826	1381423	1384018	1386611	1389202	1391791	1394378	1396963	1400000	1402534	1405066	1407596	1410123	1412648	1415171	1417692	1420211	1422728	1425243	1427756	1430267	1432776	1435283	1437788	1440291	1442792	1445291	1447788	1450283	1452776	1455267	1457756	1460243	1462727	1465208	1467686	1470162	1472636	1475107	1477575	1480041	1482504	1484964	1487421	1489876	1492329	1494780	1497229	1499675	1502118	1504559	1506997	1509432	1511864	1514293	1516719	1519143	1521564	1523982	1526397	1528809	1531218	1533624	1536028	1538429	1540827	1543222	1545614	1548003	1550389	1552772	1555152	1557529	1559903	1562274	1564642	1567007	1569369	1571728	1574084	1576437	1578787	1581134	1583478	1585819	1588157	1590492	1592824	1595153	1597479	1599802	1602122	1604439	1606752	1609062	1611369	1613673	1615974	1618272	1620567	1622859	1625148	1627434	1629717	1632000	1634279	1636556	1638831	1641103	1643372	1645639	1647903	1650164	1652422	1654677	1656929	1659178	1661424	1663667	1665907	1668144	1670378	1672609	1674837	1677062	1679284	1681503	1683719	1685932	1688142	1690349	1692552	1694752	1696949	1699143	1701334	1703522	1705707	1707889	1710068	1712244	1714417	1716587	1718754	1720917	1723077	1725234	1727387	1729537	1731684	1733827	1735966	1738102	1740235	1742365	1744492	1746616	1748737	1750854	1752967	1755076	1757182	1759285	1761385	1763482	1765576	1767667	1769754	1771837	1773916	1775991	1778062	1780129	1782192	1784251	1786306	1788357	1790404	1792447	1794486	1796521	1798552	1800579	1802602	1804621	1806636	1808647	1810654	1812657	1814656	1816651	1818642	1820629	1822612	1824591	1826566	1828537	1830504	1832467	1834426	1836381	1838332	1840279	1842222	1844161	1846096	1848027	1849954	1851877	1853796	1855711	1857622	1859529	1861432	1863332	1865228	1867121	1869010	1870895	1872776	1874653	1876526	1878395	1880260	1882121	1883978	1885831	1887680	1889525	1891366	1893203	1895036	1896865	1898690	1900511	1902328	1904141	1905950	1907755	1909556	1911353	1913146	1914935	1916720	1918501	1920278	1922051	1923820	1925585	1927346	1929103	1930856	1932605	1934350	1936091	1937828	1939561	1941291	1943018	1944741	1946460	1948175	1949886	1951593	1953296	1955000	1956699	1958394	1960085	1961772	1963455	1965134	1966809	1968480	1970147	1971810	1973469	1975124	1976775	1978422	1980065	1981704	1983339	1984970	1986597	1988220	1989840	1991456	1993068	1994676	1996281	1997882	1999479	2001072	2002661	2004246	2005827	2007404	2008977	2010546	2012111	2013672	2015229	2016782	2018331	2019876	2021417	2022954	2024487	2026016	2027541	2029062	2030579	2032092	2033601	2035106	2036607	2038104	2039597	2041086	2042571	2044052	2045529	2047002	2048471	2049936	2051396	2052852	2054304	2055752	2057196	2058635	2060070	2061501	2062928	2064351	2065769	2067183	2068593	2069998	2071398	2072794	2074186	2075574	2076958	2078338	2079713	2081084	2082450	2083811	2085168	2086521	2087870	2089215	2090556	2091893	2093226	2094555	2095880	2097201	2098518	2099831	2101140	2102445	2103746	2105043	2106336	2107625	2108910	2110191	2111468	2112741	2114010	2115275	2116536	2117793	2119046	2120294	2121537	2122776	2124011	2125242	2126469	2127692	2128911	2130126	2131337	2132544	2133747	2134946	2136141	2137332	2138519	

Table XVII--continued.

R C	2800.	2900.	3000.	3100.	3200.	3300.	3400.	3500.	3600.	3700.	3800.	3900.	4000.	4100.	4200.	4300.	4400.	4500.	4600.	4700.	4800.	4900.	5000.	5100.	5200.	5300.	5400.	5500.	5600.	5700.	5800.	5900.	6000.	6100.	6200.	6300.	6400.	6500.	6600.	6700.	6800.	6900.	7000.													
Velocity f/s																																																								
1880	15202	16100	17098	18096	19094	20092	21090	22088	23086	24084	25082	26080	27078	28076	29074	30072	31070	32068	33066	34064	35062	36060	37058	38056	39054	40052	41050	42048	43046	44044	45042	46040	47038	48036	49034	50032	51030	52028	53026	54024	55022	56020	57018	58016	59014	60012	61010	62008	63006	64004	65002	66000	67000	68000	69000	70000
1890	15203	16101	17099	18097	19095	20093	21091	22089	23087	24085	25083	26081	27079	28077	29075	30073	31071	32069	33067	34065	35063	36061	37059	38057	39055	40053	41051	42049	43047	44045	45043	46041	47039	48037	49035	50033	51031	52029	53027	54025	55023	56021	57019	58017	59015	60013	61011	62009	63007	64005	65003	66001	67001	68001	69001	70001
1900	15204	16102	17100	18098	19096	20094	21092	22090	23088	24086	25084	26082	27080	28078	29076	30074	31072	32070	33068	34066	35064	36062	37060	38058	39056	40054	41052	42050	43048	44046	45044	46042	47040	48038	49036	50034	51032	52030	53028	54026	55024	56022	57020	58018	59016	60014	61012	62010	63008	64006	65004	66002	67002	68002	69002	70002
1910	15205	16103	17101	18099	19097	20095	21093	22091	23089	24087	25085	26083	27081	28079	29077	30075	31073	32071	33069	34067	35065	36063	37061	38059	39057	40055	41053	42051	43049	44047	45045	46043	47041	48039	49037	50035	51033	52031	53029	54027	55025	56023	57021	58019	59017	60015	61013	62011	63009	64007	65005	66003	67003	68003	69003	70003
1920	15206	16104	17102	18100	19098	20096	21094	22092	23090	24088	25086	26084	27082	28080	29078	30076	31074	32072	33070	34068	35066	36064	37062	38060	39058	40056	41054	42052	43050	44048	45046	46044	47042	48040	49038	50036	51034	52032	53030	54028	55026	56024	57022	58020	59018	60016	61014	62012	63010	64008	65006	66004	67004	68004	69004	70004
1930	15207	16105	17103	18101	19100	20098	21096	22094	23092	24090	25088	26086	27084	28082	29080	30078	31076	32074	33072	34070	35068	36066	37064	38062	39060	40058	41056	42054	43052	44050	45048	46046	47044	48042	49040	50038	51036	52034	53032	54030	55028	56026	57024	58022	59020	60018	61016	62014	63012	64010	65008	66006	67006	68006	69006	70006
1940	15208	16106	17104	18102	19101	20099	21097	22095	23093	24091	25089	26087	27085	28083	29081	30079	31077	32075	33073	34071	35069	36067	37065	38063	39061	40059	41057	42055	43053	44051	45049	46047	47045	48043	49041	50039	51037	52035	53033	54031	55029	56027	57025	58023	59021	60019	61017	62015	63013	64011	65009	66007	67007	68007	69007	70007
1950	15209	16107	17105	18103	19102	20100	21098	22096	23094	24092	25090	26088	27086	28084	29082	30080	31078	32076	33074	34072	35070	36068	37066	38064	39062	40060	41058	42056	43054	44052	45050	46048	47046	48044	49042	50040	51038	52036	53034	54032	55030	56028	57026	58024	59022	60020	61018	62016	63014	64012	65010	66008	67008	68008	69008	70008
1960	15210	16108	17106	18104	19103	20101	21099	22097	23095	24093	25091	26089	27087	28085	29083	30081	31079	32077	33075	34073	35071	36069	37067	38065	39063	40061	41059	42057	43055	44053	45051	46049	47047	48045	49043	50041	51039	52037	53035	54033	55031	56029	57027	58025	59023	60021	61019	62017	63015	64013	65011	66009	67009	68009	69009	70009
1970	15211	16109	17107	18105	19104	20102	21100	22098	23096	24094	25092	26090	27088	28086	29084	30082	31080	32078	33076	34074	35072	36070	37068	38066	39064	40062	41060	42058	43056	44054	45052	46050	47048	48046	49044	50042	51040	52038	53036	54034	55032	56030	57028	58026	59024	60022	61020	62018	63016	64014	65012	66010	67010	68010	69010	70010
1980	15212	16110	17108	18106	19105	20103	21101	22100	23098	24096	25094	26092	27090	28088	29086	30084	31082	32080	33078	34076	35074	36072	37070	38068	39066	40064	41062	42060	43058	44056	45054	46052	47050	48048	49046	50044	51042	52040	53038	54036	55034	56032	57030	58028	59026	60024	61022	62020	63018	64016	65014	66012	67012	68012	69012	70012
1990	15213	16111	17109	18107	19106	20104	21102	22101	23099	24097	25095	26093	27091	28089	29087	30085	31083	32081	33079	34077	35075	36073	37071	38069	39067	40065	41063	42061	43059	44057	45055	46053	47051	48049	49047	50045	51043	52041	53039	54037	55035	56033	57031	58029	59027	60025	61023	62021	63019	64017	65015	66013	67013	68013	69013	70013
2000	15214	16112	17110	18108	19107	20105	21103	22102	23100	24098	25096	26094	27092	28090	29088	30086	31084	32082	33080	34078	35076	36074	37072	38070	39068	40066	41064	42062	43060	44058	45056	46054	47052	48050	49048	50046	51044	52042	53040	54038	55036	56034	57032	58030	59028	60026	61024	62022	63020	64018	65016	66014	67014	68014	69014	70014
2010	15215	16113	17111	18109	19108	20106	21104	22103	23101	24099	25097	26095	27093	28091	29089	30087	31085	32083	33081	34079	35077	36075	37073	38071	39069	40067	41065	42063	43061	44059	45057	46055	47053	48051	49049	50047	51045	52043	53041	54039	55037	56035	57033	58031	59029	60027	61025	62023	63021	64019	65017	66015	67015	68015	69015	70015
2020	15216	16114	17112	18110	19109	20107	21105	22104	23102	24100	25100	26098	27096	28094	29092	30090	31088	32086	33084	34082	35080	36078	37076	38074	39072	40070	41068	42066	43064	44062	45060	46058	47056	48054	49052	50050	51048	52046	53044	54042	55040	56038	57036	58034	59032	60030	61028	62026	63024	64022	65020	66018	67016	68016	69016	70016
2030	15217	16115	17113	18111	19110	20108	21106	22105	23103	24101	25101	26099	27097	28095	29093	30091	31089	32087	33085	34083	35081	36079	37077	38075	39073	40071	41069	42067	43065	44063	45061	46059	47057	48055	49053	50051	51049	52047	53045	54043	55041	56039	57037	58035	59033	60031	61029	62027	63025	64023	65021	66019	67017	68017	69017	70017
2040	15218	16116	17114	18112	19111	20109	21107	22106	23104	24102	25102	26100	27098	28096	29094	30092	31090	32088	33086	34084	35082	36080	37078	38076	39074	40072	41070	42068	43066	44064	45062	46060	47058	48056	49054	50052	51050	52048	53046	54044	55042	56040	57038	58036	59034	60032	61030	62028	63026	64024	65022	66020	67018	68018	69018	70018
2050	15219	16117	17115	18113	19112	20110	21108	22107	23105	24103	25103	26101	27099	28097	29095	30093	31091	32089	33087	34085	35083	36081	37079	38077	39075	40073	41071	42069	43067	44065	45063	46061	47059	48057	49055	50053	51051	52049	53047	54045	55043	56041	57039	58037	59035	60033	61031	62029	63027	64025	65023	66021	67019	68019	69019	70019
2060	15220	16118	17116	18114	19113	20111	21109	22108	23106	24104	25104	26102	27100	28098	29096	30094	31092	32090	33088	34086	35084	36082	37080	38078	39076	40074	41072	42070	43068	44066	45064	46062	47060	48058	49056	50054	51052	52050	53048	54046	55044	56042	57040	58038	59036	60034	61032	62030	63028	64026	65024	66022	67020	68020	69020	70020
2070	15221	16119	17117	18115	19114	20112	21110	22109	23107	24105	25105	26103	27101	28099	29097	30095	31093	32091	33089	34087	35085	36083	37081	38079	39077	40075	41073	42071	43069	44067	45065	46063	47061	48059	49057	50055	51053	52051	53049	54047	55045	56043	57041	58039	59037	60035	61033	62031	63029	64027	65025	66023	67021	68021	69021	70021
2080	15222	16120	17118	18116	19115	20113	21111	22110	23108	24106	25106	26104	27102	28100	29098	30096	31094	32																																						

Table XVII—continued.

2000.	3000.	4000.	5000.	6000.	7000.	8000.	9000.	10000.	11000.	12000.	13000.	14000.	15000.	16000.	17000.	18000.	19000.	20000.	21000.	22000.	23000.	24000.	25000.	26000.	27000.	28000.	29000.	30000.	31000.	32000.	33000.	34000.	35000.	36000.	37000.	38000.	39000.	40000.	41000.	42000.	43000.	44000.	45000.	46000.	47000.	48000.	49000.	50000.	51000.	52000.	53000.	54000.	55000.	56000.	57000.	58000.	59000.	60000.	61000.	62000.	63000.	64000.	65000.	66000.	67000.	68000.	69000.	70000.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
10795	11501	12233	12987	13765	14570	15401	16254	17128	18030	18960	19918	20901	21909	22942	24002	25090	26208	27356	28536	29750	30997	32284	33614	34987	36404	37865	39370	40919	42512	45228	48064	48826	49613	50434	51298	52205	53155	54148	55185	56266	57391	58559	59770	61021	62314	63650	65029	66451	67916	69425	70978	72575	74217	75904	77636	79413	81235	83102	85014	86971	88973	91020	93112	95250	97434	99664	101940	104262	106631	109047	111510	114019	116584	119205	121882	124615	127404	130249	133150	136107	139120	142189	145314	148495	151732	155025	158374	161778	165238	168753	172323	175948	179628	183363	187153	190998	194898	198853	202863	206928	211049	215225	219456	223742	228083	232479	236930	241436	246007	250643	255344	260110	264941	269837	274798	279824	284915	290071	295292	300578	305929	311345	316826	322372	327983	333659	339400	345206	351077	357013	363014	369080	375211	381407	387668	394004	400415	406901	413462	420098	426809	433595	440456	447392	454403	461489	468650	475886	483197	490583	498044	505580	513191	520877	528638	536474	544385	552370	560430	568564	576772	585054	593410	601841	610347	618928	627584	636315	645121	654002	662958	671989	681095	690276	699532	708863	718269	727750	737306	746937	756643	766424	776280	786211	796217	806300	816460	826696	837008	847405	857887	868454	879106	889843	900665	911572	922564	933641	944802	956047	967376	978789	990287	100591	101800	103014	104233	105457	106686	107920	109159	110403	111652	112906	114165	115429	116698	117972	119251	120535	121824	123118	124417	125721	127030	128344	129663	130987	132316	133650	134989	136333	137682	139036	140395	141759	143128	144502	145881	147265	148654	150048	151447	152851	154260	155674	157093	158517	159946	161380	162819	164263	165712	167166	168625	170089	171558	173032	174511	175995	177484	178978	180477	181981	183490	184994	186503	188017	189536	191059	192587	194120	195658	197201	198749	200302	201860	203423	204990	206562	208139	209721	211308	212900	214497	216099	217706	219318	220935	222557	224184	225816	227453	229095	230742	232394	234051	235713	237380	239052	240729	242411	244098	245790	247487	249189	250896	252608	254325	256047	257774	259506	261243	262985	264732	266484	268241	270003	271770	273542	275319	277101	278888	280680	282477	284279	286086	287898	289715	291537	293364	295196	297033	298875	300722	302574	304431	306293	308160	310032	311909	313791	315678	317570	319467	321369	323276	325188	327105	329027	330954	332886	334823	336765	338712	340664	342621	344583	346550	348522	350499	352481	354468	356460	358457	360459	362466	364478	366495	368517	370544	372576	374613	376655	378702	380754	382811	384873	386940	389012	391089	393171	395258	397350	399447	401549	403656	405768	407885	409997	412114	414236	416363	418495	420632	422774	424921	427073	429230	431392	433559	435731	437908	440090	442277	444469	446666	448868	451075	453287	455504	457726	459953	462185	464422	466664	468911	471163	473420	475682	477949	480221	482498	484780	487067	489359	491656	493958	496265	498577	500894	503216	505543	507875	510212	512554	514901	517253	519610	521972	524339	526711	529088	531470	533857	536249	538646	541048	543455	545867	548284	550706	553133	555565	558002	560444	562891	565343	567799	570261	572728	575200	577677	580159	582646	585138	587635	590137	592644	595156	597673	600195	602722	605254	607791	610333	612880	615432	617989	620551	623118	625690	628267	630849	633436	636028	638625	641227	643834	646446	649063	651685	654312	656944	659581	662223	664870	667522	670179	672841	675508	678180	680857	683539	686226	688918	691615	694317	697024	699736	702453	705175	707902	710634	713371	716113	718860	721612	724369	727131	729898	732670	735447	738229	741011	743798	746590	749387	752189	754996	757808	760625	763447	766274	769106	771943	774785	777632	780484	783341	786203	789070	791942	794819	797701	800588	803480	806377	809279	812186	815098	818015	820937	823864	826796	829733	832675	835622	838574	841531	844494	847462	850435	853413	856396	859384	862377	865375	868378	871386	874399	877417	880440	883468	886501	889539	892582	895630	898683	901741	904804	907872	910945	914023	917106	920194	923287	926385	929488	932596	935709	938827	941950	945078	948211	951349	954492	957640	960793	963951	967114	970282	973455	976633	979816	982994	986177	989365	992558	995756	998959	100215	100531	100847	101164	101481	101798	102115	102432	102749	103066	103383	103700	104017	104334	104651	104968	105285	105602	105919	106236	106553	106870	107187	107504	107821	108138	108455	108772	109089	109406	109723	110040	110357	110674	110991	111308	111625	111942	112259	112576	112893	113210	113527	113844	114161	114478	114795	115112	115429	115746	116063	116380	116697	117014	117331	117648	117965	118282	118599	118916	119233	119550	119867	120184	120501	120818	121135	121452	121769	122086	122403	122720	123037	123354	123671	123988	124305	124622	124939	125256	125573	125890	126207	126524	126841	127158	127475	127792	128109	128426	128743	129060	129377	129694	130011	130328	130645	130962	131279	131596	131913	132230	132547	132864	133181	133498	133815	134132	134449	134766	135083	135399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Table XVII—continued.

(9263)

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07188	07701	08241	08803	09385	09994	10629	11284	11965	12670	13399	14151	14928	15731	16558	17410	18288	19189	20116	21069	22050	23059	24095	25157	26247	27369	28520	29691	30911	32154	33433	34741	36087	37467	38884	40342	41835	43367	44937	46553	48214	49921
07135	07644	08181	08739	09317	09924	10555	11206	11884	12596	13311	14060	14833	15632	16456	17305	18178	19075	20000	20949	21926	22932	23963	25022	26107	27224	28372	29539	30754	31991	33267	34570	35912	37286	38698	40151	41638	43165	44730	46341	47996	49697
07082	07587	08121	08676	09250	09854	10481	11129	11804	12502	13224	13969	14739	15534	16354	17200	18069	18962	19884	20830	21803	22805	23832	24888	25968	27080	28224	29398	30598	31830	33102	34400	35737	37106	38513	39961	41443	42964	44524	46119	47759	49474
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06925	07420	07943	08488	09053	09647	10263	10902	11566	12255	12966	13701	14460	15244	16054	16889	17746	18629	19541	20476	21436	22428	23443	24480	25555	26654	27786	28941	30134	31353	32611	33907	35218	36572	37965	39397	40864	42369	43914	45503	47134	48811
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06430	06894	07382	07893	08431	08980	09571	10180	10810	11467	12143	12844	13572	14322	15096	15894	16719	17568	18441	19339	20262	21215	22196	23210	24253	25328	26434	27570	28738	29936	31163	32428	33733	35078	36463	37888	39353	40858	42403	43988	45613	
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06156	06599	07070	07565	08080	08619	09180	09773	10384	11021	11679	12360	13066	13796	14551	15330	16132	16961	17815	18694	19598	20528	21485	22468	23474	24511	25577	26671	27794	28948	30134	31351	32601	33885	35199	36548	37936	39360	40826			
06112	06553	07020	07512	08023	08550	09123	09707	10315	10949	11603	12281	12984	13711	14462	15238	16037	16862	17713	18589	19490	20416	21369	22349	23351	24384	25446	26536	27655	28805	29987	31200	32445	33725	35034	36379	37762	39181	40642			
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05938	06368	06823	07303	07801	08325	08877	09448	10043	10663	11305	11971	12660	13375	14114	14878	15668	16474	17311	18173	19060	19973	20911	21876	22864	23883	24929	26003	27107	28241	29406	30602	31831	33092	34384	35712	37076	38475				
05895	06323	06775	07252	07747	08268	08816	09384	09973	10593	11232	11895	12580	13292	14028	14787	15571	16378	17212	18070	18954	19863	20798	21759	22744	23759	24794	25840	26907	27994	29102	3										

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